COMPACT FORM AND EVALUATION OF TRUBNIKOV'S PLASMA DIELECTRIC TENSOR

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Trubnikov's plasma dielectric tensor by means of integral transform is reduced to more simple and transperant form. Due that the problem evaluating this tensor for arbitrary plasma temperature and the angle of wave propagation is resolved.

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1. INTRODUCTION

The study of EC waves even in respectively low temperature laboratory plasma requires taking into account relativistic effects, especially for the quasi-perpendicular waves propagation. Two original equivalent integral forms of the exact relativistic dielectric tensor for Maxwellian plasma were given by Trubnikov [1] but were singular and as a result proved difficult to evaluate numerically.

For low temperature plasma, in which relativistic effects are relatively weak, gradually was developed the theory of weakly relativistic approximation that widely used now in applications. A review of vast number of works in that direction is presented in the recent book of Brambilla [2].

For plasma with more high temperature the theory is developed in less detail. One approach was suggested by Tamor [3], who simplified the problem studding only non-singular anti-Hermitian parts of dielectric tensor. This way, using recursive properties of anti-Hermitian parts, allows him numerically to estimate those parts for plasma with temperature below 30 KeV. For more high temperatures this way failed due numerical instability of the recursion.

Later, at the same way Bortnatici and Ruffina gave exact analytical expressions for anti-Hermitian parts of dielectric tensor for harmonics with numbers $n \ge 0$ and estimate Hermitian parts [4]. However, their results for anti-Hermitian parts were not quite general and estimations for Hermitian parts were approximate. That didn't allow them to derive whole dielectric tensor in the exact closed form.

Recently, a new way to evaluate relativistic tensor was given by Swanson [5]. It allows for the case of perpendicular waves propagation writing the exact relativistic tensor elements in terms of five single singular integrals over hyper-geometric functions. Those integrals are relatively easy evaluated numerically. However, for oblique propagation on this way double singular integrals are necessary to calculate, that requires some approximations.

The primary purpose of the present work is giving the way to evaluate the relativistic dielectric tensor of Trubnikov for arbitrary plasma temperature and angle waves propagation. It is made on the ground of transformation of this tensor to more simple and transparent form. On this way dielectric tensor is presented in the manner of the non-relativistic approximation as an expansion in the Larmor radius in terms of specially introduced in the work [6] the exact relativistic PDFs to reduce evaluation of tensor to evaluation of these PDFs.

2. MUTUALLY CONSISTENT FORMS OF NON-RELATIVISTIC AND RELATIVISTIC DIELECTRIC TENSORS

Using analytical properties of modified Bessel functions and non-relativistic PDF, which appear in non-relativistic dielectric tensor, it is possible to write that tensor in different analytical forms. The choice of the concrete form is defined either by tendency to extremely possible simplicity of final expression [7] or by somewhat other considerations [2]. In particular, one can verify that this tensor can be presented as well in the following form:

$$\varepsilon = 1 + Z_0\left(\frac{\omega_p}{\omega}\right)^2 \sum_{n=-\infty}^{\infty}\left[P_n Z(Z_n) + R_n Z'(Z_n) + S_n Z''(Z_n)\right], \quad (1)$$

where $Z(Z_n)$, $Z'(Z_n)$, $Z''(Z_n)$ is non-relativistic PDF of the argument $Z_n = (\omega - n\Omega_c)\big/(\sqrt{2}k_{//}V_T)$ and its first and second derivatives in this argument, V_T is thermal electron velocity and

$$P_n = \begin{pmatrix} n^2 A_n/\lambda & -inA'_n & 0 \\ inA'_n & n^2 A_n/\lambda - 2\lambda A'_n & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$R_n = \begin{pmatrix} 0 & 0 & nA_n/\sqrt{2\lambda} \\ 0 & 0 & -iA'_n\sqrt{\lambda/2} \\ nA_n/\sqrt{2\lambda} & iA'_n\sqrt{\lambda/2} & 0 \end{pmatrix},$$

$$S_n = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_n/2 \end{pmatrix},$$

$A_n(\lambda) = \exp(-\lambda) I_n(\lambda)$, $A'_n(\lambda) = dA_n(\lambda)/d\lambda$.

