1. INTRODUCTION

One of the pronounced dynamic effects in the dispersion of free X-rays in a crystal is the effect of abnormal law of photoabsorption. For the first time this effect has been experimentally found out in the experiments on free X-rays dispersion in a crystal by G. Borrmann. The physics of this effect consists in formation of a standing wave by incident and scattered waves, which antinodes are located in the middle of space between next nuclear planes, where the density of atomic electrons (and, accordingly, the photoabsorption) is minimal. Under these conditions two waves are formed in the crystal, one of which is abnormal strong and another one is abnormal weak absorbed into the crystal. The linear absorption factor for both waves looks as follows [2]

\[ \eta = \eta_0 \left( \frac{1 \pm C(s) \chi_{g}^{\infty}}{\chi_{0}^{\infty}} \right), \]  

(1)

where sign "+" corresponds to the abnormal strong absorption, and "−" to abnormal weak absorption, \( \chi_{g}^{\infty} \) and \( \chi_{0}^{\infty} \) are imaginary parts of factors in expansion of the dielectric susceptibility in the Fourier series by reciprocal lattice vectors \( g \). For \( \sigma \)-polarization \( C^{(1)} = 1 \) for \( \pi \)-polarization \( C^{(2)} = |\cos 2\theta_B| \). \( \theta_B \) is the angle between an incident X-ray beam and a set of crystal planes (Bragg angle). It is obvious from the formula (1) that Borrmann effect manifests itself for \( \sigma \) -polarization more brightly. The necessary condition of observation of this effect is \( \chi_{g}^{\infty} \cdot C^{(s)} / \chi_{0}^{\infty} = 1 \).

The question of defining the existence of the similar effect in the parametric X-ray radiation (PXR) [3-5] realized in pseudo-photons Bragg diffraction process of Coulomb fields of a fast particle moving in a crystal is of importance. This task is interesting, in connection with a problem of creation of quasi-monochromatic tunable alternative sources of X-ray radiation, because the photoabsorption suppression would allow an essential increase in the intensity of the quasi-monochromatic X-ray sources based on the PXR mechanism. In Ref. [6] the influence of Borrmann effect on diffracted transitive radiation (DTR) was considered with Laue geometry. The influence of Borrmann effect on the PXR characteristics was studied in [7-8]. In the present paper the conditions of experimental observation of Borrmann effect in the X-radiation of relativistic particles in a crystal are investigated in Bragg geometry. Using the dynamic approach [9], expressions for the DTR and PXR amplitudes are extracted from a radiation amplitude general formula. The DTR contribution to the radiation is investigated depending upon orientation of the in-surface of the thick crystal. The contribution of diffracted transition radiation is investigated depending on a cut angle of the input surface of the plate-shaped single crystal target.

2. SPECTRAL-ANGULAR DISTRIBUTION OF PXR AND DTR OF AN RELATIVISTIC ELECTRON IN DYNAMIC APPROXIMATION

Let us consider the radiation of a fast charged particle moving with constant velocity \( \mathbf{v} \) through a crystal of a thickness \( L \) (Fig.1). Let us consider the equations for Fourier direct image for an electromagnetic field. Since the Coulomb field of an ultrarelativistic particle can be transverse with a good degree of accuracy, an incident \( E_0 \) and diffracted \( E_1 \) electromagnetic waves are defined by two amplitudes with different values of transverse polarization

\[ E_0 = E_0^{(1)} e_0^{(1)} + E_0^{(2)} e_0^{(2)}, E_1 = E_1^{(1)} e_1^{(1)} + E_1^{(2)} e_1^{(2)}. \]  

(2)

The unit vectors \( e_0^{(1)}, e_0^{(2)}, e_1^{(1)}, e_1^{(2)} \) are chosen in such a way. The vectors \( e_0^{(1)} \) and \( e_0^{(2)} \) are perpendicular to the vector \( k \), and the vectors \( e_1^{(1)} \) and \( e_1^{(2)} \) are perpendicular to the vector \( k + g \). The vectors \( e_0^{(2)}, e_1^{(2)} \) are situated on a plane of vectors \( k \) and \( k + g \).

