

OPERATING CHARACTERISTICS OF THE ELECTROSTATIC MONITORS OF ELLIPTIC, RECTANGULAR AND CIRCULAR CROSS-SECTION

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The various methods of approximation of operating characteristics of electrostatic beam position monitors were investigated. The new method that allows on order to increase the precision of approximation of horizontal coordinate is found. The simulation of measurement of frequencies of betatron oscillations is carried out.

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1. INTRODUCTION

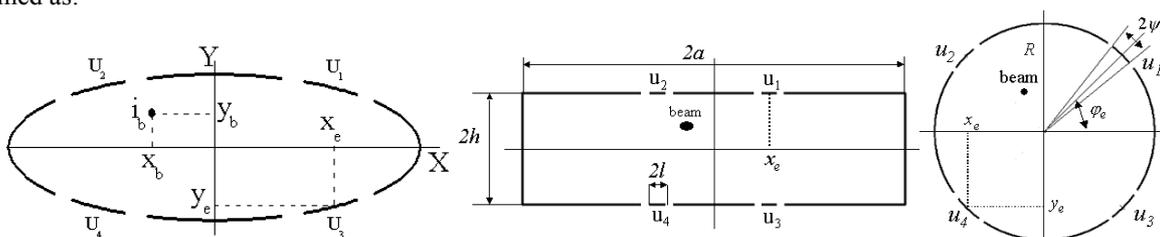
For measurement of the parameters of betatron tune and coordinates of closed orbit (CO) of electron beam in storage rings (SR) usually apply four-electrode electrostatic monitors (EM). On their basis create betatron tune monitors (BTM) and beam position monitors (BPM) with using corresponding combinations of four measured signals. The measurement accuracy of BPM should be much higher, than precision of positioning of magnetic elements of SR ($\ll 100 \mu\text{m}$).

In the articles [1,2] were shown that nonlinearity of operating characteristics (OC) of the EM of elliptic cross-section leads to errors of measured parameters of a beam. One of the sources of measurement errors of beam coordinates is the approximation error of monitors operating characteristics. The traditional methods of the approximation allow to execute the measurement with high accuracy only in very small region near the center of the monitor. In the article [3] the new method of approximation of OC of BPM that allows on the order to increase measurement accuracy in comparison with a traditional method was offered.

Because in practice are widely applied BPM of rectangular and circular cross-section, in the given paper of possibility of approximating of their operating characteristics by methods described in [2] for BPM of an elliptic cross-section were researched. The measurement simulation of spectrums of betatron oscillations and transverse beam coordinates by the EM of storage ring N-100M (NESTOR) is carried out also.

2. SIMULATION OF BPM CROSS-SECTION

Coordinates of a closed orbit of a beam versus the signals of the BPM (Figure) in the Cartesian frame are defined as:



The electrostatic monitor layouts of elliptic, rectangular and circular cross-section, u_1, u_2, u_3, u_4 is signals of monitor electrodes

$$x_{bk} = f_1(u_{1k}, u_{2k}, u_{3k}, u_{4k}) = \Phi_1(H_{tk,ck}, V_{tk,ck}), \quad (1)$$

$$y_{bk} = f_2(u_{1k}, u_{2k}, u_{3k}, u_{4k}) = \Phi_2(H_{tk,ck}, V_{tk,ck}), \quad (2)$$

here: k – is ordinal number of measurement point, $H_{tk,ck}, V_{tk,ck}$ – are the normalized differential linear combinations of the measured signals:

$$H_{tk} = \frac{u_{1k} + u_{3k} - u_{2k} - u_{4k}}{u_{1k} + u_{2k} + u_{3k} + u_{4k}}, \quad (\text{traditional}) \quad (3)$$

$$V_{tk} = \frac{u_{1k} + u_{2k} - u_{3k} - u_{4k}}{u_{1k} + u_{2k} + u_{3k} + u_{4k}}$$

$$H_{ck} = \frac{1}{2} \left[\frac{u_{1k} - u_{4k}}{u_{1k} + u_{4k}} + \frac{u_{3k} - u_{2k}}{u_{3k} + u_{2k}} \right], \quad (\text{combined}) \quad (4)$$

$$V_{ck} = \frac{1}{2} \left[\frac{u_{1k} - u_{3k}}{u_{1k} + u_{3k}} + \frac{u_{2k} - u_{4k}}{u_{2k} + u_{4k}} \right]$$

