STUDY OF SELF-ORGANIZING REGIME OF NUCLEAR BURNING WAVE IN FAST REACTOR

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An approach for description of the space-time evolution of self-organizing nuclear burning wave regime in a critical fast neutron reactor has been developed in the effective multigroup approximation. It is based on solving the non-stationary neutron diffusion equation together with the fuel burn-up equations and the equations of nuclear kinetics for delayed neutron precursor nuclei. The calculations have been carried out in the plane one-dimensional model for a two-zone homogeneous reactor with the metal U-Pu fuel, the Na coolant and constructional material Fe. The temperature effects and heat sink were not considered.

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1. INTRODUCTION

The work is devoted to developing the physical foundations of the new perspective conception of the safe fast reactor (FR). This FR is a new type of long-life operation reactor with the so-called inner safety. This type of inherent safety prevents the appearance of reactivity-initiated accidents in FR by virtue of the physical principles that underlie its design. The FR in distinction from conventional fast reactors has no initial excess reactivity. Therefore, it does not need any reactivity control. Another important peculiarity of FR under consideration is that it operates till the end of its life without any refueling or fuel shuffling. In FR of this type natural or depleted uranium can be used as its fuel, except for the active zone of the initial critical assembly that serves as an ignition region. Thorium can also be useable instead of uranium. The level of fuel burn-up can be essentially high. The FR also has an important merit from the point of view of nuclear proliferation. It does not need a human access during its operation time and so can be placed underground.

The FR operation is based on the non-linear selforganizing regime of the nuclear burning wave (NBW) that arises owing to a high conversion ratio from fertile to fissile materials in the FR. Feoktistov [1,2] was the first to show up this regime in the framework of a schematic one-dimensional model of FR with the U-Pu fuel cycle. In a self-similar approach he proved the existence of the NBW and estimated the velocity of its propagation. However, Feoktistov's model of FR has essential faults. In this model, the equilibrium and critical plutonium densities were considered as certain phenomenological parameters whereas in reality they depend on both the neutron cross-sections and characteristic neutron spectrum. This scheme was restricted to a maximally simplified set of burn-up equations that involved only four components of the nuclear transformation chain and did not take into account the fission products, constructional materials and coolant.

The concept of NBW was further developed in the framework of multigroup diffusion approximation in [3,4]. A stationary equilibrium regime of nuclear burnup (named CANDLE) was studied in the cylindrical model of FR using the self-similar solution approach. Calculations were carried out for a lead-bismuth-eutectic cooled FR with metallic uranium fuel employing a long nuclide chain in the burn-up equations. It was shown that in this steady-state regime the NBW moved with a constant velocity along the reactor without changing its shape.

Teller et al. [5,6] proposed a long-life FR of a prolate form with fuel of the U-To cycle. They sought for a numerical solution of the corresponding non-stationary problem and used a large amount of specially composed software. The burning process had the form of a running NBW that started in the ignition zone. Then this wave propagated beyond the ignition zone in an axial direction during the life of reactor.

The mechanism of self-organizing regime in the FR, was also explored in the framework of the quasidiffusion approach with the help of mathematical modeling in [7,8]. In the one-dimensional model of FR it was shown that this regime (called as self-adjusting neutron-nuclide regime) could be initiated inside its core at certain conditions. However, as it follows from the results [8], this regime does not go over into the NBW regime in the non-stationary scheme considered. The reactor starts to damp earlier than the wave is created.

The possibility of creating the NBW regime in a linear FR was also confirmed in Ref. [9] by calculations carried out for the simple model [1] both in the self-

similar solution approach and in the corresponding nonstationary one.

In our previous work [10], the possibility of creation of a self-organizing regime in the form of a running NBW was proved for a model of critical FR close to a real situation. The requirements of the wave initiation and evolution in space and time were studied. To describe the space-time evolution of NBW, an approach has been developed in the framework of one-group approximation. It includes the non-stationary diffusion equation for neutron transport, the burn-up equations for fuel components and the equations of nuclear kinetics for precursor nuclei of delayed neutrons. Certain properties of the NBW regime were studied, such as the mechanism of reactivity feedback and stability of the NBW regime relative to distortions of the neutron flux.

