SELF-FOCUSING OF RADIATION IN NON-UNIFORM MEDIUMS

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The possibility of self-focusing in propagating the rays in the non-uniform mediums is shown. This self-focusing is similar to a Veksler-Mac-Millan's autophasing in the theory of accelerators. The self-focusing of the rays are able to weaken essentially an effect of random non-uniformities and to increase the threshold of development of stochastic instability.

PACS: 42.15.-i

INTRODUCTION

In many cases in the non-uniform mediums like laboratory plasma, ionosphere, ocean, or fiber communication lines, the dimensions of the non-uniformities are able to exceed considerably a wavelength which propagates in these mediums. In order to analyze the propagation of the waves in this case it is necessary to apply an approximation of geometrical optics. Taking into account that the geometrical optics so concerns to the wave optics, as the classical mechanics concerns to the quantum mechanics we are able to expect that many important features of classical dynamics of charged particles in electromagnetic fields can develop in the dynamics of the rays too. In particular, there is a phenomenon of the autophasing of accelerated particles in physics of charged particles. This phenomenon was discovered by Veksler and Mac-Millan and is a fundamental one in the theory of the accelerators. We can hope that analogical phenomenon will take place also in propagating electromagnetic rays in the non-uniform mediums, for example, in the non-uniform plasma. It is necessary to note that a phenomenon of the self-focusing of the rays was studding in [1].

PARAXIMAL APPROXIMATION

The simplest we can see analogy between dynamics of charged particles in the external electromagnetic fields and dynamics of the electromagnetic rays in the non-uniform dielectric mediums if the paraxial approximation for describing dynamics of the rays will be used:

$$n_0 \frac{d^2 r}{dz^2} = \nabla n_1 = \frac{\partial n_1(r,z)}{\partial r}, \qquad (1)$$

where $n(r,z) = n_0(r) + n_1(r,z)$; $n_0(r)$ - independent from longitudinal co-ordinate z - component of the refraction coefficient; $n_1(r,z)$ - slowly changing along axis z component. As seen from (1), we have confined the simplest case when dynamics of the rays is considered at the plane (r,z). For comparison let us consider the non-relativistic motion equation of charged particle in potential $V(r,t)$

$$m\frac{d^2 r}{dt^2} = F = -\nabla V(r,t). \qquad (2)$$

By comparing the equation (1) with motion equation of a material point (2), we can see that these equations are completely analogical. The analogua of the potential $-V$ in the geometrical optics is the coefficient refraction n_1, and analogue of mass is n_0. If we will choose the potential as

$$V(r,t) = \alpha \cdot r^2/2 + \beta \cdot r^4/4 - r \cdot \varepsilon \cdot \sin(\Omega \cdot t),$$

we will obtain the equation that will be similar to the equation of the Duffing's oscillator which is acted an external periodical force having amplitude ε and frequency Ω on. Before the dynamics of the Duffing oscillator that is subjected both the similar external effect and parametrical perturbation action have been investigated in [2]. In particular, the condition of appearance of moving particles stochastic instability has been defined in this paper. It has been shown that dynamics of the rays in propagating in the medium having the refraction coefficient (4) should be similar dynamics of the charged particles.

SELF-FOCUSING OF THE RAYS

The paraxial approximation in the geometrical optics corresponds to the non-relativistic moving of charged particles. It means that the generalized pulse of the rays can not become by sufficiently major. In order to describe dynamics of the rays having great values of the pulse we should refuse from the paraxial approximation. In this case it is necessary to use more common equations of the geometric optics which are similar to the relativistic motion equations of charged particles. These equations may be written

$$\dot{p} = -\frac{\partial H_0}{\partial x} \qquad \dot{x} = \frac{\partial H_0}{\partial p}, \qquad (3)$$

where $H_0(x,p,z) = -\sqrt{n^2(x,z) - p^2}$ is hamiltonian, x, p are the generalized coordinate and pulse correspondently. Let us consider the first case when the rays propagate in the medium. The parameters of the medium depend only on transverse coordinate x ($n = n(x)$). From the equations (5) we can get the following equations for a generalized coordinate x.

$$\ddot{x} = -\frac{\dot{p}}{H_0} + \frac{p \cdot \dot{H}_0}{H_0} = \frac{n(x)}{H_0^2} \cdot \frac{\partial n(x)}{\partial x} = F(x). \qquad (4)$$

The equation (4) is one of the non-linear pendulum. In particular, if the refraction coefficient of the medium will be given as

$$n^2 = n_0^2 + \mu^2 / ch^2 \frac{x}{a}, \qquad (5)$$

the qualitative view of dynamics of the rays will be similar to a view that is presented in Fig. 1.