The expressions (1), (2) are operating characteristics of BPMs. They are measured on precision test bench and traditionally approximated by degree polynomials:

$$x_b = \sum_{m=0}^K \sum_{n=0}^m A_{2m-2n+1,2n} H_{t,c}^{2m-2n+1} V_{t,c}^{2n}, \quad (5)$$

$$y_b = \sum_{m=0}^K \sum_{n=0}^m B_{2m-2n+1,2n} V_{t,c}^{2m-2n+1} H_{t,c}^{2n}. \quad (6)$$

Approximation of OC (1) by new empirical function

$$x_b = C \cdot \text{acth}[H_{t,c}] + D \cdot \text{sh} \left[\sum_{m=0}^K \sum_{n=0}^m E_{2m-2n+1,2n} H_{t,c}^{2m-2n+1} V_{t,c}^{2n} \right], \quad (7)$$

was considered in work [3].

Table 1. Average errors of an approximation calculated by various methods for the operation characteristics of the elliptic BPM (N-100M). The size of BPM: the big axis is 79 mm, a small axis is 28.7 mm, the coordinates of the electrode centers is $X_e = \pm 10$ mm, $Y_e = \pm 13.89$ mm, the size of the electrodes is 5 mm

Region of BPM, mm	Error→ ↓ Formula→	σ_x (5) – polynomial	σ_x (7) – empirical	σ_y (6) – polynomial
-8<x<8	(3) - traditional	0.002662	0.001351	0.001756
-4<y<4	(4) - combined	0.001995	0.000102	0.000789

Table 2. Average errors of an approximation calculated by various methods for the operation characteristics of the rectangular BPM. The size of BPM is 100 mm×30 mm, the coordinates of the electrode centers is $X_e = 14.4$ mm, $Y_e = 15$ mm, the size of the electrodes is 5 mm

Region of BPM, mm	Error→ ↓ Formula→	σ_x (5) – polynomial	σ_x (7) – empirical	σ_y (6) – polynomial
-10<x<10	(3) - traditional	0.005730	0.001255	0.001711
-5<y<5	(4) - combined	0.004874	0.000037	0.000835

Table 3. Average errors of an approximation calculated by various methods for the operation characteristics of the circular BPM. Diameter of BPM is 41 mm, the coordinates of the electrode centers is $X_e = 14.49$ mm, $Y_e = 14.49$ mm, the size of the electrodes is 5 mm

Region of BPM, mm	Error→ ↓ Formula→	σ_x (5) – polynomial	σ_x (7) – empirical	σ_y (6) – polynomial
-9.5<x<9.5	(3) - traditional	0.003305	0.002940	0.003738
-5.5<y<5.5	(4) - combined	0.001877	0.000558	0.000501

Table 4. The amplitudes of harmonics of signal H_β [μV]

Harmonics	ω_0	$(k \pm q_x) \omega_0$	$(k \pm q_y) \omega_0$	$(k \pm 2q_x) \omega_0$	$(k \pm 2q_y) \omega_0$	$(k \pm q_x \pm q_y) \omega_0$
Coordin. of CO						
$x_b = 9$ mm, $y_b = 7$ mm	1637	33.6	91.1	5	4.5	1.3
$x_b = 4$ mm, $y_b = 3$ mm	627	71.8	19.7	1.3	1.7	2.2
$x_b = 0$ mm, $y_b = 0$ mm	0	74	0	0	0	0

