TRANSFER MATRIX FOR A HIGH ENERGY COOLING SYSTEM

***A. Dolinskii*[a,b], *P. Zenkevich*[c]**
[a] *Institute for Nuclear Research, Kiev, Ukraine*
[b] *Gesellschaft für Schwerionenforschung, Darmstadt, Germany*
[c] *Institute of Theoretical and Experimental Physics, Moscow, Russia*
e-mail: A. Dolinskii@gsi.de

A transfer matrix for a high-energy electron cooling system, which describes the linear incoherent effects, is derived. Knowing this matrix we can treat an electron cooling section in the ring as an additional focusing element deforming the beam optic functions due to space charge forces This matrix can be implemented in different codes (MAD, MIRKO, SixTrack et al.) for precise calculation of the beam optic and dynamic aperture in storage rings, where a high energy electron cooling system is planed to install. As an example, an optic of the high-energy storage ring [6] calculated by MAD code with derived matrix is given.
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1. INTRODUCTION

The electron cooling method is based on the heat exchange between the beam of charged heavy particles (ions) circulating in the storage ring and the beam of electrons having the same average velocity [1,2]. By combining an ion beam and a high-intensity electron beam having a small momentum spread in a rectilinear storage section, it is possible to achieve efficient energy exchange between them. The electron beam passing through the cooling section is lost in the collector and takes some of the ion beam thermal energy with it, which leads to an effective reduction in the transverse dimension and the momentum spread in the initial ion beam. At low ion beam intensities electron cooling is successfully used in many devices and a space charge focusing of the electron beam may not significantly perturb the lattice functions. However, on transition to higher energy ion beam (more then 1 GeV/u) there is tendency to enhancement parameters of an electron cooling section (ECS) to have reasonable small cooling time. In this case the ECS in the ring may treated as an additional focusing element deforming the beam optic functions due to space charge forces.

To write the equation of motion of ions in electron beam, from which the transfer matrix for an electron cooling section can be derived, we consider the hydrodynamic approximation assuming that ion and electron temperatures are zero and the beams undergo simultaneous coherent transverse motion. The particle density in the beams is radially uniform and the beam radii are the same. The conducting wall is removed to infinity and image charges have no influence on the beam dynamic. The beams are matched for a time shorter than all the other characteristic times of the problem. Outside the cooling section the beam propagates in the storage ring with azimuthally focusing symmetry. We use a simple model to analyse the influence of the finite time of joint beam motion on the interaction of the electron and ion beams.

2. EQUATION

The equation of linear motion of the circulation antiproton beam in the electron cooling section with account of the space charge field and longi-tudinal magnetic field has the following form [3]

$$\frac{d^2u_c^{\pm}}{ds^2} \mp i\Omega_c^L \frac{du_c^{\pm}}{ds} + \Omega_c^2 u_c^{\pm} = 0\,. \tag{1}$$

Here subscript c refers to the circulating beam, $u^{\pm}=x\pm iy$ describes transverse displacement of the particles in complex notation motion (x, y are the horizontal and vertical displacements respectively),

$$\Omega^L = \frac{ZeB_s}{Am_pc^2\beta\gamma}$$

is the Larmour frequency of the circulating beam, Z, A are the charge and mass of the ion, B_s is the longitudinal magnet field in the cooler, m_p is the mass of the proton, e is the electron charge, c is the light velocity, $\beta=v/c$, v is the ion velocity, γ is the relativistic parameter.

$$\Omega_c^2 = (1-\gamma^2\eta+\eta_c)\Omega_{c,e}^2\,,\quad \Omega_{c,e}^2 = \frac{2\pi n_e r_p Z}{A\beta^2\gamma^3}$$

are circulating beam frequencies, where $\eta_c=I_c/I_e$ characterises the focusing of ions due to space charge forces in the cooler section (I_c , I_e are ion and electron currents respectively), η is the neutralization factor [4], $\gamma^2\eta$ describes the focusing due to neutralization of ions, n_e is the electron density in the cooler, r_p is the classical proton radius. Eugenfrequencies of the Eq. (1) are

$$\Omega_{1,2}^2 = \pm\left(\frac{\Omega^L}{2}\right) \pm \sqrt{\left(\frac{\Omega^L}{2}\right)^2 + \Omega_c^2} \tag{2}$$

