

# QUANTUM FIELD METHODS IN THE ELECTRON COOLING

*O. V. Khelemelya\**, *R. I. Kholodov*,

*Institute of Applied Physics NAS of Ukraine, 40000, Sumy, Ukraine*

(Received Marth 22, 2013)

Quantum field methods are proposed to describe the electron cooling process. The influence of the magnetic field and the anisotropy of the temperature distribution of the electron gas are considered in the framework of quantum field theory. It is shown the longitudinal component of the thermal motion of electrons acts the main role in the electron cooling of heavy ions in the presence of a sufficiently strong magnetic field. Quantum effects in the electron cooling are estimated.

PACS: 52.30.-q, 52.25.Xz

## 1. INTRODUCTION

In 1967 G.I. Budker [1] proposed the electron cooling method of charge massive particle beam (damp volume space).

One combines of moving “hot” ion and “cold” electron beams at a some location of the storage ring. Due to the Coulomb interaction beams temperatures equalized then “warmed up” electrons are removed from the system drive. The cooled ion beam continues to move in a storage ring.

In modern facilities, such HESR (High Energy Storage Ring, Germany, FAIR Collaboration), electron cooling method is a necessary part of the process of accumulation of antiprotons. Charmonium spectroscopy, which is one of the main items in the experimental program of HESR, requires antiproton momentum up to 8.9 GeV/c with a resolution  $\Delta p/p \sim 10^{-5}$ . This can be achieved only with electron cooling.

Originally Coulomb theory of binary collisions is used to describe the process of electron cooling. The advantage of this method is evident in the construction of the theory, but the total result can be obtained only by means of numerical simulations [2, 3, 4].

Also, the friction force can be obtained by the methods of plasma physics (dielectric model). Here the properties of the medium are given by the dielectric constant of the plasma, which determines the form of the expression for the energy loss of ions moving in the electron gas [5].

The theory of binary collisions and the dielectric model have a significant disadvantage associated with the introduction of empirical values such as the cutoff parameters in the Coulomb logarithm.

The quantum field theory method is devoid of this disadvantage, the Coulomb logarithm contains values determined from the first principles.

The question of energy loss of the interaction charged particles with a plasma without a magnetic field was investigated in the framework of quantum field theory by Larkin [6].

The interaction of a nonrelativistic charged particle with a plasma in a magnetic field was described within quantum field theory in Akhiezer’s work [7]. The energy losses were obtained with approximation of zero temperature.

In this paper the effect of the anisotropic electron-gas temperature on the energy losses are taken into account in the linear approximation.

## 2. ENERGY LOSSES OF PARTICLE IN A MAGNETIZED ELECTRON GAS

Let consider the basic steps of quantum field theory to the description of electron cooling.

The Hamiltonian of the system of charged particles interacting by Coulomb’s law with the passing through of the particles, can be written as

$$H = H_0 + H'(t), \quad (1)$$

where  $H_0$  is the main Hamiltonian of nonperturbative system of plasma particles, the second term  $H'(t)$ , the Hamiltonian of interaction, describes the perturbation introduced by the projectile particle:

$$H'(t) = \int d\vec{r} J_0(\vec{r}, t) a_0(\vec{r}, t), \quad (2)$$

$$a_0(\vec{r}, t) = (4\pi)^{-1} \int d\vec{r}' |\vec{r} - \vec{r}'|^{-1} j_0(\vec{r}', t).$$

$a_0$  is operator of a scalar potential,  $j_0$  and  $J_0$  are operators of the charge density of the system and of the projectile particle, respectively, the operators are taken in the interaction representation. The elements

\*Corresponding author E-mail address: xvdm@mail.ru

of the scattering matrix

$$S = T \left\{ -i \int_{-\infty}^{\infty} H'(t) dt \right\} \quad (3)$$

connects the various states of the original system and external particle. These states are characterized by the quantum numbers  $\alpha, n$ , where  $\alpha \equiv (\nu, p_z, q)$  are quantum numbers of the projectile particle in a magnetic field  $\vec{H}$  and  $n$  is set of quantum numbers describing the state of the environment with a certain energy  $E_n$  and a certain number of particles  $N_n$ .