Table 5. The amplitudes of harmonics of signal V_β [μV]

Harmonics	ω_0	$(k \pm q_y) \omega_0$	$(k \pm q_x) \omega_0$	$(k \pm 2q_x) \omega_0$	$(k \pm 2q_y) \omega_0$	$(k \pm q_x \pm q_y) \omega_0$
Coordin. of CO						
$x_b = 14$ mm, $y_b = 7$ mm	983	112	132	12.9	4.7	6
$x_b = 9$ mm, $y_b = 7$ mm	1364	111	26.4	3.6	1.7	4
$x_b = 4$ mm, $y_b = 3$ mm	415	68.4	14.6	0.6	0.2	2.6
$x_b = 0$ mm, $y_b = 0$ mm	0	59.4	0	0	0	0

The simulation of bench measuring of operating characteristics (1), (2) of rectangular and round cross-section BPM were carried out and their approximations by expressions (5), (6), (7) for two variants of arguments (3), (4) are obtained, by methods described in [2]. The geometrical parameters of the said monitors were chosen similar to monitor of an elliptic cross-section considered in [2]. BPM designed for N100-M is investigated also. In all cases the approximation of OC was carried out by polynomials of 5-th order for which $K=2$. The results of calculations are brought together in Tables 1-3 as relative root-mean-square errors of approximation averaged on fixed region of BPMs - σ_x and σ_y .

One can see, for all forms of a cross-section of the monitors the error of approximation σ_x of coordinate x_b by empirical expression (7) with the combined arguments H_c , V_c about ten times is less than at approximation by the polynomial (5) with traditional arguments H_i , V_i (3).

Use of combined arguments H_c , V_c at approximation of OC (2) by the polynomial (6) too reduces in some

times error of approximation σ_y in comparison with the use traditional arguments H_i , V_i .

Thus, the approximation of operation characteristics (1), (2) by expressions (6), (7) with use of arguments (4) gives considerably smaller general error of measuring of coordinates of the beam for any shape of cross section of monitors in comparison with traditionally used approximations. Use of empirical expression (7) and the combined arguments H_c , V_c will allow to fulfill precision measurements of the beam coordinates of SR N-100M with accuracy up to ± 10 μm ($\leq \pm 10^{-3}$).

3. SIMULATION OF OC OF THE STORAGE RING N-100M BTM

Operating characteristics of BTM are frequency spectrum of horizontal and vertical signals of beam deviation. In practice these signals are extracted with the help of the corresponding differential combinations of four EM signals.

Signals of horizontal H_β and vertical V_β beam deviation are:

$$H_\beta = (u_{1,n} + u_{3,n}) - (u_{2,n} + u_{4,n}) = \sum_{n=0}^{M-1} i_{b,n} Z_{H,n}(G, X_{b,n}, Y_{b,n}),$$

$$V_\beta = (u_{1,n} + u_{2,n}) - (u_{3,n} + u_{4,n}) = \sum_{n=0}^{M-1} i_{b,n} Z_{V,n}(G, X_{b,n}, Y_{b,n}),$$

$$i_{b,n} = (I_{0,n} / 2) + \sum_{k=1}^{\infty} I_{k,n} \cos[k(\omega_0 t - 2\pi n / M)],$$

$$Z_{H,n} = \sum_{m,j=0}^{\infty} S_{m,j} X_{b,n}^m Y_{b,n}^j,$$

$$Z_{V,n} = \sum_{m,j=0}^{\infty} D_{m,j} Y_{b,n}^m X_{b,n}^j,$$

$$X_{b,n} = x_b + x_n, \quad Y_{b,n} = y_b + y_n,$$

$$x_n = a_{x,n} \cos[q_x(\omega_0 t - 2\pi n / M)],$$

$$y_n = a_{y,n} \cos[q_y(\omega_0 t - 2\pi n / M)],$$

$i_{b,n}$ is a current of corresponding n -th bunch of beam; ω_0 is rotation frequency of bunch in storage ring; $Z_{H,n}$, $Z_{V,n}$ are coupling impedances; G is geometric parameters of the BTM; x_b , y_b are CO coordinates; x_n , y_n are horizontal and vertical components of beam deviation from the CO; q_x , q_y are fractional parts of horizontal and vertical betatron tune, M is quantity of the bunches.