The calculations have shown that the regime of running NBW can be observed when one uses the effective one-group cross sections obtained by averaging the group cross sections over the group neutron spectrum integrated over the length of the initial critical assembly of FR. It should be noted that the NBW regime did not arise if for this purpose the group neutron fluxes at each space point of the initial critical assembly were used (cf. [8]).

In both cases the effective one-group microscopic cross sections remained unchanged during the lifetime of the FR. As a matter of fact, the group neutron fluxes essentially change inside the FR during its operation time. Therefore, the effective one-group microscopic cross sections must also be changed according with this space-time alteration of the neutron spectrum with time.

The purpose of the present paper is to study the influence of the space-time alteration of the group neutron fluxes on the process of NBW initiation and propagation inside the FR under consideration. This can be done considering the multigroup criticality problem at every moment of the FR operation time. The corresponding calculations will be performed in the framework of effective multigroup approximation, which is a modified version of the approach developed in [10].

2. THE CALCULATION SCHEME

For the description of space-time evolution of the self-organizing NBW in a critical FR it is necessary to solve the set of equations in partial derivatives that includes the non-stationary diffusion equation, the equations of fuel burn-up and the equations of nuclear kinetics.

Hereafter, these equations are written in the onegroup approximation for the case of the plane onedimensional model of FR under consideration.

The non-stationary diffusion equation with taking into account delayed neutrons can be written as

$$\frac{1}{v}\frac{\partial\Phi}{\partial t} - \frac{\partial}{\partial x}(D\frac{\partial\Phi}{\partial x}) + \Sigma_{a}\Phi - (1-\overline{\beta})(v_{f}\Sigma_{f})\Phi$$
$$= \sum_{i}\sum_{i}\lambda_{i}^{i}C_{i}^{i}, \qquad (1)$$

where $\Phi(x,t)$ is the scalar neutron flux, $\Sigma_{\alpha}(x) = \Sigma_j \sigma_{\alpha}^j N_j(x)$ is the macroscopic cross section of the neutron reaction

of the α -type (the index α corresponds to the reactions of neutron absorption (*a*) and fission (*f*)), $N_j(x)$ is the concentration of *j*'th nuclide at the point *x*; $\sigma_{\alpha}^{\ j}$ is the corresponding effective one-group microscopic cross section for the *j*'th nuclide; $v_i \Sigma_j = \Sigma_j v_j^i \sigma_j^j N_j(x)$, v_j^j is the mean number of neutrons produced at a single nuclear fission event for the *j*'th fissile nuclide; $\overline{\beta} = \Sigma_j \beta_j (v_j \Sigma_f) j / v_j \Sigma_f$ is the effective fraction of delayed neutrons, $\beta_j = \Sigma_i \beta_j^i$, and β_j^i , C_j^i and λ_j^i are the portion of delayed neutrons, the concentration and decay constant of the precursor nuclei in the *i*'th group of the *j*'th fissile nuclide, correspondingly; $D(x) = 1/(3\Sigma_{tr}(x))$ is the diffusion coefficient, $\Sigma_{tr}(x)$ is the macroscopic transport cross-section, *v* is the one-group neutron velocity.

We use the boundary conditions of the third kind for the flux $\Phi(x,t)$ that take into account the presence of an external neutron flux j_{ex} falling onto the left boundary of FR while the right boundary is free:

$$\Phi(0,t) - 2D(0,t) \frac{\partial \Phi(x,t)}{\partial x} \Big|_{x=0} = 2j_{ex}, \qquad (2)$$

$$\Phi(L,t) + 2D(L,t) \frac{\partial \Phi(x,t)}{\partial x} \bigg|_{x=L} = 0.$$
(3)

These conditions are valid for any moment of time within the considered time interval $0 \le t \le T$. Besides, the scalar neutron flux in the corresponding critical assembly $\Phi_0(x)$ is chosen as an initial condition for $\Phi(x,t)$ at the moment t = 0 for all values *x* from the space interval $0 \le x \le L$.

