SPHERICAL DETECTOR DEVICE MATHEMATICAL MODELING WITH TAKING INTO ACCOUNT DETECTOR MODULE SYMMETRY

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Mathematical model for spherical detector device accounting to symmetry properties is considered. Exact algorithm for simulation of measurements procedure with multiple radiation sources developed. Modelling results are shown to have perfect agreement with calibration measurements.

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1. INTRODUCTION

Spherical detector (SD) device created in ISP NPP was successfully used for gamma radiation angular distribution studies for the "Shelter" object (SO) and can be used for the similar purposes on other nuclear power facilities. In our paper we consider mathematical modelling of the gamma radiation angular distribution measurement procedure for multiple radiation sources with different intensities, which are placed at the different distances.

2. MATHEMATICAL MODEL

The proposed method is based on the so-called response function describing SD device detectors values for point source with fixed position. In fact, they constitute vector-function for the source angular coordinates

$$\mathbf{F_i} = \mathbf{F_i}(\theta_i, \varphi_i) \equiv (H_1^{\det}, \ldots, H_{32}^{\det}). \quad (1)$$

Calculation of the response functions provides the way for attenuation coefficients corrention and calibration results verification. Mathematical model of the SD device required for calculations with GEANT-3 software was developed at NSC KIPT and numeric computations were performed.

Response functions obtained by the mathematical modeling can be used for the more precise method of point source angular coordinates measurements and for angular distribution procedure simulation in the case of multiple radiation sources. Significant accuracy improvement requires a considerably large set of response functions for various radiation source positions. Direct calculations for total spatial angle are crucial enough due to large amount of numerical calculations. We can essentially reduce number of calculations taking into account the symmetry of detection module.

Detection module construction implies detector collimating holes placement the icosahedron's vertices (12 holes) and dodecahedron's vertices (20 holes) with entire symmetry corresponding to icosahedron's spatial symmetry group. It follows that applying symmetry transformations to response function for one source position we obtain response function for another source location corresponding to the first location's symmetry transformation

$$\mathbf{F}' = \widetilde{R}\,\mathbf{F}, \quad (2)$$

where $\widetilde{R}$ is as symmetry transformation. Applying symmetry transformation to the detector coordinates we can find the rule for detector's positions interchange during detector module rotations.

So we can limit the set of response functions to those belong to the single icosahedron's face while the others could be obtained using the symmetry transformations (2) from this only set. During our calculations this base face was 1-7-11 (the numbers denote vertices defining the vertex according to Table 1). In the center of this face collimating hole 2 is located.

***Table 1**. Collimation holes angular coordinates*

№	θ, deg.	φ, deg.	№	θ, deg.	φ, deg.
1	0	0	17	100.8	36
2	37.4	0	18	100.8	108
3	37.4	72	19	100.8	180
4	37.4	144	20	100.8	252
5	37.4	216	21	100.8	324
6	37.4	288	22	116.6	0
7	63.4	36	23	116.6	72
8	63.4	108	24	116.6	144
9	63.4	180	25	116.6	216
10	63.4	252	26	116.6	288
11	63.4	324	27	142.6	36
12	79.2	0	28	142.6	108
13	79.2	72	29	142.6	180
14	79.2	144	30	142.6	252
15	79.2	216	31	142.6	324
16	79.2	288	32	180	0

Symmetry transformation R corresponds to some spatial rotation of the detector module. Such rotation [3] is defined by three independent parameters. In our case it is efficient to use 3×3 nonsingular matrix that is coordinate system rotation matrix.

For the response functions calculations we need to develop an algorithm to generate rotation matrix that transforms icosahedron's face 1-7-11 to any other face preserving face's orientation. We shall show this is possible when both center and one of the vertices of the new face are known. Table 2 contains numbers for centers of planes together with corresponding vertices numbers.

