FORMULATION OF CRITERION FUNCTIONAL AND SET OF CONSTRAINTS IN PROBLEMS OF PHYSICAL SETTINGS DESIGNING

I.M. Prokhorets, S.I. Prokhorets, M.A. Khazhmuradov

National Science Center "Kharkov Institute of Physics and Technology" KIPT, Kharkov, Ukraine
e-mail: khazhm@kipt.kharkov.ua, iprokhorets@kipt.kharkov.ua

Characteristics of semiconductor spectrometer and neutronography setting were investigated using system analysis methods.

PACS: 61.20ja

1. INTRODUCTION

It is impossible to develop at NSC KIPT researches in different branches of physics, technics and medicine without creation new experimental settings at existing and planned accelerators. At development new settings it is necessary to choose from the set of feasible solutions the best or optimal one. Problem of obtaining of optimal solution exists at all development stages. Mathematically, search for optimal solution of such tasks reduces to finding parameters that give maximal or minimal value of criterion functional [1].

There are different types of physical settings and hence, there are different methods of their optimization. But in spite of this there are stages common to developing of all physical settings. They are:

- ➢ problem statement;
- ➢ creation of physical model of design object;
- ➢ optimization problem definition;
- ➢ creation of mathematical model that depicts interrelationships between object main features;
- ➢ problem solving on the basis of the used mathematical model;
- ➢ obtained results analyses, correction of conceptual and mathematical models.

Data about object purposes and its operating regimes are source information for optimal parameters finding. They determine main design aim and requirements to designed object. Influence of many factors on the design object can be found using mathematical optimization methods, which are subject of operation research or, widely, of systems analysis [2].

System analysis methods are used in different branches: military science, economics, agriculture, medicine, etc. In spite of qualitative difference tasks in all these branches of human activity reduce to choosing of modus operandi and design parameters, that is, to decision-making. It is concerned also such complex objects as technological and physical settings, where radiation technologies are used. Till recently system analyses of their characteristics wasn't made, and used for this purposes figure of merit characterize in most cases only one feature. So it is actual to develop methods of physical settings and their systems characteristics defining at the design stage using computer experiment methods.

The aim of this article is to investigate characteristics of specific physical settings and their systems using system analysis methods.

2. PHYSICAL MODEL OF EXPERIMENTAL SETTING

Generally nearly each physical setting can be represented as set of subsystems with interrelationships (Fig. 1), caused by system functional features [3,4]. Semiconductor spectrometer is one of the simplest systems. It consists from detector, prime- and shaping amplifier, bias voltage supply unit, converter of signal amplitude or charge to digital value. More complex system is modern multichannel semiconductor detector- pixel, drift or strip detector. Last one consists from many expanded p-n junctions, each of them is separate detector element with prime- and shaping amplifiers. Signal from each amplifier is put to memory, read out, converted to digital value and put to intermediate or PC memory. Distinctive feature of such complex detector system allocated at single semiconductor plane is relations between them.

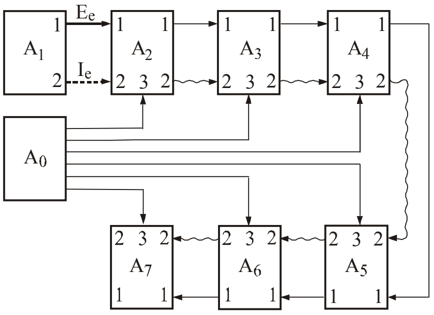

***Fig. 1**. Simplified structure of physical setting. A_1 … A_7 – physical setting subsystems. A_0 – environment*

Experimental setting can consist from several devices based on different physical principles. Then it can be represented as set of subsystems, each of them consists from unit etc. On Fig. 1 environment influence (temperature, moisture, irradiation) also is represented as several unit A_0. If each system doesn't depend on the next one, then optimization problem of its characteristics reduces to autonomous modeling of subsystems. Modeling can begin from any subsystem (unit), which receive information from the single source. Representation of complex system as set of subsystems allows to use both mathematical models and information in form of tables, diagrams etc. for characteristics of its subsystems.