The main peculiarity of such presentation is total separation of perpendicular and longitudinal dispersion. The perpendicular dispersion is described by tensors P_n, R_n, S_n and the longitudinal one is described by plasma dispersion function, $Z(Z_n)$, and its first and second derivatives in the parameter Z_n.

Let us show using the exact relativistic PDFs, $Z^0_{q+3/2}(a, z, \mu)$ and indications from the work [6] that the Trubnikov's dielectric tensor can be presented in the form, which is similar to the form (1). First, we'll prove that functions, $Z^1_{q+3/2}(a,z,\mu)$, appearing in the elements of dielectric tensor $\varepsilon_{13},\varepsilon_{31},\varepsilon_{23},\varepsilon_{32}$ and functions, $Z^2_{q+3/2}(a,z,\mu)$, appearing in element ε_{33} can be expressed in terms of the exact PDFs, $Z^0_{q+3/2}(a,z,\mu)$. Really, using differentiation the expression (4) from the work [6] in argument z one can prove the next identities:

$$2\sqrt{a}\, dZ^0_{q+3/2}(z,a\mu)/dz = -\sqrt{2}Z^1_{q+1/2}(z,a\mu), \qquad (2)$$

$$2a\, d^2 Z^0_{q+3/2}(z,a\mu)/d^2z + Z^0_{q+1/2}(z,a\mu) = Z^2_{q-1/2}(z,a\mu), \quad (3)$$

Note that the identities (2), (3) take place for both cases $0 \le N_{//} < 1$ and $N_{//} > 1$. Now let us change argument z in relativistic PDFs into $Z_n = z/(2\sqrt{a})$. Then it is obviously, that $Z_n = (\omega - n\Omega_c)/(\sqrt{2}k_{//}V_T)$. The advantage of such a change consists of the fact that relativistic PDFs depend now of the same argument that the non-relativistic PDF. It is not difficult to verify now term by term that the relativistic tensor (1) can be written in the form

$$\varepsilon = 1 - \mu\left(\frac{\omega_p}{\omega}\right)^2 \sum_{n=-\infty}^{\infty}\left[P_n * \bar{Z}(Z_n) + R_n * \bar{Z}'(Z_n) + S_n * \bar{Z}''(Z_n)\right], \quad (4)$$

where $\bar{Z}(Z_n)$ is an infinite vector containing all relativistic PDFs with gradually increasing index starting from $q = 5/2$:

$$\bar{Z}(Z_n) = \left[Z^0_{5/2}(Z_n,a,\mu), Z^0_{7/2}(Z_n,a,\mu), Z^0_{9/2}(Z_n,a,\mu), \ldots\right],$$

here the sign $*$ denotes the scalar product of the infinite vector $\bar{Z}(Z_n)$ with every left-side matrix element considered as an infinite vector resulting from finite Larmor radius (FLR)- expansion beginning from the term of zeroth order. So, for instance

$$\left[P_n * \bar{Z}(Z_n)\right]_{11} = n^2 \bar{A}_n/\lambda * \bar{Z}(Z_n),$$

where $\bar{A}_n/\lambda = \left[A^0_n/\lambda, A^1_n/\lambda, A^2_n/\lambda, \ldots\right]$. Here the superscript k in A^k_n/λ denotes the order in FLR-expansion of A_n/λ. The order in FLR-expansion in tensor R_n, that is connected with elements of tensor $\varepsilon_{13},\varepsilon_{31},\varepsilon_{23},\varepsilon_{32}$, must be considered after dividing these elements by $\sqrt{\lambda}$.

3. EVALUATING OF THE EXACT RELATIVISTIC PDFs

From the form (4) of Trubnikov's dielectric tensor and the formulas for the 1st and 2nd PDFs derivatives (20) from the work [6] it follows that the problem of evaluation of this tensor is reduced to the problem of evaluation of the exact PDFs, $Z^0_{q+3/2}(a,z,\mu)$.