Table 2. *Plane centers and corresponding vertices numbers*

Plane №	Center number	Vertex number
1	2	1, 7, 11
2	3	1, 7, 8
3	4	1, 8, 9
4	5	1, 9, 10
5	6	1, 11, 10
6	12	7, 11, 22
7	13	8, 7, 23
8	14	9, 8, 24
9	15	10, 9, 25
10	16	11, 10, 26
11	17	7, 23, 22
12	18	8, 24, 23
13	19	9, 25, 24
14	20	10, 26, 25
15	21	11, 22, 26
16	27	22, 26, 32
17	28	23, 22, 32
18	29	24, 23, 32
19	30	25, 24, 32
20	31	26, 25, 32

Firstly, let us fix the initial coordinate system. Initial coordinate system has the center in the icosahedron's geometrical center with Z-axis directed to hole 1 (see Table 1) and hole 2 belongs to XZ coordinate plane. Let n_z and n_0 denote vector from the detector block's geometrical center to the vertex and to the center of new plane correspondently. From these we can generate three coordinate orts e'_x, e'_y and e'_z for the rotated coordinate system

$$\vec{e}'_z = \frac{\vec{n}_z}{|\vec{n}_z|},\quad \vec{e}'_y = \frac{\vec{e}_z \times \vec{n}_0}{|\vec{e}_z \times \vec{n}_0|},\quad \vec{e}'_x = \vec{e}_y \times \vec{e}_z. \tag{2}$$

Thus we define the new coordinate system that has Z-axis coinciding with n_z and n_0 vector belongs to XZ plane. Rotation matrix R in this case could be constructed from e'_x, e'_y and e'_z orts

$$R = \begin{pmatrix} e'_{x1} & e'_{y1} & e'_{z1} \\ e'_{x2} & e'_{y2} & e'_{z2} \\ e'_{x3} & e'_{y3} & e'_{z3} \end{pmatrix}. \tag{3}$$

Matrix R constructed this way defines transformation from the initial coordinate system to new coordinate system with vector transformation rule

$$r'_i = \sum_{k=1}^{3} R_{ik} r_k, \tag{4}$$

where r_k and r'_i are components of the arbitrary vector in initial and rotated coordinate systems.

According to the definition response function gives detector counts for the point source for the known position relative to detector module. This point source could be thought as a model source. From the above it follows that response function symmetry transformations provide us with response functions for other source's positions being in turn the result of those symmetry transformations. This fact could be used for simulation of measurement procedure with multiple radiation sources.

In the case of multiple sources we need a set of precalculated response functions for the 1-7-11 plane. Using symmetry transformations we can find correspondence between the simulated source and one of the response functions and the inverse transformation gives us detector counts for the source. Total radiation rate from multiple sources is additive so we can obtain it by adding detector counts for each point source. We can use weighted detectors counts to simulate sources with varying intensity. The exact algorithm is presented below.

The following algorithm is applied for each source. Starting parameters are source angular coordinates and its relative intensity. Also we need to switch from angular to Cartesian coordinates for detectors and sources. Here we assume the corresponding points in the Cartesian coordinate system to lie on the unit sphere. Hence we can introduce unit vector s pointing to source and a set of unit vectors $S = \{S_i, i = 1 \ldots m\}$, pointing to the locations that response functions were calculated.

1. Among the detectors located in plane centers (see Table 2) find the one closest to the given source. The closeness criterion is distance minimum between source and plane center. Cartesian coordinates of the selected vertex give us vector n_z.
2. From the Table 2 choose vertex detectors for the plane selected on the previous step and find the closest to source. Cartesian coordinates of the selected vertex give us vector n_0.
3. Using (3) and (4) construct e'_x, e'_y and e'_z orts and transformation matrix R.
4. Apply transformation R to vector s according to (4) to obtain rotated vector s', which belongs to 1-7-11 plane.
5. Using minimum distance criterion select from the set S the closest vector s''. This one will be prototype for the source.
6. Using the inverse transformation R^{-1} on vector s'' get the model source coordinates and the corresponding detectors counts.
7. Multiply model detectors counts obtained on the previous step by the model source relative intensity and add to the resulting data.

The result of this algorithm applied to all sources it total response function for the set of point sources with known positions and relative intensities. For the actual calculations this algorithm was implemented using C++ language.