We assume the speed of the moving particle  $V$  is large enough that  $(e^2 V^{-1} \hbar^{-1} \ll 1)$  its interaction with the particles of the medium can be considered by perturbation theory. In the linear approximation in  $H'(t)$  probability of transition from the initial state  $\alpha, n$  to the final state  $\alpha', n'$  has the form

$$W_{if} = 2\pi \delta(E_i - E_f) |H'|^2 \quad (4)$$

or

$$W_{if} = 2\pi \delta(E_i - E_f) \int d\vec{r} d\vec{r}' \langle \alpha' | \hat{J}_0(\vec{r}) | \alpha \rangle, \quad (5)$$

$$\langle \alpha | \hat{J}_0(\vec{r}') | \alpha' \rangle \langle n' | \hat{\varphi}(\vec{r}) | n \rangle \langle n | \hat{\varphi}(\vec{r}') | n' \rangle,$$

where  $E_n - E_{n'} + \varepsilon_\alpha - \varepsilon_{\alpha'} \equiv \varepsilon_{\nu, p_z} = \eta(\nu + 1)m/M + p_z/2M$  is the energy of projectile particle,  $M$  is its mass,  $\eta = eH/mc$  is the Larmor frequency of electron in magnetic field  $\vec{H}$  ( $\hat{\alpha}_0, \hat{J}_0$  are operators in the Schrodinger representation).

Let sum expression for the probability of the final states environment and average over the initial states of the density matrix  $\rho_0 = \exp\{\beta(\Omega + \mu N - E_n)\}$  to get the full transition probability of a particle from a state with energy  $\varepsilon_{\nu, p_z}$  in a state of the energy  $\varepsilon_{\nu', p'_z}$

$$W_{\alpha, \alpha'} = \sum_n \exp\{\beta(\Omega + \mu N - E_n)\} \sum_{n'} W_{if}. \quad (6)$$

Passing to Fourier components

$$W_{\alpha, \alpha'} = 2\pi \int d^3 k \Phi(\vec{k}, \varepsilon_\alpha - \varepsilon_{\alpha'}) U(\vec{k}), \quad (7)$$

where  $\Phi(\vec{k}, \varepsilon_\alpha - \varepsilon_{\alpha'})$  and  $U(\vec{k})$  are components of Fourier function,

$$\Phi(\vec{r}_1 - \vec{r}_2, \omega) = \sum_{n, n'} \exp\{\beta(\Omega + \mu N - E_n)\} \times \quad (8)$$

$$\times \langle n' | \hat{\alpha}_0(\vec{r}_1) | n \rangle \langle n | \hat{\alpha}_0(\vec{r}_2) | n' \rangle \delta(E_n - E_{n'+\omega}),$$

$$U(\vec{r}_1 - \vec{r}_2) = \sum_{q, q'} \langle \alpha' | \hat{J}_0(\vec{r}_1) | \alpha \rangle \langle \alpha | \hat{J}_0(\vec{r}_2) | \alpha' \rangle.$$

The energy losses per unit of time of the particle is connected with the probability

$$-\frac{dE}{dt} = \sum_{\alpha'} (\varepsilon_\alpha - \varepsilon_{\alpha'}) W_{\alpha, \alpha'}. \quad (9)$$

The final form of this equation can be written as

$$-\frac{dE_{\nu, p}}{dt} = \frac{2e^2 m \omega_B}{(2\pi)^2} \sum_{\nu'} \int_{-\infty}^{\infty} \frac{\omega d\omega}{(1 - e^{-\beta\omega})} \times \quad (10)$$

$$\times \int \frac{d^3 k}{k} \Lambda_{\nu, \nu'} \left( \frac{k_t}{\sqrt{2m\omega_B}} \right) \times$$

$$\times Im \frac{\kappa(\vec{k}, \omega)}{1 + \kappa(\vec{k}, \omega)} \delta(\varepsilon_{\nu, p} - \varepsilon_{\nu', p-k_z} - \omega),$$

where the dielectric susceptibility  $\kappa(\vec{k}, \omega)$  can be determine through polarized operator:

$$\kappa(\vec{k}, i\omega) = -k^{-2} P(\vec{k}, i\omega). \quad (11)$$

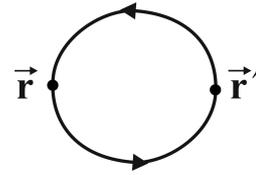
The graphic technique is applied to calculate the polarization operator. In the one-loop approximation (first Born approximation) the polarization operator are described by the Feynman diagram shown in Fig. 1.