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polarization), while the vectors \( e_0^{(1)} \) and \( e_1^{(3)} \) are perpendicular to this plane (\( \vec{\sigma} \) -polarization); \( g \) is a reciprocal lattice vector defined as a set of reflecting atomic planes.

![Fig. 1. The radiation process geometry and a system of notation](image)

Using the two-wave approximation of the dynamic theory of diffraction [2], we will write down a well-known system of equations for Fourier transform images for intensity of the incident and diffracted waves

\[
\begin{align*}
\omega^2(1 + I_0) - k^2)E_0^{(s)} + \frac{\omega}{2} I_0 C^{(s)} E_1^{(s)} &= \frac{4\pi}{2\pi} \delta (\omega - kV), \\
\omega^2 I_0 C^{(s)} E_0^{(s)} + (\omega^2 (1 + I_0) - (k + g)^2)E_1^{(s)} &= 0 , \quad (3)
\end{align*}
\]

where \( I_g = I_{01} = I_{10} = I_{gg} = \frac{\omega^2}{2} + iI_g \) (for a symmetrical crystal), \( I_0 = I_{00} + 2I_g = \frac{\omega^2}{2} + iI_0 \), \( C^{(s)} = e_0^{(s)} e_1^{(s)} \), \( P^{(s)} = \frac{e_0^{(s)}}{\rho} \), \( \rho = k - \frac{\omega V}{V} \) is a momentum component of a virtual photon, perpendicular to the particle velocity \( V \) (\( \rho = \frac{\omega \vec{\sigma}}{V} \), where \( \vec{\sigma} \ll 1 \) is an angle between the vectors \( k \) and \( V \), \( g = \frac{2\omega g \sin \theta}{V} \) - the reciprocal lattice vector defined as the set of reflecting layer atomic planes. The solution to the equation system (3) in the crystal looks like:

for the diffracted field

\[
E_1^{(s)} = \frac{8\pi}{2\pi} \frac{2ieV}{\rho} \frac{\chi_y}{\chi_z} \frac{\rho P^{(s)} C^{(s)}}{Z} \delta (\rho V) , \quad (4a)
\]

for the incident field

\[
E_0^{(s)} = \frac{8\pi}{2\pi} \frac{2ieV}{\rho} \frac{\chi_y}{\chi_z} \frac{P^{(s)} C^{(s)}}{Z} \delta (\rho V) , \quad (4b)
\]

and for the field in a vacuum

\[
E_{\text{vac}}^{(s)} = \frac{8\pi}{2\pi} \frac{2ieV}{\rho} \frac{\chi_y}{\chi_z} \frac{P^{(s)} C^{(s)}}{Z} \delta (\rho V) , \quad (4c)
\]

where

\[
Z = i 2^2 C^{(s)} \left( I_{00} - \gamma^2 - \rho^2 \right) \left( I_{00} - \gamma^2 - \rho^2 \right) \left( 1 - 2\gamma^2 - \rho^2 - k^2 \right) ,
\]

\( \gamma = (1 - \nu^2)^{1/2} \) is Launrce factor of the particle, \( P^{(s)} = \sin \gamma \), \( \rho P^{(s)} = \cos \gamma \), \( \theta \) is the angle counted from the plane formed by the vectors \( V \) and \( g \).

Let us consider the case of dispersion in geometry of Bragg, when the diffracted field in Bragg direction comes out through the input surface of the crystal. For this purpose it is necessary for the angle between the vector \( g + \omega V/V^2 \) and the normal to the crystal outer surface to be obtuse. The diffracted radiation passes along the vector \( g + \omega V/V^2 \). The amplitude of the radiation field in considered geometry is

\[
E^{(s)} = E_1^{(s)} + (E_{\text{vac}}^{(s)} - E_0^{(s)}) Q^{(s)} , \quad (5)
\]

where

\[
Q^{(s)} = - \frac{\beta \pm (\beta^2 - 4\nu g C^{(s)} \epsilon)}{2\pi g C^{(s)} ,}
\]

\[\beta = \chi - (\gamma^2 - 2\gamma (1 + \epsilon), \quad \chi = \frac{1}{\gamma^2 - 2\gamma} (k^2 + C)^2) ,\]

\[\epsilon = \frac{\cos(\Psi g)}{\sin(\theta - \delta)} + \frac{\sin(\theta + \delta)}{\sin(\theta + \delta)} \]