3. MATHEMATICAL DEFINITION OF OPTIMIZATION PROBLEM

Each physical device has set of features that determine its purpose and can be changed or calculated. Physical device features are called characteristics. Characteristics can be independent each from another, but they depend on directed parameters and external factors which forms environment where device works. Directed parameters and external factors are independent variables and characteristics depend on them.

Parametric optimization deals with calculation of parameters $\mathbf{X} = (x_1, x_2, ..., x_m)$. It results in values x_i, at which criterion functional $Q(\mathbf{X})$ is maximum or minimum. As criterion functional we can use, for example, spatial, time, amplitude resolution or efficiency of radiation registration. Let's criterion functional has to be minimal

$$\min_{X \in X_g} Q(\mathbf{X}), \tag{1}$$

where X_g – region of allowed parameters. Such problem must be solved when following inequation system is satisfied

$$x_j^- \le x_j \le x_j^+,\ j = 1,2,\dots,n, \tag{2}$$

$$\varphi_i^- \le \varphi_i(\mathbf{X}) \le \varphi_i^+,\ i = 1,2,\dots,m, \tag{3}$$

where x_j^-, x_j^+ – values of j-th directed value characterizing its allowed values range, and φ_i^-, φ_i^+ – limit value of characteristics.

Optimization problem (1-3) is solved using linear (non-linear)-programming technique if optimality criterion – criterion functional and constraints are linear (non-linear) functions of parameters. If there are no non-linear constraints (3), then solving of optimization problem (1) reduces to minimum search of the criterion functional (1) with constraints (2) that simplifies the problem. Depending on number of variables optimization problems can be one-dimensional ($n = 1$) or multidimensional (multiparametric) ($n \ge 2$).

When during design there is a need to obtain the best values for several object characteristics it is necessary to find such values of directed parameters, which give minimum of criterion functional that satisfies all criterions simultaneously. It is necessary to find compromise solving. Mathematically let's introduce vector criterion of optimality [5]

$$\mathbf{Q}(\mathbf{X}) = \mathbf{Q}(Q_1(\mathbf{X}), Q_2(\mathbf{X}), ..., Q_s(\mathbf{X})). \tag{4}$$

Compromise solving of such multicriterion problem is point $\mathbf{X}^* \in X_g$, which satisfies inequation $\mathbf{Q}(\mathbf{X}^*) \le \mathbf{Q}(\mathbf{X})$.

Practically point $\mathbf{X}^*$ search reduces to search for set of partial optimization criterions, satisfying Pareto criterion [2,5]. This criterion says that none of the partial criterions can be diminished without increasing of the others.

Let's consider methods of determining of optimal parameters of spectrometric channel with planar semiconductor detector and beam formation system for neutronography setting. Direct search method can be used to solve such optimization problem. Used for these purposes algorithms allow to solve such tasks in a following sequence: next variant generating, variant rating of merit and decision-making.

4. OPTIMIZATION OF PLANAR SEMICONDUCTOR DETECTOR CHARACTERISTICS

Planar semiconductor detectors are widely used for registration and spectrometry of radiation of different types. Noise level is one of the characteristics, determining quality of channel for information read-out from such device. If we consider that planar detector from wide-gap semiconductor (CdTe, CdZnTe, GaAs) behaves as ionization chamber, then dark current [6]

$$I_d = \frac{VA}{d\rho}, \tag{5}$$

where V – potential difference applied to the parallel contacts of the detector, d, A – detector thickness and area and ρ – resistivity. Dark current defines parallel noise in detector – charge sensitive preamplifier system

$$ENC_p = \sqrt{\frac{I_d \tau}{q}}, \tag{6}$$

where τ – integration time of the amplifier. Second noise component – series noise