$$P(\vec{r} - \vec{r}', i\omega) = \frac{2e^2}{\beta} \sum_{p_4} G(\vec{r}, \vec{r}', p_4) (\vec{r}', \vec{r}, p_4 - i\omega), \quad (12)$$

where  $G(\vec{r}, \vec{r}', p_4)$  is Green's function of an electron in a magnetic field that has the form

$$G(\vec{r}, \vec{r}', p_4) = \sum_{\alpha} \Psi_{\alpha}(r_1) \frac{1}{\varepsilon_{\alpha} - \mu + ip_4} \Psi_{\alpha}^*(r_2), \quad (13)$$

where  $\Psi_{\alpha}(r)$  is the wave function of a particle in a magnetic field. The factor of inverse temperature  $\beta = 1/T$  are contained in the resulting formula for the energy loss (10).



**Fig. 1.** The Feynman diagram of the polarization operator in the one-loop approximation

Special function  $\Lambda_{\nu, \nu'}(a)$  is

$$\Lambda_{\nu, \nu'}(a) = \int_0^{\infty} ds J_0(2\sqrt{as}) L_{\nu}(s) L_{\nu'}(s) \exp(-s), \quad (14)$$

where  $a = \frac{(\hbar k_t)^2}{2m\hbar\omega_B}$  is the ratio of the transverse energy  $\frac{(\hbar k_t)^2}{2m}$  and the distance between adjacent Landau levels  $\hbar\omega_B$ ,  $L_{\nu}(x) = \frac{e^{-x}}{n!} \frac{d^n}{dx^n} (e^{-x} x^n)$  is Laguerre polynomial,  $J_0(x)$  is Bessel function.

In the case of strong magnetic fields ( $a \ll 1$ )

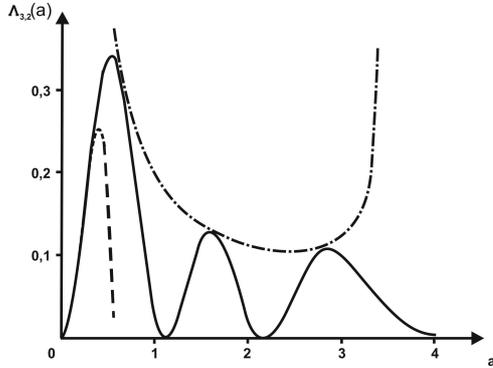
$$\begin{aligned}
\Lambda'_{\nu,\nu'} &\approx \delta_{\nu,\nu'} + \\
a [(\nu+1)\delta_{\nu+1,\nu'} - (2\nu+1)\delta_{\nu,\nu'} + \nu\delta_{\nu-1,\nu'}] &+ \\
\frac{a^4}{4} [(\nu+1)(\nu+2)\delta_{\nu+2,\nu'} - 4(\nu+1)^2\delta_{\nu+1,\nu'} + \\
2(3\nu^2+3\nu+1)\delta_{\nu,\nu'} - 4\nu^2\delta_{\nu-1,\nu'} + \\
\nu(\nu-1)\delta_{\nu-2,\nu'}], & \quad (15)
\end{aligned}$$

where  $\delta$  is the delta function [10].

In the case of weak magnetic fields ( $a \gg 1$ ):

$$\Lambda''_{\nu,\nu'} \approx \frac{m\hbar\omega_B}{\pi\Delta}, \quad (16)$$

where  $\Delta$  is the area of the triangle which was built on the segments  $\hbar k_{\perp}$ ,  $p_{\perp} = 2m\hbar\omega_B(\nu + 1/2)$ ,  $p'_{\perp} = 2m\hbar\omega_B(\nu' + 1/2)$ .