\( \Psi_g \), \( \Psi_g \) are the angles between the wave vectors of the incident wave, back wave and the external normal to the crystal. The amplitude reflection coefficient of the X-waves field by the X-waves in the crystal. Sarsity reflection coefficient of the X-waves field by the X-waves in the crystal.

\[E^{(s)} = E^{(s)}_{\text{PX}} + E^{(s)}_{\text{DTR}} \]

\[E^{(s)}_{\text{PX}} = \frac{8\pi}{2\pi} \frac{2ieV}{g} \frac{\rho P^{(s)} C^{(s)} \left( I_{00} - \gamma^2 - \rho^2 \right) \left( 1 - 2\gamma^2 - \rho^2 - k^2 \right) }{Z} , \quad (6a)
\]

\[E^{(s)}_{\text{DTR}} = \frac{8\pi}{2\pi} \frac{2ieV}{g} \frac{P^{(s)} C^{(s)} \left( I_{00} - \gamma^2 - \rho^2 \right) \left( 1 - 2\gamma^2 - \rho^2 - k^2 \right) }{Z} Q^{(s)} , \quad (6c)
\]

Spectral-angular radiation distributions can be written as [9]

\[
d^2W/d\theta d\Omega = \frac{\omega^2}{2(2\pi)^{\nu}} \sum_{s} |E^{(s)}|^2 . \quad (7)
\]

If we substitute the radiation amplitudes (6,b) and (6,c) into (7) we will get expressions for the PXR and DTR spectral-angular distributions.
\[
\frac{d^2W^{\text{PXR}}}{d\theta d\varphi} = \frac{e^2}{\pi^2} \theta^2 \sum_{x=1}^{\nu} P^{(x)} \left( \chi_x \left( C^{(x)}(\theta,\varphi) \sqrt{\chi_x C^{(x)}} \right)^2 + \frac{1}{\gamma^2 + \theta^2} \right),
\]
\[
\frac{d^2W^{\text{DTR}}}{d\theta d\varphi} = \frac{e^2}{\pi^2} \theta^2 \sum_{x=1}^{\nu} P^{(x)} \left( 1 - \frac{1}{\gamma^2 + \theta^2} \right),
\]
Equations (8,a) and (8,b). As 
\[
\frac{2\chi_x \chi_y \chi_z}{\chi_x \chi_y \chi_z C^{(x)}} - \frac{\chi_x \chi_y \chi_z (\theta^2 - \theta_0^2 - \theta_0^2 - a)}{\chi_x \chi_y \chi_z (\theta^2 - \theta_0^2 - \theta_0^2 - a)} \approx \frac{\chi_x \chi_y \chi_z}{\chi_x \chi_y \chi_z} \ll 1,
\]
we will take the well-known approximation 
\[
\left( \frac{\chi_x \chi_y \chi_z}{\chi_x \chi_y \chi_z C^{(x)}} - \frac{\chi_x \chi_y \chi_z (\theta^2 - \theta_0^2 - \theta_0^2 - a)}{\chi_x \chi_y \chi_z (\theta^2 - \theta_0^2 - \theta_0^2 - a)} \right)^2 + \left( 2\chi_x \chi_y \chi_z C^{(x)} - \chi_x \chi_y \chi_z (\theta^2 - \theta_0^2 - \theta_0^2 - a) \right)^2 \approx \frac{\chi_x \chi_y \chi_z}{\chi_x \chi_y \chi_z} \cdot \delta \left( \chi_x \chi_y \chi_z C^{(x)} - \chi_x \chi_y \chi_z (\theta^2 - \theta_0^2 - \theta_0^2 - a) \right).
\]
From (8,a) we can obtain the expression for the radiated X-ray photons number:
\[
\frac{d^2N^{\text{PXR}}}{d\theta d\varphi} = \frac{e^2}{\pi^2} \theta^2 \sum_{x=1}^{\nu} P^{(x)} \left( \frac{\theta + \gamma^2}{k_0} + 1 - \frac{\chi_x \chi_y \chi_z C^{(x)}}{\chi_x \chi_y \chi_z} \right)^2 + \left( 1 - \frac{\chi_x \chi_y \chi_z C^{(x)}}{\chi_x \chi_y \chi_z} \right)^2 \frac{\chi_x \chi_y \chi_z}{\chi_x \chi_y \chi_z} \cdot \delta \left( \chi_x \chi_y \chi_z C^{(x)} - \chi_x \chi_y \chi_z (\theta^2 - \theta_0^2 - \theta_0^2 - a) \right).
\]
(10)