$$ENC_s = \frac{1}{q}\left(C_g + C_d + C_{st}\right)\sqrt{\frac{4kT}{g_m \tau}}, \tag{7}$$

where C_d – planar detector capacitance, C_g – capacitance of the gate of the input FET, C_{st} – stray capacitance associated with the connection of the amplifier, g_m – transconductance of the readout FET, k – Boltzmann constant, T – temperature. If two above-mentioned noise components are considered to be statistically independent then resulting electron noise

$$ENC^2 = ENC_p^2 + ENC_s^2. \tag{8}$$

So, if we consider electronic noise value as spectrometer characteristics, then mathematically minimizing of the noise can be considered as criterion functional of such system. Criterion functional depends in our example mainly on detector thickness and area, electric field intensity in the detector volume, capacitance of the gate of the input FET and FET parameters, integration time of the amplifier and external factors – temperature, irradiation, humidity. Integration time τ depends, in turn, on drift time to outer contacts of the charges – electrons and holes – born in the detector, i.e. on electron and hole mobility, detector thickness and electric field intensity in the detector volume. Hence, criterion functional depends on the following directed parameters:

– semiconductor detector thickness, mm – ≤ 10;

– semiconductor crystal area, cm^2 – ≤ 100;

– electric field intensity, kV/cm – ≤ 2;

– capacitance of the gate of the input FET, pF – ≤ 2,5;

– transconductance of the readout FET, mS – ≥ 4;

– stray capacitance associated with the connection of the amplifier, pF – ≤ 10;

– integration time of the amplifier, µs – ≤ 20.

On Fig. 2 it is shown results of spectrometer modeling with CdZnTe detectors with thickness 10 mm and volume 1 and 10 cm^3. Calculations was made for $V = 1$ kV, $C_g = 2{,}5$ pF, $C_s = 10$ pF and $g_m = 6$ mS. Noise in electrons (rms) was converted to keV (*FWHM*) using formula

$$FWHM = 2{,}35 \cdot \xi \cdot ENC\,, \qquad (9)$$

where $\xi = 5$ eV – mean energy required to create an electron-hole pair in CdZnTe.

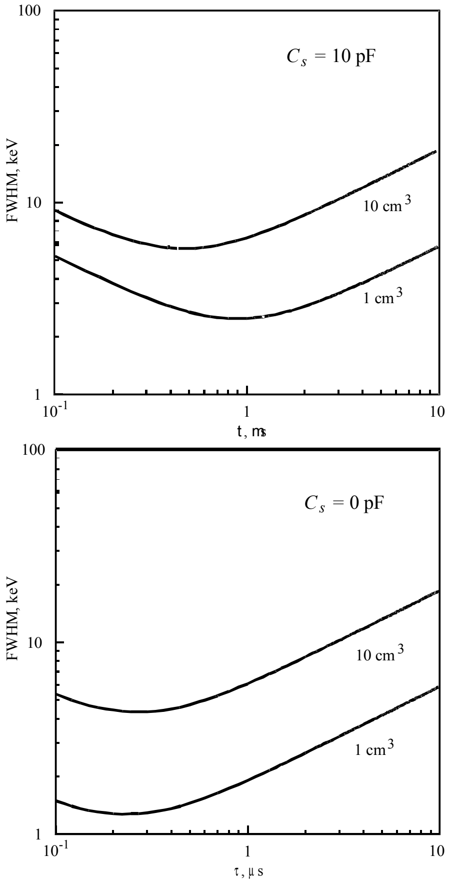

Fig. 2. *Noise (FWHM) versus integration time τ for spectrometric channel with CdZnTe detectors with volumes 1 and 10 cm^3*

From Fig. 2 it is clear that electronic noise allows to obtain resolution near 7 keV (*FWHM*) with detector 1 cm and 1 cm^2, that gives ~1 % at the source ^{137}Cs (662 keV). Modern technologies of detector production don't allow obtain such resolution, as there are additional noise sources, which increase *FWHM*. Minimum points of the curve at Fig. 2 are solving of optimization problem

$$\underset{X \in X_g}{FWHM \to \min, \tau \to \min} \qquad (10)$$

Direct search method was used for obtaining this minimum. In the minimum point we obtain compromise between amplitude resolution and system operating speed.