The burn-up equations describe changing the fuel components with time according to the chain of nuclear transformations. In the case of FR with the U-Pu fuel cycle we consider the chain including only 10 nuclides, whose numeration is presented in the table, to facilitate writing down corresponding equations:

$$\frac{\partial N_{l}}{\partial t} = -\left(\sigma_{al}\Phi + \Lambda_{l}\right)N_{l} + \left(\sigma_{cl}\Phi + \Lambda_{(l-1)}\right)N_{(l-1)}, \quad l = 1...8,$$
(4)

$$\frac{\partial N_9}{\partial t} = \Lambda_6 N_6, \qquad (5)$$

$$\frac{\partial N_{10}}{\partial t} = \sum_{l=1,4,5,6,7} \sigma_{jl} N_l \Phi \quad , \tag{6}$$

where $\sigma_{al} = \sigma_{cl} + \sigma_{fl}$, σ_{cl} is the microscopic cross section of radiation neutron capture by the *l*'th nuclide, $\Lambda_l = ln2/T_{1/2}^{i}$ and $T_{1/2}^{i}$ are the decay constant and half-life of the *l*'th nuclide.

The numeration of nuclei in the $^{238}U - ^{239}Pu$ transformation chain

N⁰	1	2	3	4	5
Nucleus	²³⁸ U	²³⁹ U	²³⁹ Np	²³⁹ Pu	²⁴⁰ Pu
N⁰	6	7	8	9	10
Nucleus	²⁴¹ Pu	²⁴² Pu	²⁴³ Am	²⁴¹ Am	FP

The pair of fission fragments produced at every fission event is considered to be one nuclide that we denote by the symbol FP (fission products).

At the initial moment of time the values of nuclide concentrations are

$$N_l(x,t=0) = N_l^0(x) . (7)$$

In the scheme studied we neglect the burn-up of nuclei ²³⁹U, ²³⁹Np, ²⁴¹Am, ²⁴³Am ($\sigma_{a2}=\sigma_{a3}=\sigma_{a8}=\sigma_{a9}=0$) because the decrease of their concentrations due to the absorption reactions is small as compared with the processes that have been considered. The changes of the fission fragments as a result of neutron absorption also were not considered.

The NBW regime is a slow process in which the scalar neutron flux Φ varies very weakly during the characteristic decay time of the precursor nuclei that emit delayed neutrons (see, for example, [10]). In this case, for the nuclear kinetic equations can be used the approximation of one equivalent group of delayed neutrons

$$\frac{\partial C_l}{\partial t} = -\lambda_l C_l + \beta_l (v_f \Sigma_f)_l \Phi , \qquad (8)$$

$$C_l(x,t=0) = C_l^0(x),$$
 (9)

where $\lambda_l = \beta_l / \sum_i \beta_i^i / \lambda_l^i$.

The complete statement of the non-stationary problem considered above includes the set of 16 non-linear partial differential equations and the corresponding initial and boundary conditions for them as well. We have solved this set of the equations using the finite-difference method. To apply the finite-difference technique, we introduce a rectangular mesh with steps h and τ (uniform for x and variable for t) in the range of the variables x and t. The FR length L was divided into M intervals of a spatial calculation mesh. We have found the solutions of the set of algebraic equations obtained from Eq. (1) in this way using, like in Ref. [10], the implicit Crank-Nicolson difference scheme [11]. This symmetric-in-time scheme shows an unconditional stability at any relation between space and time steps. It has the approximation of the second order of accuracy in h and τ .

The solution of the burn-up equations (4)-(6) and equations of nuclear kinetics (8) has been simplified assuming that the effective one-group cross sections and the neutron flux Φ are constant during the time intervals τ . As for the constancy of cross-sections, this assumption is well fulfilled for the FR conditions because of a weak sensitivity of the effective cross-sections to changes in the fast neutron energy spectrum. The assumption for Φ can be easily satisfied by choosing sufficiently small time intervals τ , within which the flux value should be taken as $\Phi = (\Phi_n + \Phi_{n+1})/2$ (Φ_n is the flux value at the time layer n). This fact allowed us to obtain an approximate analytical solution of Eqs. (4)-(6) and (8) for the concentrations of the corresponding nuclides at every node of the space mesh for the new time layer *n*+1 via the solution for layer *n* (see for details [10]).