Solution of Eq.(1) can be written in matrix forms:

$$\begin{pmatrix} u^+ \\ (u^+)' \end{pmatrix} = M \begin{pmatrix} u_0^+ \\ (u_0^+)' \end{pmatrix}, \qquad \begin{pmatrix} u^- \\ (u^-)' \end{pmatrix} = \widetilde{M} \begin{pmatrix} u_0^- \\ (u_0^-)' \end{pmatrix}$$

Here M is the second order complex matrix, $\widetilde{M}$ is the complex conjugated matrix. The elements of this matrix are given by the following formulae ($K = \sqrt{(\Omega^L/2)^2 + \Omega_c^2}$):

$$M_{11}^{\pm}(s) = \exp\left(\mp i\frac{\Omega^L}{2}s\right)\left[\pm i\frac{\Omega^L}{2K}\sin(Ks) + \cos(Ks)\right], \quad (3a)$$

$$M_{12}^{\pm}(s) = \exp\left(\mp i\frac{\Omega^L}{2}s\right)\frac{\sin(Ks)}{K}, \quad (3b)$$

$$M_{21}^{\pm}(s) = \exp\left(\mp i\frac{\Omega^L}{2}s\right)\frac{\Omega_c^2}{K}\sin(Ks), \quad (3c)$$

$$M_{22}^{\pm}(s) = \exp\left(\mp i\frac{\Omega^L}{2}s\right)\left[\pm i\frac{\Omega^L}{2K}\sin(Ks) + \cos(Ks)\right], \quad (3d)$$

If $K^2<0$ one should substitute in Eq. (3) instead of $\cos(Ks)$ and $\sin(Ks)/K$ the expressions $\cosh(|K|s)$ and $\sinh(|K|s)/|K|$. One can see that the elements of the complex matrix depend on the longitudinal variable s and K, Ω^L, Ω_c parameters. For practical simulations it is necessary to write a real transfer matrix for the cooling section. The expressions (3) allows us to find the real matrix T_{cool} of fourth order related with real coordinates by:

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = T_{cool}\begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \end{pmatrix}. \quad (4)$$

The matrix T_{cool} is expressed through the matrix M using the following formula:

$$T_{cool} = \begin{pmatrix} A & B \\ -B & A \end{pmatrix}, \quad (5)$$

where $A=Re(M)$, $B=-Im(M)$. The elements of this matrix are given by formulae

$$A_{11}(s) = \cos(Ks)\cos\left(\frac{\Omega^L}{2}s\right) + \frac{\Omega^L}{2K}\sin(Ks)\sin\left(\frac{\Omega^L}{2}s\right),$$

$$A_{12}(s) = \cos\left(\frac{\Omega^L}{2}s\right)\frac{1}{K}\sin(Ks),$$

$$A_{21}(s) = \frac{\Omega_c^2}{2K}\sin(Ks)\sin\left(\frac{\Omega^L}{2}s\right),$$

$$A_{22}(s) = \cos(Ks)\cos\left(\frac{\Omega^L}{2}s\right) - \frac{\Omega^L}{2K}\sin(Ks)\sin\left(\frac{\Omega^L}{2}s\right), \quad (6)$$

$$B_{11}(s) = \cos(Ks)\sin\left(\frac{\Omega^L}{2}s\right) - \frac{\Omega^L}{2K}\sin(Ks)\cos\left(\frac{\Omega^L}{2}s\right),$$

$$B_{12}(s) = \sin\left(\frac{\Omega^L}{2}s\right)\frac{\sin(Ks)}{K},$$

$$B_{21}(s) = \frac{\Omega_c^2}{2K}\sin(Ks)\sin\left(\frac{\Omega^L}{2}s\right),$$

$$B_{22}(s) = \cos(Ks)\sin\left(\frac{\Omega^L}{2}s\right) + \frac{\Omega^L}{2K}\sin(Ks)\cos\left(\frac{\Omega^L}{2}s\right).$$