**Fig.2.** The function  $\Lambda_{\nu,\nu'}(a)$  (solid curve), its asymptotic for  $a \ll 1$  (dash-dotted curve) and for  $a \gg 1$  (dotted curve) with numbers of Landau levels  $\nu = 3$  and  $\nu' = 2$

In Fig.2. the explicit form of special functions  $\Lambda_{\nu,\nu'}(a)$  (14) and its approximations in the case of strong  $\Lambda'_{\nu,\nu'}(a)$  (15) and weak  $\Lambda''_{\nu,\nu'}(a)$  (16) magnetic fields for the numbers of Landau levels  $\nu = 3$  and  $\nu' = 2$  are presented [10].

### 3. THE APPROACHING OF ZERO TEMPERATURE OF ELECTRON GAS

If the electron temperature is  $T = 0$  in [7], a simple expression for an arbitrary angle of the projectile particle relative to the external magnetic field of arbitrary strength was obtained.

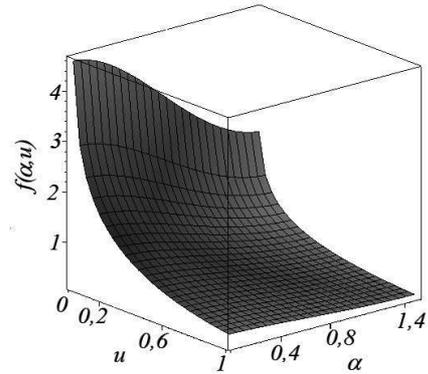
$$-\frac{dE_p}{dt} = \frac{e^2\omega_p^2}{4\pi V} \left[ L_C - f\left(\alpha, \frac{\omega_p}{\omega_B}\right) \right], \quad (17)$$

where  $L_C = \ln\left(\frac{2mMV^2}{(M+m)\omega_B}\right)$  is Coulomb logarithm, the addition term to the friction force relating to the

influence of the magnetic field is written as

$$\begin{aligned}
f\left(\alpha, \frac{\omega_p}{\omega_B}\right) &= \frac{1}{\pi} \left(\frac{\omega_p}{\omega_B}\right)^2 \left\{ \int_0^{z_1} g(z)dz - \int_{z_2}^{z_3} g(z)dz \right\}, \\
g(z) &= \frac{z(1-z)}{\sqrt{z(z-z_1)(z-z_2)(z_3-z)}}, \\
z_{1,2} &= \frac{1}{2} \left( z_3 + \text{sqrt}z_3^2 - 4\left(\frac{\omega_p}{\omega_B}\right)^2 \sin^2\alpha \right), \\
z_3 &= 1 + \left(\frac{\omega_p}{\omega_B}\right)^2.
\end{aligned}$$

The resulting analytical expression corresponds expression derived in the framework of the classical theory of binary collisions and the dielectric model (Such result is shown in Fig.3.)



**Fig.3.** Dependence  $f(\alpha, u)$ , on the angle  $0 \leq \alpha \leq \pi/2$  of a projectile particle with respect to the external magnetic field  $0 \leq u = \omega_p/\omega_B$

When the magnetic field is turned off the addition part  $f(\alpha, \omega_p/\omega_B)$  and we get a supplement known expression for the polarization losses without a magnetic field when the temperature is neglected, that is, the initial velocity of the incoming particle is much greater than the speed of random motion of particles in the plasma.

### 4. THE ACCOUNTING OF THE ANISOTROPIC TEMPERATURE OF THE ELECTRON GAS IN THE LINEAR APPROXIMATION

Taking into account of the temperature of the electron gas is essential problem of electron cooling.

The temperature is met in the expression for the energy loss of the projectile particle in the form  $\beta\varepsilon_{\alpha}$

$$\beta\varepsilon_{\alpha} = \beta\varepsilon_{\alpha\perp} + \beta\varepsilon_{\alpha\parallel} = \frac{\omega_B}{T} \left( \nu + \frac{1}{2} \right) + \frac{p^2}{2mT}. \quad (18)$$

An important point in accelerator technology is the effect of "plated" of the electrostatically accelerated beam of charged particles. This effect is a consequence of Liouville's theorem. After accelerating the electron velocity distribution is essentially anisotropic. The transverse temperature is a thousand times greater than the longitudinal component

(for the characteristic values  $T_{\perp}e \approx T_{cath} \sim 1000K$ ,  $U \sim 10kV$ ,  $T_{\parallel} = T_{cath}^2/E \sim 1K$ .)