It should be noted that the implicit finite-difference scheme that was used for solving the set of algebraic equations under consideration is non-linear. The neutron flux has been found using the method of successive approximations, in which its values at a new time layer were determined by an iteration procedure (see [10]). The expression for the equilibrium concentration of plutonium in a stationary state is written as

$$N_{eq} = \frac{\sigma_{c}^{1} N_{1}}{\sigma_{f}^{4} + \sigma_{c}^{4} + \sigma_{(n,2n)}^{4}},$$
(10)

where $\sigma^{4}_{(n,2n)}$ is the cross section of the (n,2n) reaction for 239 Pu.

At the initial stage of the nuclear burning inside the FR the following expression for N_{eq} is more correct

$$\overline{N_{eq}} = \frac{\Lambda_{3}N_{3}}{(\sigma_{f}^{4} + \sigma_{c}^{4} + \sigma_{(n,2n)}^{4})\Phi} .$$
(11)

For calculations of the effective one-group microscopic cross sections we used the group neutron fluxes Φ^{g} (g is the number of neutron energy group) for the initial critical assembly, found from solving the stationary multigroup problem. The calculations were performed in the 26-group approximation using the group neutron constants from Ref. [12]. The method of averaging the group cross-sections when passing from a greater number of energy groups to a smaller one is well known (see, for example, [13]). We have used the approach that takes into account the requirement of conservation of rates of corresponding reactions during this procedure.

The scheme of passing from the group microscopic cross sections to the effective one-group cross sections is defined by the relations

$$\sigma_{\alpha}^{\ l} = \sum_{g=1}^{26} \frac{\sigma_{\alpha}^{\ g/\Phi} \, g}{\Phi_{S}} , \ \Phi_{S} = \sum_{g=1}^{26} \Phi^{\ g} , \qquad (12)$$

where Φ_s is the neutron flux summed over 26 groups, the index α corresponds to the reactions of neutron capture (c), fission (f) and scattering (s).

The one-group neutron velocity is given by

$$\frac{1}{v} = \frac{1}{\Phi_s} \sum_{g=1}^{26} \frac{\Phi^g}{v^g},$$
(13)

where v^{g} is the neutron velocity for the group g.

When averaging the transport cross section σ_{tr} , the requirement of conservation of the neutron leakage value was taken into account, which leads to the following expression

$$\sigma_{tr}^{l} = \sum_{g=1}^{26} \sigma_{tr}^{gl} D^{g} \Phi^{g} / \sum_{g=1}^{26} D^{g} \Phi^{g} .$$
(14)

3. RESULTS AND DISCUSSION

We have carried out a series of calculations of the NBW regime in the critical FR using the developed procedure of numerical solution of the set of non-linear algebraic equations under consideration. The FR core length, $0 \le x \le L$, is divided into M=200 intervals of the spatial calculation mesh. We impose boundary conditions (3), (4) on the scalar neutron flux $\Phi(x,t)$. To create a neutron field in the system, that has to initiate the process of nuclear burning, we assume that the left boundary of the system is exposed to an external neutron flux coming from a source of certain intensity j_{ex} .

We consider a two-zone homogeneous FR with a metal U–Pu fuel of porosity p=1 (see Fig. 1). In the first zone (near the left edge of the reactor) the fuel consists of uranium enriched with plutonium (the ignition regi-

on). The second zone (the breeding zone), adjacent to the ignition one, is filled with 238 U (the presence of 235 U was neglected in the present FR model). Both zones also include the constructional material Fe and the Na coolant. The plutonium concentration in the enriched zone was chosen so as to be less than the equilibrium plutonium concentration and was equal to 10% of the concentration of fuel nuclei. The isotope composition of plutonium in the fuel was chosen to be as follows:

 239 Pu : 240 Pu : 241 Pu : 242 Pu = 0.70 : 0.22 : 0.05 : 0.03.

Composition: U-Pu fuel - 40 %, Na - 25 % Fe - 35 % Zone 1 Zone 2 j_{ex} Fuel: $p_{U} - 10 \%$ $^{238}U - 90 \%$ \rightarrow 0 X_B L X



The sharp boundary between the zones that exists at the initial moment of time would be quickly diffused with time. Therefore, to simplify the calculations, the U–Pu fuel distribution is assumed in the form of the Fermi step function with the parameters $x_{\rm B}$ and Δ defining the distribution width and diffuseness, respectively (see [10]).

The initial space distribution of the neutron field corresponding to the neutron flux of the critical FR was normalized so that the averaged energy production density in the enriched zone was equal to 10^{-8} kW cm⁻³. The intensity of the external neutron flux falling onto the left boundary of FR, which initiates the burning process, has been chosen to be $j_{ex} = 6 \cdot 10^{11}$ cm⁻² s⁻¹ for all variants of calculations.

We have chosen the following volume fractions of components in each zone: for the nuclear fuel $F_{\rm fu} = 40$ %, the constructional material $F_{\rm Fe} = 35$ % and the coolant $F_{\rm Na} = 25$ % (see Fig.1). These values of volume fractions were taken to correspond to the composition of actual reactors.

In one of the variants that we have calculated in our paper [10] the corresponding effective one-group microscopic cross sections $\sigma_{\alpha}(x)$ were calculated (see (12)) using the multigroup neutron fluxes $\Phi_0^{g}(x)$ obtained for the initial critical assembly under study. During the reactor operation period considered in our non-stationary calculation, the values of effective microscopic cross sections $\sigma_{\alpha}'(x)$ (dependent on the space variable) considered to be unchanged, as it was done in [7]. In this case the variation of the effective one-group cross sections according to the change of neutron spectrum with time was not taken into account. As result the running NBW does not arise, although a little shift of maxima in the space distributions of neutron flux and power is observed. In the reactor, the self-organized regime of nuclear burning is established for a certain time period, being similar to the so-called self-adjusting neutron-nuclide regime that was described in [7].



Fig. 2. Calculations for critical assembly of the FR: a) the neutron flux Φ (in an arbitrary units) both summed over 26 groups (Φ_s in (12), solid curve) and obtained in the one-group approach (dashed curve) versus x (in cm); b) the spatial distribution of group neutron fluxes Φ^g : short dashes are for g = 5($0.8 < E_n < 1.4$), solid curve is for g = 7 ($0.2 < E_n < 0.4$), long dashes are for g = 8 ($0.1 < E_n < 0.2$), dotted curve is for g = 10 ($0.0215 < E_n < 0.0465$), the bounds of energy groups are presented in parentheses; c) the group neutron spectrum χ^g averaged over the FR volume via lethargy $u = ln(10.5/E_n)$, E_n is the neutron energy in MeV

In the present paper we take into consideration the variation of neutron flux and the effective one-group cross sections in the FR with time. Therefore, at each time layer we solved the multigroup problem for fluxes in the critical FR assembly whose composition changes according to the burn-up equations. In the calculations that are described below, the group neutron fluxes $\Phi^{g}(x)$

found for the corresponding critical assemblies were used to obtain the effective one group cross sections. Thus, during the whole lifetime of FR the cross sections were corrected in the correspondence to the neutron spectrum alteration that occurred.

Fig. 2 presents the results of calculations of the scalar neutron fluxes in a critical assembly of FR that were carried out in the 26-group approximation with the parameters $x_{\rm B} = 64$ cm, $\Delta = 4.3786$ cm and L = 200 cm. These parameter values correspond to $k_{eff} = 1$ for the given variant of FR.

A comparison of the calculation results for the summed neutron flux $\Phi_s(x)$ with the corresponding flux $\Phi(x)$ calculated in the one-group approximation with the effective cross sections $\sigma_{\alpha}^{\prime}(x)$ obtained using the 26group fluxes $\Phi^{g}(x)$ is made in Fig. 2a. As can be seen, the results of calculations in the multigroup approach and in the considered variant of the one-group calculations are close to each other. Therefore, this one-group calculation reproduces the result obtained for the neutron flux in the multigroup approximation in a sufficiently accurate way.