The matrix elements satisfy the relations: $\det(T_{cool})=1$, $|\det(M)|=1$. The transfer matrix of a circular accelerator is $T_{per}=T_1T_{cs}$, where T_1 is the transfer matrix from the Electron Cooling Section (ECS) exit to the ECS entrance, T_{cs} is the transfer matrix of the cooler section including fringing fields of the solenoind. The transfer matrix for the cooler section taking account fringing fields of solenoid is written by $T_{cs}=T^-_{sf}T_{cool}T^+_{sf}$, where $T^{\pm}_{sf}$ are matrices for the fringing fields.

3. APPLICATION TO THE HESR

A planned high energy storage ring (HESR) for internal experiments with antiprotons at a maximum kinetic energy of 14.5 GeV is part of the GSI (Gesellschaft für Schwerionenforschung) future project [5]. The beam quality of $2x10^{-5}$ and the high average luminosity of $2x10^{32}$ $c^{-1}cm^{-2}$ are main objectives of the HESR [6]. Obviously, such parameters can by achieved by strong electron beam cooling, which is mandatory for high luminosity as well as for high energy and angular resolution. A cold electron beam at energies up to 8 MeV (corresponding to an antiproton energy of 14 GeV) with an electron current of up to 1 A are required with radius of 3 mm. A high cooling rate can be achieved only by the co-called magnetized cooling. It requires a strong longitudinal magnetic field (B=0.5 T) that guides the electron beam along the entire interaction region of up to 30 m length. The electron cooling system with such parameters is a straight-forward from a physical point of view although the technical realisation seems to be rather challenging. In a storage ring such electron cooling system will be as additional focusing element influencing on the beam dynamic of antiprotons beams due to space charge forces.

In order to determine the complete dynamics of the beam in the storage ring one needs to supplement the matrix of the cooling section with the matrix describing the ion beam motion in the storage ring. That can be done by using ion-optical codes (for instance MAD, TRANSPORT) where the external matrix is possible to introduce.

We calculate the lattice functions of the HESR with and without ECS by MAD code, introducing the transfer matrix, which elements are calculated by formulae (6). The parameters used to derive proper transfer matrix of the ECS are given in the Table.

Antiproton energy [GeV]	1.3
Magnet rigidity, BR, [Tm]	6.17
Neutralization factor, η	0.01
Solenoid field strength, B_s [T]	0.0
Electron beam radius,a [mm]	3
Electron beam current, I_e, [A]	1
Length of cooling section, L_{cool} [m	30
$\Omega^L=B_s/BR$	0.044

The lattice functions over the HESR, where the ECS is treated as a drift space, are shown in Fig. 1. This is a desired beta function distribution over the ring. A small value of beta functions (1 m) in both planes is required at the internal target and large ones (100 m) at the ECS. At high energy of antiprotons (more than 3 GeV) the ECS influences on the lattice functions is weak. Hence there is no need to make additional matching of the cooler section with optic of the rest ring. If the energy of antiprotons is below 3 GeV the lattice function of the ring is changed drastically. To demonstrate the influence of the ECS on the lattice functions only due to space charge focusing of the electron beam in the ECS we exclude the longitudinal magnetic field from our simulation.

