STABILIZATION OF THERMAL BREAKDOWN DEVELOPMENT IN SEMICONDUCTOR FILMS

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The analysis of fixed points of the evolution equation for temperature in thermal fluctuation area in semiconductor film is done. It is shown that there exists a stable fixed point being more than the threshold of the breakdown regime development.

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1. INTRODUCTION

In this communication, it is analysed the stabilisation condition for the evolution regime which can appear when the direct electrical current getting through the thin semiconductor film. Such a regime generates the so-called thermal breakdown. This phenomena consists of the localisation of the extracted Joel heat in those areas on the film where sufficiently large thermal fluctuation are concentrated, it gives the strong heating of these areas. Due to increasing dependence of the electrical conductivity of the material on the temperature. Such positive feedback may give the local increasing of the temperature up to the melting temperature of the material or to its eutectic point in the case of an intrinsic semiconductor. This physical process is called {\it the thermal breakdown}, it is developed during several microseconds and gives the functional destruction of the material.

2. THE EVOLUTION EQUATION AND THE WAGNER APPROXIMATION

It has been obtained in Ref. [1] the evolution integral and differential equation of the temperature field on the film, which describes the breakdown development and also one-dimensional solutions of this equation have been analysed. It has been used such an approximation when the effect of the voltage transfer during the breakdown regime is neglected. It may; really, neglect this effect at the origin stages of the breakdown regime and if the circuit resistance being external to the film is not sufficiently large. In this work, we shall analyse the evolution equation taking into account the influence of the voltage transfer. We shall show that if the external resistance is sufficiently large then it is possible the stabilisation of the evolution regime. As a result, the melting temperature (the eutectic one) is not attained.

We start from the thermal conductivity equation in the form

$$\dot{T} = \nabla(\kappa(T)\nabla T) + Q(T), \tag{1}$$

where

$$Q(T) = \frac{E^2\sigma(T)}{(1 + (\bar{\sigma} S d)^{-1}\int \sigma(\vec{r}\,',t)d\vec{r}\,')^2}.$$

It describes the temperature distribution $T(\vec{r}\,',t)$ on the film when there are non-linear dependencies of the temperature conductivity coefficient κ(T) and the electrical conductivity $\sigma(T)$. The integral denominator in the second summand takes into account the effect of the change of the voltage applying to the film during the evolution process. In Eq. (1) E is the voltage on the film in the equilibrium state, $\bar{\sigma}$ is the average electrical conductivity characterising the external resistance, S is the film square, d and is its thickness. Further, we shall consider in Ref. [2] that κ(T)=κ is constant and

$$\sigma(T) = \sigma_0(1 + \nu(T - T_m)^2/2),$$

T_m is the temperature of the electrical conductivity minimum.

We realise the investigation of the solution stability of Eq. (1) using the approximation, which we name the Wagner one. It was used in the thermal breakdown theory in dielectrics. We consider that the heat localisation may be modelled by introducing of the thermal channels having a critical diamete D. These channels are passing through the film. The temperature in these channels is approximately constant when the spatial point changing in them. It is changed essentially in each of them only in thin boundary layer having the thickness l. We put that the temperature is equal T(t)

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in channels outside these boundary layers. Outside channels, it is put to constant temperature T_0 of the thermal surrounding.

The breakdown originates at $T_0 = T_m$. At these conditions, the equation for relative temperature fluctuation

$$\sqrt{2/\nu}\,\Theta(\vec{r},t) = T(\vec{r},t) - T_0$$

is obtained on the basis of Eq. (1), if there is one cylindrical channel with diameter D on the film. This value is not equal to zero only in the channel. The equation has the form

$$\dot{\Theta} = \kappa\Delta\Theta + \sqrt{\nu/2}\,\frac{\mathrm{E}^2\sigma_0(1+\Theta^2)}{(1+\eta(1+\Theta^2(t)))^2}. \qquad (2)$$

Here, we introduce non-dimensional parameter

$$\eta = (\sigma_0/\bar{\sigma})(\pi D^2/4S)$$

characterising the speed of the voltage transfer and we denote $\Theta(t) = T(t) - T_0$. Averaging Eq. (2) over the channel domain having the volume V and using the transformation of integral on the volume to the integral on the surface (it is considered that the heat flow aside of the channel is absent), we obtain

$$V^{-1}\int \Delta\Theta\, dv = V^{-1}\oint(\nabla\Theta\, d\vec{s}) =$$
$$= -\frac{\pi dD\Theta}{lV} = -\frac{4\Theta}{lD}.$$

Here, we take into account that the heat flow through cylinder side surface having the square πDd is directed along the radial temperature gradient. This gradient we change approximately by the finite difference $\Theta(t)/l$. At last, we simplify the averaging equation changing the temporal scale $t\sqrt{\nu/2}\mathrm{E}^2 \Rightarrow t$ and introducing the decrement $\alpha = \sqrt{2/\nu}\,(4\kappa_0/lD\sigma_0\mathrm{E}^2)$,

$$\dot{\Theta}(t) = -\alpha\,\Theta(t) + \frac{1+\Theta^2(t)}{(1+\eta(1+\Theta^2(t)))^2}. \qquad (3)$$

Our further analysis is reduced to the investigation of the solution stability of this equation.