5. DETERMINING OF CHARACTERISTICS OF NEUTRONOGRAPHY SETTING

Neutron flux density in full energy range and in separate energy intervals (thermal, fast and so on) are the main characteristics of setting for neutron radiography (NR). If NR setting is planned at the electron accelerator base theses characteristics depend on such directed parameters as accelerated electrons energy and current at neutron-producing target, target thickness and material, collimator-moderator material and geometrical sizes (Fig. 3).

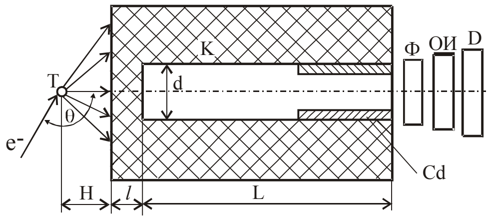

Fig. 3*. Neutron beam shaping system (in a simplified way)*

As criterion functional for neutronography setting we can consider maximum number of neutrons with specified energy spectrum when ratio of the hollow collimator length to its outer diameter is given. For this aim we calculated neutron flux and energy spectrum after collimator which we considered as cylinder with outer diameter 40 cm and inner channel in the form of hollow cylinder with diameter $d = 10$ cm. In our calculations we used method of statistical testing [7]. During our calculations we determined how moderator front wall thickness, inner hollow cylinder length, distance from object of researches to collimator out, neutron-producing target, cadmium inset influence on the beam characteristics. We considered two targets: lead ball with diameter 60 mm, placed at collimator axes and lead plate with thickness 6 cm and diameter 10 cm, placed at angle 45° to electron beam and collimator axis. Both ball and plate were isotropic neutron sources.

On Fig. 4 it is shown influence of hollow cylinder length on the thermal neutron flux at the output plane of the collimator-shaper. Modeling results show that neutron–producing target in form of plane disk scanned with electron beam gives in considered geometry larger neutron flux, compared with ball and point beam. If we consider that at energy 23 MeV neutron yield at 4π angle is $4 \cdot 10^{10}$ neutron/µA·s, such target allows to obtain 240 neutron/s of thermal neutrons with energies 0,025… 0,1 eV for $d/L = 0{,}025$. Influence of cadmium insert (cylinder with wall thickness 1 mm and length 50 cm inside hollow cylinder of collimator) is shown on Fig. 5. It is clear that influence of cadmium on the neutron flux is insignificant when $d/L = 0{,}025$.

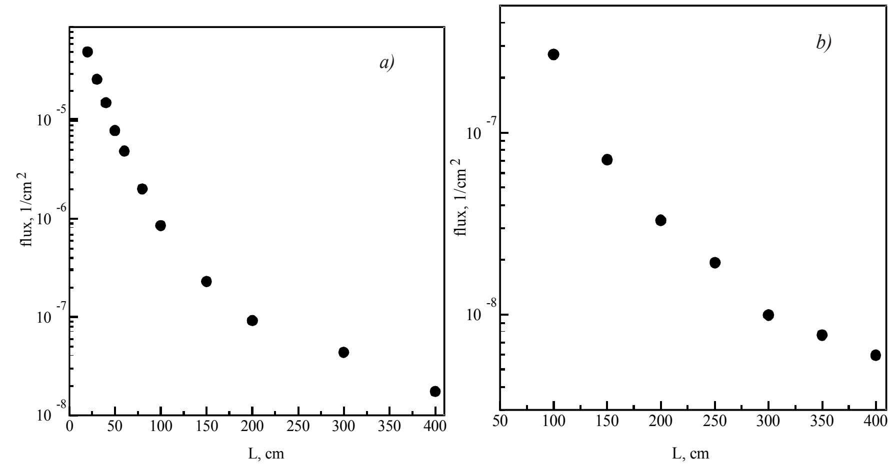

Fig. 4. *Neutron flux from neutron-producing target in form of ball (a) and plane disk with diameter 10 cm (b) versus length of collimator-shaper hollow cylinder*