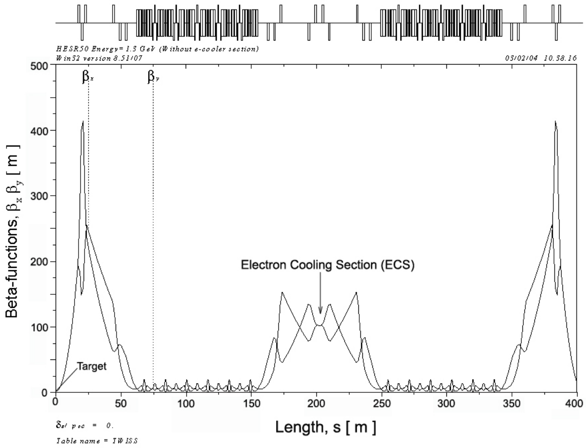

Fig. 1. *Lattice functions of the HESR. The ECS is treated as drift space*

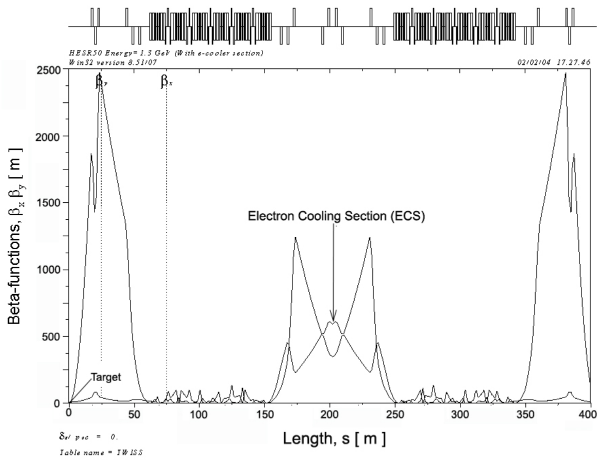

Fig. 2. *Lattice functions of the HESR. The ECS with electron beam but without longitudinal magnetic field*

Fig. 2 shows the beta function perturbation over the ring at the energy of antiproton of 1.3 GeV. It should be noted that according to the matrix elements given by formulae (6) the betatron motion in the ring is coupled. Therefore the Twiss in MAD are computed in coupled functions in the sense of Edwards and Teng parameterisation [7]. In this case the additional matching between arcs and a straight section, where the ECS is placed, is needed.

4. CONCLUSIONS

The transfer matrix for cooling section with a high electron density and strong longitudinal magnetic field is presented. It was shown on the example of the HESR that space charge focusing may significantly perturb the lattice functions causing a big enough tune shift and acceptance reduction of the circular accelerator. In this case additional matching of the ring optic functions with the ECS depending on the energy should be considered on the designing stage of the machine.

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МАТРИЦА ПЕРЕХОДА ДЛЯ ВЫСОКОЭНЕРГЕТИЧЕСКОЙ СИСТЕМЫ ЭЛЕКТРОННОГО ОХЛАЖДЕНИЯ

А. Долинский, П. Зенкевич

Выводится матрица перехода для высокоэнергетической системы электронного охлаждения, которая описывает линейные некогерентные эффекты. Такая матрица позволяет рассматривать секцию электронного охлаждения в кольце, как дополнительный фокусирующий элемент, искажающий оптические функции вследствие сил пространственного заряда. Полученная матрица может использоваться в различных программах (MAD, MIRKO, SixTrac и др.) для проведения прецизионных расчетов оптики пучка и динамической апертуры в накопительных кольцах, где планируется установка высокоэнергетических систем электронного охлаждения. В качестве примера приведен расчет оптики высокоэнергетического накопительного кольца [6], выполненный с помощью программы MAD и выведенной матрицы.

МАТРИЦЯ ПЕРЕХОДУ ДЛЯ ВИСОКОЕНЕРГЕТИЧНОЇ СИСТЕМИ ЕЛЕКТРОННОГО ОХОЛОДЖЕННЯ

А. Долинський, П. Зенкевич

Виводиться матриця переходу для високоенергетичної системи електронного охолодження, яка описує лінійні некогерентні ефекти. Така матриця дозволяє розглядати секцію електронного охолодження у кільці, як додатковий фокусуючий елемент, який спотворює оптичні функції внаслідок сил просторового заряду. Отримана матриця може використовуватися у різних програмах (MAD, MIRKO, SixTrac та інших) для проведення прецизійних розрахунків оптики пучка та динамічної апертури нагромаджувальних кілець, де планується планується установка високоенергетичних систем электронного охлаждения. Як приклад наве-

дено розрахунок оптики високоенергетичного нагромаджувального кільця [6], виконаний за допомогою програми MAD та виведеної матриці.