While processing the formula (10) it is necessary to take into consideration that the inequality 
\[
\gamma \geq \theta^2 + \gamma^2 - \theta_0^2 > \chi_x \chi_y \chi_z C^{(x)}
\]
must be hold true.

It is convenient to write the spectral-angular distribution of diffracted transition radiation photon numbers as:
\[
\frac{d^2N^{\text{DTR}}}{d\theta d\varphi} = \frac{e^2}{\pi^2} \theta^2 \sum_{x=1}^{\nu} P^{(x)} \left( 1 - \frac{\chi_x \chi_y \chi_z C^{(x)}}{\chi_x \chi_y \chi_z} \right)^2 \left( 1 - \frac{\chi_x \chi_y \chi_z C^{(x)}}{\chi_x \chi_y \chi_z} \right)^2 \frac{\chi_x \chi_y \chi_z}{\chi_x \chi_y \chi_z} \cdot \delta \left( \chi_x \chi_y \chi_z C^{(x)} - \chi_x \chi_y \chi_z (\theta^2 - \theta_0^2 - \theta_0^2 - a) \right).
\]
(11)

Where,
\[
\gamma = \frac{\chi_x \chi_y \chi_z}{\chi_x \chi_y \chi_z} \cdot \delta \left( \chi_x \chi_y \chi_z C^{(x)} - \chi_x \chi_y \chi_z (\theta^2 - \theta_0^2 - \theta_0^2 - a) \right).
\]
(12)

Since within the X-ray frequency region \(2\sin^2 \theta_0 \frac{\chi_x \chi_y \chi_z C^{(x)}}{\chi_x \chi_y \chi_z} > 1\) holds true, \(\gamma \) depends strongly on frequency, and thus it is convenient to regard this variable as a spectral one. Note, all the terms proportional to \(\gamma^2\) are omitted in (11) as the X-ray region is in consideration.

### 3. NUMERICAL CALCULATIONS OF PXR INTENSITY OF THE RELATIVISTIC ELECTRON IN A SILICON CRYSTAL

The dynamical formulas (10), (11) can be used for studies into the Borrmann effect conditions in the PXR and contribution of the DTR of relativistic electrons in the single crystals. Like in the case of free X-rays in the crystals, the necessary conditions for the Borrmann effect in the PXR is the closeness of imaginary parts of a corresponding expansion coefficient of crystal dielectric susceptibility to the reciprocal lattice vectors \(\chi_x \chi_y \chi_z C^{(x)}/\chi_x \chi_y \chi_z \) to unity, the stronger dynamical effect manifests itself in the PXR. As it was pointed above, these effects are better observed in the case of \(\sigma\)-polarization because in this case \(C^{(x)} = 1\). For the Borrmann effect manifestation it is necessary to produce the PXR on one of the main set of the crystal plane, i.e. the condition \(\chi_x \chi_y \chi_z C^{(x)}/\chi_x \chi_y \chi_z = 1\) should hold true. Because the denominator in the PXR angle distribution formula decreases under the conditions of the Borrmann effect manifestation and, ideally, approaches \(\theta^2 + \gamma^2 \cdot |\chi_x \chi_y \chi_z|\), the maximum of the radiation angle density will shift towards small values of the angle \(\theta\). The angle density maximum of the PXR will be below \(\theta = \gamma^{-1}\), just as the transition radiation maximum. It becomes apparent that the Borrmann effect can be considered as a suppression of the density effect in the PXR.