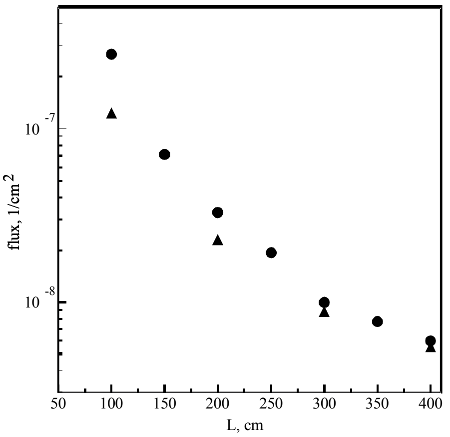

Fig. 5. *Influence of cadmium insert on the neutron flux at the output of the collimator-shaper hollow cylinder. • – without cadmium insert; ▲ – with cadmium*

6. CONCLUSIONS

In this work it is shown that problems of the physical settings development can be formulated as optimization problems of complex systems. It was formulated criterion functional and defined main parameters in the set of constraints. It was solved several practical tasks, appearing while developing semiconductor spectrometers and settings for neutron researches.

REFERENCES

1. I.V. Beyko, B.N. Bublik, P.N. Sinko. *Methods and algorithms of optimization problem solving*. Kiev: “Vyscha shkola”, 1983, 512 p. (in Russian).
2. N.N. Moiseev. *Mathematical tasks of system analysis*. M.: “Nauka”, 1981, 488 p. (in Russian).
3. N.P. Buslenko. *Complex system modeling*. M.: “Nauka”, 1968, 356 p. (in Russian).
4. L.N. Dychnenko, N.P. Kuzmin, E.G. Petrov. *Basis of complex systems modeling*. Kiev: “Vyscha shkola”, 1981, 360 p. (in Russian).
5. D.I. Batischev. *Search methods of optimal design.* M.: “Sovetskoe radio”, 1975, 215 p. (in Russian).
6. J.C. Lund, J.M. Van Scyoc III, R.B. James et al. Large volume room temperature gamma-ray spectrometers from $Cd_xZn_{1-x}Te$ // *Nucl. Instr. and Methods.* 1996, A380, p. 256-261.
7. I.M. Prokhorets, S.I. Prokhorets, M.A. Khazhmuradov. Algorithms of modeling of neutron passage through matter // *Radioelectronics and informatics.* 2003, №4, p. 128-132 (in Russian).

ФОРМУЛИРОВКА ФУНКЦИИ ЦЕЛИ И СИСТЕМ ОГРАНИЧЕНИЙ В ЗАДАЧАХ ПРОЕКТИРОВАНИЯ ФИЗИЧЕСКИХ УСТАНОВОК

И.М. Прохорец, С.И. Прохорец, М.А. Хажмурадов

Рассмотрено влияние параметров полупроводникового спектрометра на основе CdZnTe на энергетическое разрешение и влияние параметров системы формирования пучка на поток нейтронов для нейтронографической установки.

ФОРМУЛЮВАННЯ ФУНКЦІЇ ЦІЛІ ТА СИСТЕМ ОБМЕЖЕНЬ В ЗАДАЧАХ ПРОЕКТУВАННЯ ФІЗИЧНИХ УСТАНОВОК

І.М. Прохорець, С.І. Прохорець, М.А. Хажмурадов

Розглянуто вплив параметрів напівпровідникового спектрометру з CdZnTe на енергетичну роздільність та вплив параметрів системи формування пучку на потік нейтронів для нейтронографічного пристрою.