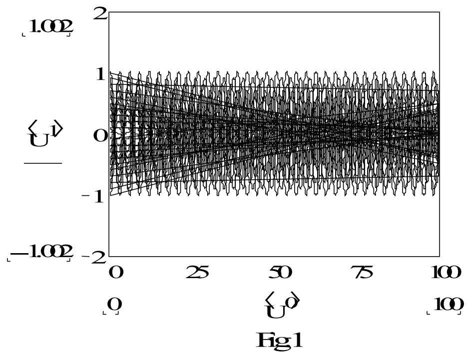

Fig. 1

In Fig. 1 the trajectories of 20 rays which were entered into the medium with different input characteristics are shown. A value $\Psi = \dfrac{x}{a}$ is called by a phase of a ray relatively of non-uniformity. As this takes place, a value a defines transversal dimensions of a canal of the medium. Let us assume that typical transversal dimensions of the canal of the medium depend on longitudinal coordinate z, e.g. $a = a(z)$. In this case a phase of a ray will be a function not only x but also z, e.g. $\Psi = \Psi(x,z)$. Taking into account an analogy with dynamics of the charged particle in our case the self-focusing of the rays will take place. In order to define the conditions of appearance of the self-focusing it is necessary to consider the equation that describes changing a phase of a ray with coordinate z:

$$\ddot{\Psi} + 2\dot{\Psi}\frac{\dot{a}}{a} + \Psi\frac{\ddot{a}}{a} - \frac{\ddot{x}}{a} = 0 \qquad (6)$$

$$\ddot{x} = \frac{n}{H_0^2}\cdot\frac{\partial n}{\partial x} + \frac{p}{H_0^2}\cdot\frac{n}{H_0}\cdot\frac{\partial n}{\partial z}.$$

A synchronous phase we can define using the following condition: $\Psi_s = x(z)/a(z) = \alpha = const$. The small deviations of the phase from the synchronous phase $\varphi = \Psi - \Psi_s$ satisfy an equation of the linear pendulum with attenuation:

$$\ddot{\varphi} + \frac{\dot{a}}{a}\cdot\dot{\varphi} + \left(\frac{\partial F}{\partial \varphi}\right)_{\Psi_s}\cdot\varphi = 0, \qquad (7)$$

where $F(\Psi) = -\dfrac{\ddot{x}}{a(z)} + \Psi\dfrac{\ddot{a}}{a(z)}$.

In particular it follows from equation (7) that if the transversal dimensions of the dielectric waveguide will increase ($\dot{a} > 0$) the phases of all rays will aim at a phase of a synchronous ray. In this case the self-focusing will take place. Hence the condition of the self-focusing in the considered model will be simple condition of growth of a typical transversal dimension of a canal in the medium. Fig. 2 and 3 show the generalized coordinates and generalized pulse vs the longitudinal coordinate z for the rays used before in Fig. 1. The difference is to change the value $a(z)$ in the last case take place in according to the following law: $a = a_0 + z$. The characteristics of the refraction coefficient have been chosen the following: $a_0 = 0.8$; $\mu = 3$; $n_0 = 1$. It is seen from Figs. 2 and 3, that in growing $a(z)$ dynamics of the rays are organized.

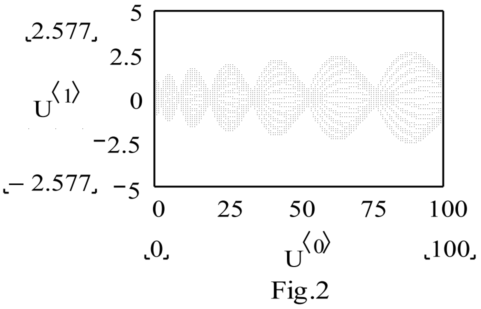

Fig.2

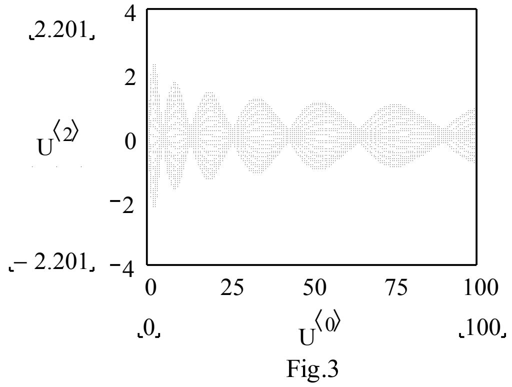

Fig. 3

A value of the generalized impulse decreases, e.g. the angle of incidence of the rays to the axis z decreases. The absolute values of deviations of the rays are grown.

INFLUENCE OF FLUCTUATION UPON DYNAMICS OF RAYS

There are fluctuations in the real mediums always. In order to take into account the fluctuations the refraction coefficient can be written as:

$$n^2 = n_0^2 + n_1(x,z) + q(z) \qquad q << 1. \qquad (8)$$

It is to be noted that the fluctuations are by random functions, which have zero mean values, and these functions are delta-correlated:

$$\langle q(z)\cdot q(z')\rangle = D\cdot\delta(z - z').$$

By assuming that the fluctuations are not great the generalized impulse and Hamiltonian may be presented as a sum of perturbed and unperturbed parts:

$$p = p_0 + \tilde{p} \qquad H(x,p,z) \approx H_0(x,p,z) + H_1(x,p,z),$$

where $H_0 = -\sqrt{n_0^2 + n_1 - p^2}$ $\qquad H_1 = \dfrac{q}{2H_0}$

Then for perturbed generalized impulse the following equation can be written as

$$\frac{d\tilde{p}}{dz} = -\frac{\partial H_1}{\partial x} = -\frac{q}{2}\frac{1}{H_0^2}\frac{\partial H_0}{\partial x} = \frac{q}{2}\frac{1}{H_0^2}\frac{dp_0}{dz}. \qquad (9)$$

From equation (9) it is seen that a mean value of $\tilde{p}$ equals zero, e.g. $\langle\tilde{p}\rangle = 0$.