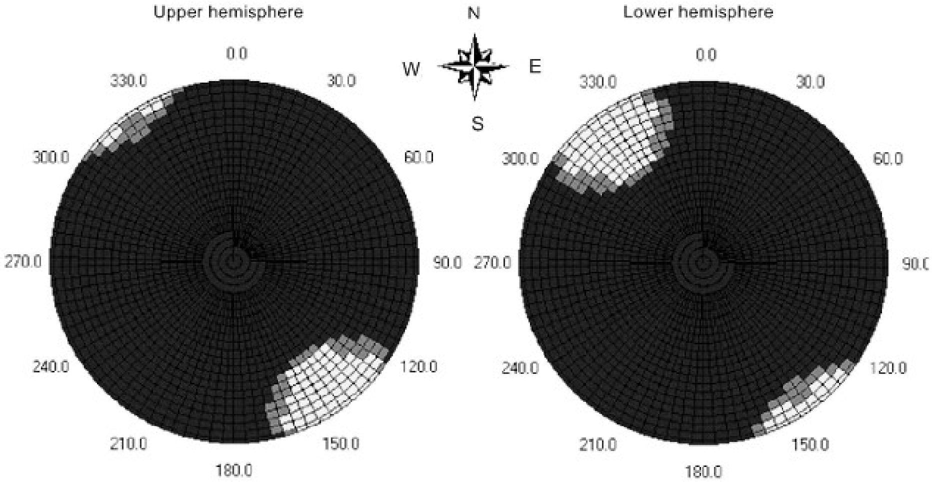

Fig. 1. *The result of modeling for two sources*

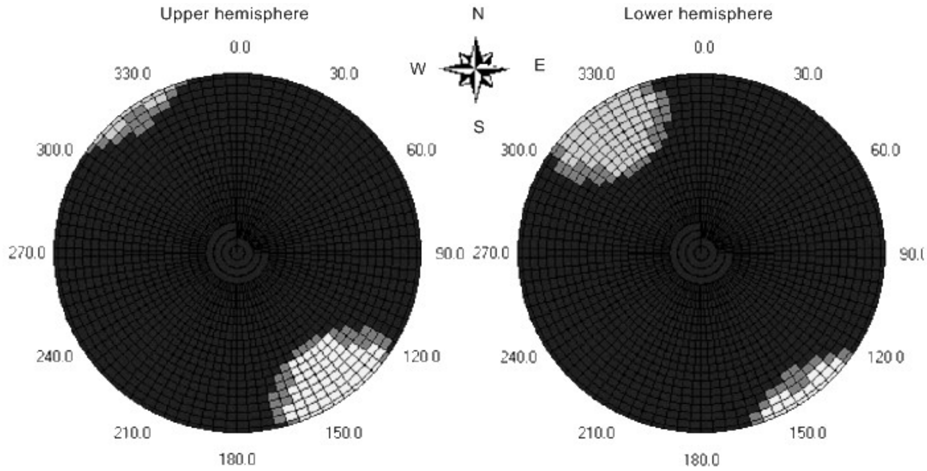

Fig. 2. *The result of calibration measurements for two sources*

3. RESULTS AND CONCLUSIONS

For testing purposes we have performed calculations for the same sources positions as those used for calibration measurements (see Figs. 1,2). Perfect agreement was achieved proving the correctness of developed modeling method. Thus one can apply it for more accurate SD device calibration procedure, detector module angular resolution survey and for SD device certification procedure.

REFERENCE

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ШАРОВОГО ДЕТЕКТОРА С УЧЕТОМ СИММЕТРИИ ДЕТЕКТОРНОГО БЛОКА

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Рассмотрено математическое моделирование шарового детектора с учетом свойств симметрии. Представлен последовательный алгоритм моделирования процедуры измерения при наличии нескольких источников излучения. Показано, что результаты моделирования имеют хорошее согласие с калибровочными измерениями.

МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ШАРОВОГО ДЕТЕКТОРА З УРАХУВАННЯМ СИМЕТРІЇ ДЕТЕКТОРНОГО БЛОКУ

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Розглянуто математичне моделювання шарового детектора з урахуванням властивостей симетрії. Представлено послідовний алгоритм моделювання процедури вимірювань за наявності декількох джерел випромінювання. Показано, що результати моделювання добре узгоджуються з калібрувальними вимірюваннями.