Fig. 2b presents, as an example, the space distributions for the group neutron fluxes calculated for four energy groups.

As can be seen from Fig. 2c, the maximum of the energy spectrum of the considered FR with the metal fuel lies in the region of 200 keV (on the axis of ordinates we plot the integral neutron flux normalized to unity). It should be noted that, depending on the reactor composition, essential distinctions in spectra can be observed, especially in the low energy part. For example, the calculations show that when passing to the oxide fuel a considerable softening of the spectrum occurs in the low energy region. It can also be seen that at neutron energies lower than 10 keV that corresponds to the energy groups with numbers less than 12 the flux rapidly drops. Therefore, the neutrons in FR do not practically get to the thermal energy region. At the energies higher than 1 MeV the form of the calculated multigroup spectrum approaches to that of the fission neutron spectrum.

We have found approximate solutions of the nonstationary problem under consideration in the effective multigroup approach. This approach is based on the above-obtained result that the neutron flux in a critical FR summed over all groups does not practically differ from the one-group flux calculated with the effective cross sections obtained as described above. In this case, the values of effective one-group cross sections at each space point are corrected at each time layer according to the fuel composition that changes with time. The effective cross sections define the coefficients of the implicit Crank-Nicolson difference scheme for the nonstationary one-group diffusion equation. Thus, the scalar one-group neutron flux was found using the procedure developed in [10] for the numerical solution of the corresponding set of non-linear algebraic equations.

Fig. 3 presents the results of calculations of the main characteristics that describe the burning process in FR.

It can be seen that the neutron flux Φ and the density of energy production *P* change with time. By the 110-th day they become approximately 400 times higher as compared with their values initiated by the external flux at the very beginning (cf. the curves for t = 1 day and t = 100 days).



Fig. 3. The NBW regime initiated by the external neutron flux $j_{ex}=6\cdot10^{11}$ cm⁻² s⁻¹ falling onto the left boundary of FR via the reactor length x (cm): a) the scalar neutron flux $\Phi(x)$ (×10¹⁶ cm⁻² s⁻¹); b) the power density P(x) (kW cm⁻³); c) the ²³⁹Pu concentration $N_{Pu}(x)$ (×10²¹ cm⁻³); d) the fuel burn-up depth B(x) (%) for $t_1=1$, $t_2=70$, $t_3=109$, $t_4=130$ and $t_5=180$ days

At the initial stage during about 70 days, we observe slow changes of the initial distribution of ²³⁹Pu and its

accumulation by means of the transformation of ²³⁸U nuclei that capture neutrons and then suffer two successive β -decays ²³⁹U \rightarrow ²³⁹Np \rightarrow ²³⁹Pu with the characteristic time $t \approx 2.5$ days. Owing to this process that occurs mainly near the right boundary of the ignition zone, the front of the future NBW is gradually formed. In this case, as can be seen from Fig. 3, the allowance for the neutron spectrum alteration effects leads to the space-time behavior of FR qualitatively different from the results of Ref. [7] obtained with the effective one-group cross sections $\sigma_{\alpha}'(x)$ for the initial critical assembly. So after the moment $t \approx 100$ days the space profiles $\Phi(x)$ and P(x) do not practically change during 50...60 days, while the maxima of the corresponding curves move along the *x*-axis.