3. ANALYSIS OF THE SOLUTION STABILITY

If Θ in Eq. (3) is small then one can consider that

$$\dot{\Theta}(t) = -\alpha\,\Theta(t) + 1$$

and, therefore, solutions are stable. If $\Theta(t)$ is sufficiently large but the value η is very small such as we may neglect by the value $\eta(1+\Theta^2(t))$ in denominator then we obtain the equation $\dot{\Theta}(t) = -\alpha\Theta(t) + \Theta^2(t)$ possessing the peaking regime which describes the thermal breakdown development, i.e. its solution

$$\Theta(t) = \alpha\,(1-(1-\frac{\alpha}{\Theta(0)}.)e^{\alpha t})^{-1}$$

$\Theta(0) > \alpha$ goes to infinity during the finite time t_∞, which is identified with the breakdown time. Let us study now the possibility of stabilisation in the case when $\eta(1+\Theta^2(t))$ is sufficiently large in comparison with the unit. In this case we neglect the unit at the denominator of second summand in Eq. (3) and after that we take away the extra parameter in the obtained equation introducing the new temperature $\eta^{1/2}\Theta \Rightarrow \Theta$, the decrement $\eta^{1/2}\alpha \Rightarrow \alpha$ and the time $\eta^{1/2}t \Rightarrow t$. As a result we obtain the equation with one parameter

$$\dot{\Theta}(t) = -\alpha\,\Theta(t) + \frac{\Theta^2(t)}{(1+\Theta^2(t))^2} \equiv f(\Theta(t)). \qquad (4)$$

Let us find some fixed points of this equation, i.e. solutions $f(\theta) = 0$. Such solutions consist with $\theta = 0$ and solution of the equation $\theta = \alpha(1+\theta^2)^2$. Let us introduce the notation $X = 1+\theta^2$. Then this equation has the form

$$P(X) = \alpha X^4 - X + 1 = 0.$$

There is the unique minimum of the polinom $P(X)$ in the point $X_* = (4\alpha^2)^{-1/3}$. It satisfies the equation $P'(X) = 0$. There are not other fixed points different from $\theta = 0$ at $P(X_*) \geq 0$ (the breakdown regime is not realised). If

$$P(X_*) = 1 - 3(4\alpha^2)^{-1/3}/4 \leq 0,$$

i.e. $\alpha^2 \leq (3/4)^3/4$, then there are two solutions $X_\pm, X_+ > X_-$ of the equation $P(X) = 0$. They are both positive since $P(0) = 1$. At this case $X_+ > 1$ is fulfilled for sure, since $X_* > 1$. Consequently, $X_- > 1$, since $P(1) = \alpha^2 > 0$. Therefore, two fixed points $\theta_\pm$ correspond to solutions $X_\pm$. It is easy checked that these fixed points tend as $\theta_- \sim \alpha, \theta_+ \sim \alpha^{-1/3}$ asymptotically at small α.

Let us analyse the stability of those found fixed points. For this, it is necessary to set the sign of the derivative $df(\theta)/d\theta$ in each of these points. It is obvious in the point $\theta = 0$ that

$$(df(\theta)/d\theta)_0 = -\alpha\ .$$

This is in accordance with the above conclusion about its stability. In points $\theta_\pm$ we obtain

$$f'(\theta_\pm) = \alpha(1 - 3\theta_\pm^2)(1 + \theta_\pm^2)$$

on the basis $\theta_\pm = 0$. Since the following conditions

$$X_+ > X_* = (4\alpha^2)^{-1/3} > 4/3,$$

$$3\theta_+^2 = 3(X_+ - 1) > 3(X_* - 1) = 1$$

take place. Consequently, the point θ_+ is stable and, the point θ_- should be unstable on the basis of some topological arguments. Just this point corresponds to the threshold of the fluctuation value from which the breakdown is developed.

4. CONCLUSIONS

Thus, the stable fixed point θ_+ places above the threshold point Θ_-. The "breakdown" solutions $\Theta(t)$ are attained to this point and if its value corresponds to the temperature T_+ being less the melting one (or the eutectic one) then the breakdown is not realised. In this case only some areas having very large temperature (the mesoplasma channels) may be occur.

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СТАБИЛИЗАЦИЯ РАЗВИТИЯ ТЕПЛОВОГО ПРОБОЯ В ПОЛУПРОВОДНИКОВЫХ ПЛЁНКАХ

Н.В. Андреева, Ю.П. Вирченко

Проведен анализ неподвижных точек эволюционного уравнения для температуры в области тепловой флуктуации на полупроводниковой плёнке. Показано, что существует устойчивая неподвижная точка, большая по величине порога возникновения режима пробоя.

СТАБІЛІЗАЦІЯ РОЗВИТКУ ТЕПЛОВОГО ПРОБОЮ У НАПІВПРОВІДНИКОВИХ ПЛІВКАХ

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Проведено аналіз нерухомих точок еволюційного рівняння для температури в області теплової флуктуації у напівпровідниковій плівці. Доведено, що існує нерухома стійка точка, яка є більшою за величиною порогу виникнення режиму пробою.