The numerical calculations of the PXR angular density were carried out for (220)-set of the Si crystal planes, which best of all satisfy the conditions pointed above.
The curves of the radiation density angle dependences plotted using the formula (10) and integrated over a frequency $\omega$ for $\sigma$-polarization are presented in Fig.2. This figure demonstrates a considerable increase (growth) in the radiation angular density under the condition of Borrmann effect.

The values for parameters of the reflecting atomic planes of the crystal, $I_{\theta}^{-\epsilon} C(x) / I_0^{-\epsilon}$ and $I_{\theta}^{+\epsilon} C(x) / I_0^{+\epsilon}$, were taken from Z. Pinsker monograph [2].

4. THE CRYSTAL INPUT SURFACE INFLUENCE ON X-RADIATION SPECTRAL-ANGULAR DISTRIBUTION

Found was a rather interesting dependence of the X-radiation characteristic of relativistic electrons in the single crystal on the orientation angle of input surface of plate-shaped crystal target under fixed angle of the relativistic electron incidence on the set of atomic plane in the crystal. The orientation of the crystal input surface is defined by the parameter $\epsilon = \sin(\theta_B - \delta) / \sin(\theta_B + \delta)$.

Under the fixed value of $\theta_B$, as the electron incidence angle $\theta_B + \delta$ decreases, the parameter $\delta$ becomes negative and its absolute value increases leading to an increase in parameter $\epsilon$. Opposite, while the angle $\theta_B + \delta$ increases, the parameter $\epsilon$ decreases (in the extreme case $\delta \rightarrow \theta_B$). The curves describing the PXR angle density plotted using the formula (10) for different orientations of the target input surface are presented in Fig.3.

Fig.2. The Borrmann effect influence on PXR angle distribution. The lower curve – angular density of the radiation dependent on the angle $\theta_i$ without taking the Borrmann effect into consideration; upper curve – the radiation angle distribution with the Borrmann effect considered. $\theta_\perp = \theta \sin \phi$.

As the parameter $\epsilon$ increases (at the decrease in the incident electron-input surface angle), the amplitude reflection coefficient decreases and the total reflection region of frequencies increases (see Fig.5).

Fig.3. The PXR angular density for different orientations of the crystal input surface under condition of Borrmann effect manifestation

One can see that the angle density of PXR under the Borrmann effect conditions depend on the input surface orientation. However, the dependence of DPR angular density on the parameter $\epsilon$ manifests itself more essentially that is shown by the curves presented in Fig.4 plotted using the formula (11) previously integrated over frequency the $\zeta(\theta)$. It can be explained by the total reflection region modification as the angle between the electron incidence direction and the crystal target input surface changes.

As the parameter $\epsilon$ increases (at the decrease in the incident electron-input surface angle), the amplitude reflection coefficient decreases and the total reflection region of frequencies increases (see Fig.5).

Fig.4. The angular density of DTR for different orientations of the crystal input surface

As the parameter $\epsilon$ increases (at the decrease in the incident electron-input surface angle), the amplitude reflection coefficient decreases and the total reflection region of frequencies increases (see Fig.5).

Fig.5. The dependences of the reflection coefficient amplitude modulus squared versus frequency for different orientations of the crystal input surface

It is easily to show that the width of the total reflection region is defined by $2\sqrt{\epsilon}$. So, if the value of $\epsilon$ is small, the angular density values of DTR and PXR will...
become comparable, and this fact is shown in Fig.6. for Ge crystal as an example.

![Fig. 6. The angular density of PXR and DTR of a relativistic electron in Ge crystal for different orientations of the crystal input surface under the Bormann effect conditions.](image)

**CONCLUSION**

Thus, in this work the analytical expressions for the spectral-angular density of PXR and DTR of a relativistic electron are obtained; the conditions for the Bormann effect manifestation in the relativistic electron parametric radiation in a single crystal are defined taking into account the orientation of semi-infinite crystal input surface; numerical calculations of the angle density of the relativistic electron coherent radiation are carried out for the set of atomic plane (220) of Si crystal; the angular density of diffracted transition radiation is studied by investigating the dependence of cut angle of the crystal target input surface as well. The results obtained allow estimating the opportunities for experimental observation of the Bormann effect in the PXR.

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