As a result of calculations we obtain a series of harmonics with frequencies $(k \pm r q_x \pm d q_y) \omega_0$, where k , r , $d = 0, 1, 2, \dots$ are coefficients of harmonics measured by BTM.

In ideal case BTM should extracted only harmonics $(k \pm q_x) \omega_0$ or $(k \pm q_y) \omega_0$, $k \omega_0$. But really always will be present higher and combination parasitic harmonics. As shown in [2], the contribution to measured signal of parasitic harmonics strongly depends on beam closed orbit coordinates in the aperture of the monitor.

Calculations of the spectrums of the signals H_β , V_β of BTM of N-100M were carried out for electron beam with parameters $M=1$, $I_k=2$ mA, $a_x=a_y=1$ mm, $q_x=0.155$, $q_y=0.082$ and the results are brought together in Tables 4,5.

One can see from Tables 4, 5 in both cases the spectrum of the measured signals are enriched by parasitic harmonics with increase of coordinates of the closed or-

bit. For example, at $x_b=9$ mm, $y_b=7$ mm the amplitude of the parasitic harmonics $(k \pm q_y) \omega_0$ of the signal H_β exceeds amplitude of measured harmonics $(k \pm q_x) \omega_0$, that will not allow correctly to execute measuring.

Similar situation arises at measuring the signal V_β at coordinates of the closed orbit $x_b=14$ mm, $y_b=7$ mm.

4. CONCLUSIONS

Thus, the approximation of operation characteristics (1), (2) by expressions (6), (7) with use of arguments (4) gives considerably smaller general error of measuring of coordinates of the beam for any shapes of cross section of monitors in comparison with traditionally used approximations. Use of empirical expression (7) and the combined arguments H , V , will allow to fulfill precision measurements of the beam coordinates of SR N-100M with accuracy up to $\pm 10 \mu\text{m}$ ($\leq \pm 10^{-3}$).

As the result of the carried out calculations it is visible that region about $x_b = \pm 4$ mm, $y_b = \pm 3$ mm for the monitor of storage ring N-100M is optimal for reliable measurements of betatron tune signals.

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РАБОЧИЕ ХАРАКТЕРИСТИКИ ЭЛЕКТРОСТАТИЧЕСКИХ ДАТЧИКОВ ЭЛЛИПТИЧЕСКОГО, ПРЯМОУГОЛЬНОГО И КРУГЛОГО ПОПЕРЕЧНОГО СЕЧЕНИЯ

В.Е. Иващенко, И.М. Карнаухов, В.И. Троценко, А.А. Щербаков

Исследованы различные методы аппроксимации рабочих характеристик электростатических датчиков положения пучка. Найден новый метод, позволяющий на порядок повысить точность аппроксимации горизонтальной координаты. Выполнено моделирование измерения частот бетатронных колебаний.

РОБОЧІ ХАРАКТЕРИСТИКИ ЕЛЕКТРОСТАТИЧНИХ ДАТЧИКІВ ЕЛІПТИЧНОГО, ПРЯМОКУТНОГО ТА КРУГЛОГО ПОПЕРЕЧНОГО ПЕРЕТИНУ

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Досліджено різні методи апроксимації робочих характеристик електростатичних датчиків положення пучка. Знайдено новий метод, що дозволяє на порядок підвищити точність апроксимації горизонтальної координати. Виконано моделювання вимірювання частот бетатронних коливань.