The NBW velocity V defined as the velocity of shifting the scalar neutron flux maximum is shown in Fig. 4a. The function $\Phi_l(t)$ presented in Fig. 4b is the scalar neutron flux integrated over the reactor length. One can observe a jump of V at the beginning of the NBW propagation, which is caused by the influence of the near ignition zone. After this initial velocity jump a stable propagation of the NBW occurs with an almost constant velocity $V \sim 1.7$ cm/d during about thirty days. This stage of NBW corresponds to the wave profiles for the moment t = 130 days in Fig. 3. The space width of NBW for this case is large enough so that the chosen length of reactor core is relatively small. Owing to this fact, the formed NBW quickly reaches the right boundary of the reactor and the stage of stable motion of the wave is rather short. Then the wave velocity starts to decrease and the stage of slow extinction of the process in the vicinity of the right boundary of the reactor begins. Duration of the stage of extinction is about 60... 70 days. Characteristic NBW profiles for this stage are presented for the moment t = 190 days, which are featured by considerably lower values of the neutron flux and energy production. Thus, in contrast to Ref. [7], the NBW arises and moves with a constant velocity towards the region with low plutonium concentration.

After the extinction of the reactor, plutonium and the nuclear fission products are distributed with a practically uniform concentration over the whole volume of the reactor core, except for the regions close to its boundaries (see Fig. 3). By the extinction moment the fuel burn-up depth reaches a high level of 30...40 % practically in the whole volume of the reactor. It can be seen from Fig. 3 that in the breeding zone an intense accumulation of plutonium occurs and the isotope that burns up is practically ²³⁸U.

It follows from Fig. 4 that the velocity V and integral neutron flux Φ_I calculated without turning the external flux j_{ex} off increase more rapidly than in the case when the source of this flux is turned off after 10 days from the moment of its turning on. The cause of this effect is demonstrated in Fig. 5.

At the moment of turning the external neutron flux off the magnitude of the integral flux in the reactor first decreases. This decrease is followed by an increase of the flux magnitude and then by oscillations that are gradually damped. Therefore, the reactor itself quenches the arising perturbation in approximately 30 days. Thus, the self-organized burning regime that arose before the moment of turning j_{ex} off is stable.

If we increase the external flux intensity by an order of magnitude, then the curves for the wave velocity and integral neutron flux in the FR are similar to those shown



Fig. 4. The NBW velocity V (cm d^{-1}) (a) and the integral neutron flux Φ_l (×10¹⁸ cm⁻¹s⁻¹) (b) versus time (days). Solid curves were calculated without turning j_{ex} off, the dashed ones are for j_{ex} turned off at $t_{off} = 10$ days. The conditions correspond to Fig. 3



Fig. 5. The integral neutron flux $\Phi_l (\times 10^{18} \text{ cm}^{-1} \text{ s}^{-1})$ at the initial stage of burning versus time (days). The solid curve was calculated without turning j_{ex} off and the dashed one is for j_{ex} turned off at $t_{off} = 10$ days. The conditions correspond to Fig. 3

in Fig. 4 but are shifted down in time. Although the perturbation of the integral neutron flux that arises when the more intense external flux is turned off is much greater, the picture of the damped oscillations of this

perturbation is similar to the one shown in Fig. 5. The amplitude and duration of these oscillations are essentially greater than in the previous case, nevertheless the stability of the regime of our interest is not violated. Turning the external flux source off at a much later time moment (after 30 days) does not practically affect the integral neutron flux behavior.



Fig. 6. The spatial distribution of equilibrium ²³⁹Pu concentration N_{eq} (×10²¹ cm⁻³), calculated by formulae (10) (dashed curves) and (11) (solid curves), for t = 20 days (1 and 2) and t = 200 days (3 and 4): (a) without and (b) with allowance for the spectrum alteration

Fig. 6 presents the space distributions of the equilibrium ²³⁹Pu concentration calculated according to formulae (10) and (11). It can be seen that these quantities have a considerable time and space variation. It should be noted that in Fig. 6b the changes of the equilibrium concentration with time occur owing to the decrease of the uranium concentration and to the change of the group neutron spectrum. The results for N_{eq} obtained with the allowance for the spectrum alteration and without it essentially differ at large *t* values (cf. Figs. 6a and 6b). The values of the equilibrium concentrations calculated by formulae (10) and (11) significantly differ from each other due to the violation of the condition of constancy of the ²³⁹U and ²³⁹Np concentrations [10].