It is necessary to note that the unperturbed value of the impulse has regular periodical variations along of axis z with a typical period equal β. In addition the value of unperturbed impulse attenuates slowly according to the law $\exp(-\delta \cdot z)$. By using these considerations from equation (9) can get the following expression for mean square of $\tilde{p}$:

$$\left\langle \tilde{p}^2 \right\rangle = \frac{\beta^2 \cdot (\Delta p_0)^2}{4H_0^4} \cdot D \cdot \frac{(1-\exp(-2\delta z))}{2\delta} . \qquad (10)$$

If self-focusing of the rays is absent then the value $\delta = 0$. In this case an expression (10) is transformed in the famous law of a diffusion:

$$\left\langle \tilde{p}^2 \right\rangle = \frac{\beta^2 \cdot (\Delta p_0)^2}{4H_0^4} D \cdot z . \qquad (11)$$

As appears from expression (11) the scattering of generalized impulse grows with increasing the distance. Eventually, the rays will abandon the canal.

As may be seen from expression (10), the presence of self-focusing can essentially confine an influence of the fluctuation in propagating of the waves.

STABILIZATION OF STOCHASTIC INSTABILITY

If parameters of the medium are constant along of axis z, hamiltonian will depend on canonical variables ($H = H_0(x,p)$). In this case the new canonical variables which include an action and angle I, Θ can be entered. With these new canonical variables initial hamiltonian will be a function only of one action. Let us assume that there have small periodical perturbations. In this case hamiltonian can be written as

$$H = H_0(I) + \varepsilon V(I, \Theta, z) , \qquad (12)$$

where

$$V = \frac{1}{2}\sum_{m,s} V_{m,s}(I) \exp\left[im\Theta + is\int_0^z \aleph \, dz \right] + k.c. .$$

The difference of given perturbation from well-known perturbation is to take place a dependence of the parameter $\aleph$ from coordinate z. In spite of hamiltonian (21) allows getting the following equations set that describe dynamics in the vicinity of one cyclotron resonance:

$$\dot{I} = \varepsilon \cdot mV_{m,s} \cdot \sin\Phi_s ,$$

$$\dot{\Theta} = \omega(I) + \varepsilon\cos\Phi_s \left(\partial V_{m,s} / \partial I \right) , \; \dot{\Phi} = m\omega(I) + s\aleph . \qquad (13)$$

It is known, that when the non-linear resonances overlap the stochastic instability develops. If the parameter $\aleph$ does not depend on z then by using (13) a condition of the stochastic instability can be found easily (see, for example, [2]). In our case dynamics of a small deviations from stationary points may be found easily by using:

$$\ddot{\varphi} + \frac{\alpha}{\lambda}\dot{\varphi} - \beta \cdot \cos(\Phi_n) \cdot \varphi = 0 . \qquad (14)$$

Here is $\varphi = \Phi - \Phi_n$, Φ_n is stationary phase. In getting (14) we assumed that the parameter $\aleph$ depend on z in according to the following law:

$$\aleph = 2\pi / (\lambda + \alpha z) , \text{ where } \lambda = -\pi s / m\omega .$$

As will be seen from (14) we have obtained an damping oscillator. In common case the presence of the attenuation allows increasing the threshold of appearance of the stochastic instability.

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САМОФОКУСИРОВКА ИЗЛУЧЕНИЯ В НЕОДНОРОДНЫХ СРЕДАХ

А.В. Буц, В.А. Буц, Г.И. Чурюмов

Показана возможность автофазировки лучей, распространяющихся в неоднородных средах. Эта автофазировка аналогична автофазировке Векслера-Мак-Миллана в теории ускорителей. Автофазировка лучей существенно ослабляет влияние случайных неоднородностей и увеличивает порог развития стохастической неустойчивости.

САМОФОКУСУВАННЯ ВИПРОМІНЮВАННЯ В НЕОДНОРІДНИХ СЕРЕДОВИЩАХ

А.В. Буц, В.А. Буц, Г.І. Чурюмов

Показана можливість автофазування променів, що поширюються в неоднорідних середовищах. Це автофазування аналогічне автофазуванню Векслера-Мак-Міллана в теорії прискорювачів. Автофазування променів істотно послабляє вплив випадкових неоднорідностей і збільшує поріг розвитку стохастичної нестійкості.