Using the expression (4) for anti-Hermitian parts of these PDFs from the work [6] one can verify the identity

$$\operatorname{Im} Z^0_{q+1/2}(a,z,\mu) = \operatorname{Im}\pi^{1/2}\int_{-\infty}^{\infty} dx \exp\left[-x^2 + \left(z - 2a^{1/2}x\right)^2/(2\mu)\right] \times F_q\left[z + x^2 - 2a^{1/2}x - \left(z - 2a^{1/2}x\right)^2/(2\mu)\right],$$

where $F_q(z)$ is Dnestrovskii function with integer index q. It is easy to see that from theory of Cauchy type integrals it follows the formula

$$Z^0_{q+1/2}(a,z,\mu) = \pi^{1/2}\int_{-\infty}^{\infty} dx \exp\left[-x^2 + \left(z - 2a^{1/2}x\right)^2/(2\mu)\right] \times F_q\left[z + x^2 - 2a^{1/2}x - \left(z - 2a^{1/2}x\right)^2/(2\mu)\right]. \qquad (5)$$

The integral form (5) provides the first way evaluating the exact PDFs and is a generalization on the fully relativistic case of the method evaluating the weakly relativistic PDFs of Airoldi and Orefice [8].

The second way is based on the direct calculating of singular integrals describing Hermitian parts of the exact PDFs using for calculation of the principal value of these integrals the exact non-singular formulas [9]

$$\mathrm{P}\int_0^{\infty}\frac{f(t)}{t-b}dt = \int_0^{b}\frac{f(t)-f(2b-t)}{t-b}dt - \int_{-\infty}^{0}\frac{f(2b-t)}{t-b}dt, \qquad (6)$$

(for $0 \le N_{//} < 1$)

$$\mathrm{P}\int_{-\infty}^{\infty}\frac{f(t)}{t-b}dt = \int_{-\infty}^{b}\frac{f(t)-f(2b-t)}{t-b}dt, \qquad (7)$$

(for the case $N_{//} > 1$).

At the figures 1, 3 of Supplement there are given graphics of the exact PDFs with indexes from q =3/2 till q =9/2 for two values of longitudinal refractive index, $N_{//}$ =0.3, $N_{//}$ =1.1, and two values of plasma temperatures, T_e = 20keV and T_e =40keV, respectively. For comparison, at the figures 2, 4 of Supplement there are presented plots of the weakly relativistic and exact PDFs for the same values of parameter $N_{//}$ and different values of temperature.

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SUPPLEMENT

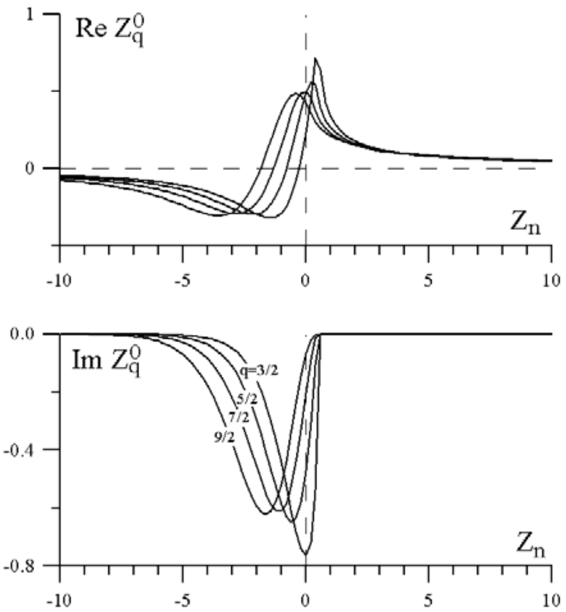

Fig. 1 PDFs for the case $T_e = 20keV$, $N_{//} = 0.3$

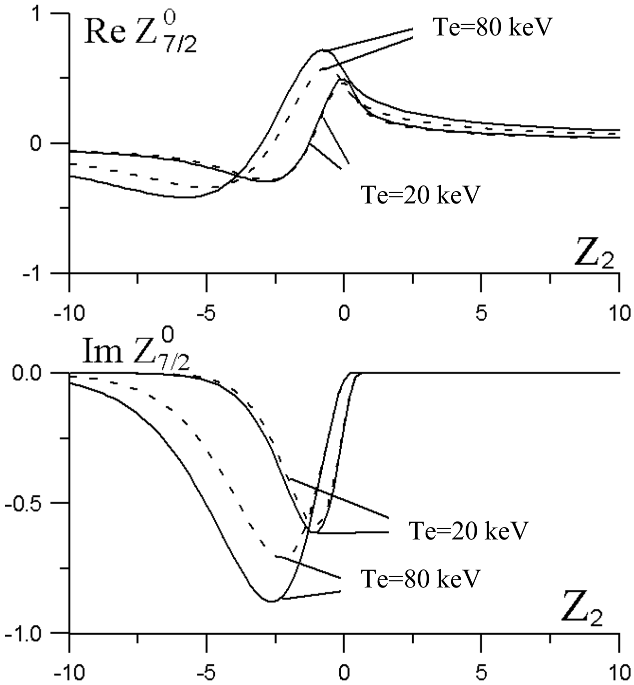

Fig. 2 Weakly relativistic (solid-line) and exact (dash-line) PDFs for $q = 7/2$ *and* $N_{//} = 0.3$ *for temperatures* $T_e = 20keV$ *and* $T_e = 80keV$