In the NBW regime the FR is automatically sustained in a state close to the critical one during a long time despite forming large amount of fission products. The self-organizing NBW regime is determined by the nuclear processes, in which the burn-up of the produced ²³⁹Pu excess occurs instantaneously, whereas the increase of its concentration by means of two successive β -decays of ²³⁹U and ²³⁹Np nuclei is a slow process developing during a long time period. Therefore, the described processes implement an intrinsic reactivity

feedback that ensures the reactor operation stability (see the discussion in [10]).

4. CONCLUSIONS

The feasibility of creating a self-organizing nonlinear regime in the form of a running wave of nuclear burning in a critical FR is studied.

The solution of the corresponding non-stationary problem was found in the framework of effective multigroup diffusion approach. The approach takes into account the variation of the effective one-group cross sections according to the alteration of the group neutron spectrum with time that corresponds to the real situation. This allowed us to describe the peculiarities of the NBW regime in the FR.

The allowance for the neutron spectrum alteration effects leads to qualitative changes of the space-time behavior of the scalar neutron flux, as compared with the results obtained in Ref. [7]. In contrast to [7], the NBW arises in the FR. Its front moves with an approximately constant velocity from the ignition zone boundary towards the region with low plutonium concentration.

In the FR of length 2 m under consideration the selforganizing nuclear burning regime is set lasting for a time period of about 200 days. In the NBW regime FR sustains in a state close to the critical one without any control during a fairly long time. An intrinsic reactivity feedback is governed by the non-linearity of the NBW regime. This feedback prevents the reactor from the runaway regime and ensures the stable evolution of the self-organizing NBW process.

The average fuel burn-up of about 40 % can be attained inside the FR.

The present results show a notable stability of the NBW regime to distortions of the neutron flux.

It should be noted that the values of the neutron flux and density of energy production in the hypothetical FR under consideration are rather high. To obtain more realistic parameters of the FR that would be applicable from the practical point of view, it is necessary to use more complete mathematical models for the description of the NBW regime. Of great interest would also be similar investigations for the FR with the thoriumuranium fuel.

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ИССЛЕДОВАНИЕ САМООРГАНИЗУЮЩЕГОСЯ ВОЛНОВОГО РЕЖИМА ЯДЕРНОГО ГОРЕНИЯ В РЕАКТОРЕ НА БЫСТРЫХ НЕЙТРОНАХ

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В эффективном многогрупповом приближении развит подход для описания пространственно-временной эволюции самоорганизующегося волнового режима ядерного горения в критическом реакторе на быстрых нейтронах. Он основан на решении нестационарного диффузионного уравнения переноса нейтронов совместно с уравнениями выгорания топлива и кинетики предшественников запаздывающих нейтронов. Расчеты проводились в плоской одномерной модели двухзонного гомогенного реактора с металлическим U-Pu топливом, Na-теплоносителем и конструкционным материалом Fe. Температурные эффекты и отвод тепла не рассматривались. Показано, что при определенных условиях в реакторе можно создать волновой режим ядерного горения, в котором реактор без управления в течение длительного времени может поддерживаться в состоянии, близком к критическому.

ДОСЛІДЖЕННЯ ХВИЛЬОВОГО РЕЖИМУ ЯДЕРНОГО ГОРІННЯ, ЩО САМООРГАНІЗУЄТЬСЯ У РЕАКТОРІ НА ШВИДКИХ НЕЙТРОНАХ

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В ефективному багатогруповому наближенні розвинуто підхід для опису просторово-часової еволюції хвильового режиму ядерного горіння, що самоорганізується у критичному реакторі на швидких нейтронах. Він заснований на розв'язанні нестаціонарного дифузійного рівняння переносу нейтронів разом з рівняннями вигоряння палива і кінетики попередників запізнілих нейтронів. Розрахунки проводилися у плоскій одновимірній моделі двохзонного гомогенного реактора з металевим U-Pu паливом, Na-теплоносієм та конструктційним матеріалом Fe. Температурні ефекти і відвід тепла не розглядались. Доведено, що за певних умов у реакторі можна створити хвильовий режим ядерного горіння, у якому реактор без керування протягом тривалого часу може підтримуватися у стані, близькому до критичного.