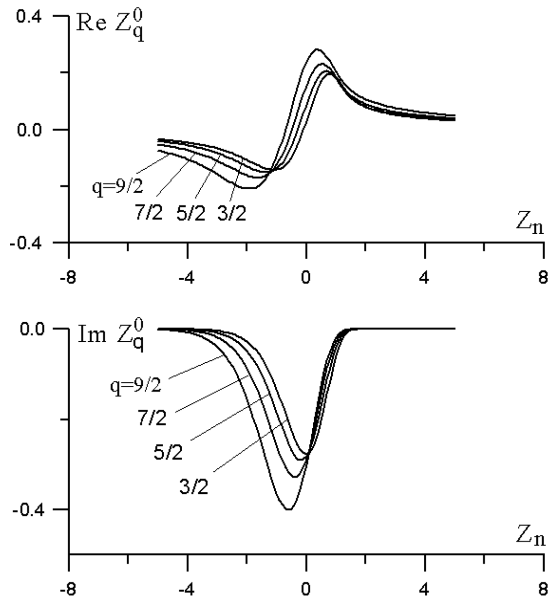

Fig. 3 Exact PDFs for the case $T_e = 40keV$ *and* $N_{//} = 1.1$

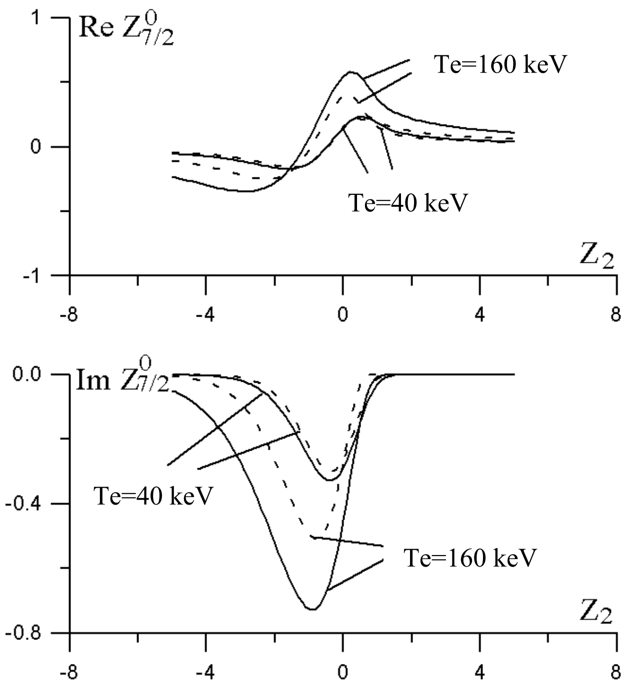

Fig. 4 Weakly relativistic (solid-line) and exact (dash-line) PDFs for $q = 7/2$ *and* $N_{//} = 1.1$ *for temperatures* $T_e = 40keV$ *and* $T_e = 160keV$

КОМПАКТНАЯ ФОРМА И ВЫЧИСЛЕНИЕ ПЛАЗМЕННОГО ДИЭЛЕКТРИЧЕСКОГО ТЕНЗОРА ТРУБНИКОВА

С.С. Павлов, Ф. Кастехон

Решается проблема точного вычисления Трубниковского плазменного диэлектрического тензора для произвольных значений температуры плазмы и угла распространения в ней волн.

КОМПАКТНА ФОРМА І ОБЧИСЛЕННЯ ПЛАЗМОВОГО ДІЕЛЕКТРИЧНОГО ТЕНЗОРУ ТРУБНІКОВА

С.С. Павлов, Ф. Кастехон

Розв'язується проблема точного обчислення плазмового діелектричного тензору Трубнікова для довільних значень температури плазми і кута розповсюдження в ній хвиль.