Let use replacement to account for the temperature anisotropy

$$\beta\varepsilon_{\alpha} = \beta_{\perp}\varepsilon_{\alpha\perp} + \beta_{\parallel}\varepsilon_{\alpha\parallel} = \frac{\omega_B}{T_{\perp}} \left( \nu + \frac{1}{2} \right) + \frac{p^2}{2mT_{\parallel}}. \quad (19)$$

In the linear approximation of the temperature the dielectric susceptibility one can write as [10]

$$\kappa(\omega, k, T) = \kappa(\omega, k, 0) + AT_{\parallel} + BT_{\perp} + C, \quad (20)$$

where

$$\kappa(\omega, k, 0) = -\frac{\omega_P^2}{k^2} \left( \frac{k_z^2}{\omega^2} + \frac{k_{\perp}^2}{\omega^2 - \omega_B^2} \right). \quad (21)$$

The first term in (20) is the well-known expression in the plasma physics obtained in hydrodynamics approximation [5]

$$A = -\frac{\omega_P^2 k_z^2}{mk^2} \left( \frac{3k_z^2}{\omega^4} + k_{\perp}^2 \frac{3\omega^2 + \omega_B^2}{(\omega^2 - \omega_B^2)^3} \right), \quad (22)$$

$$B = -\frac{\omega_P^2 k_{\perp}^2}{mk^2} \times \left( \frac{k_z^2}{\omega^2} \frac{3\omega^2 - \omega_B^2}{(\omega^2 - \omega_B^2)^2} + \frac{3k_{\perp}^2}{(\omega^2 - 4\omega_B^2)(\omega^2 - \omega_B^2)} \right). \quad (23)$$

Next two terms in (20) give a correction for temperature in the linear approximation [10].

$$C = -\frac{\omega_P^2}{k^2} \left( \frac{\hbar^2 k_z^2}{4m\omega^2} + \frac{\hbar^2 k_{\perp}^4 k_z^2}{8m^2\omega^2\omega_B^2} \left( 1 + \frac{3T_{\perp}}{\hbar\omega_B} \right) \right) - \frac{\omega_P^2}{k^2} \left( \frac{\hbar^2 k_z^4 k_{\perp}^2}{4m^2} \left( \frac{3\omega^2 + \omega_B^2}{\omega^2 - \omega_B^2} \right) \right). \quad (24)$$

Expression (24) takes into account the quantum corrections [10]. Let consider the case of longitudinal motion of the particles in a magnetic field. In the weak and strong magnetic fields one can obtain simple analytical expressions for the energy loss. Energy losses are written in approximation of a weak magnetic field ( $\omega_B/\omega_P \ll 1$ ) as

$$-\frac{dE}{dt} = \frac{q^2\omega_P^2}{V_i} \left[ L_C - \frac{\tau}{2} - \frac{1}{2} \left( \frac{\omega_B}{\omega_P} \right) \right]. \quad (25)$$

where  $\tau = \frac{3v_e^2}{V_i^2}$ ,  $L_C$  is Coulomb logarithm (17)

If the value of the magnetic field one comes to the result obtained in the case without the magnetic field [6]  $\vec{H} = 0$  that means the correspondence principle is performed.

In approximation of a weak magnetic field they are ( $\omega_B/\omega_P \gg 1$ )

$$-\frac{dE}{dt} = \frac{q^2\omega_P^2}{V_i} \left[ \left( 1 - \frac{4\tau_{\parallel}}{3} \left( \frac{\omega_B}{\omega_P} \right)^4 - \frac{2\tau_{\perp}}{3} \left( \frac{\omega_B}{\omega_P} \right)^2 \right) L_C - \ln \left( \sqrt{1 + \left( \frac{\omega_B}{\omega_P} \right)^2} + \frac{2\tau_{\parallel}}{3} \left( \frac{\omega_B}{\omega_P} \right)^4 - \frac{\tau_{\perp}}{3} \left( \frac{\omega_B}{\omega_P} \right)^2 \right) \right]. \quad (26)$$

The term with the Coulomb logarithm yields main contribution to the expression (26). It is proportional

$$\left( 1 - \frac{4\tau_{\parallel}}{3} \left( \frac{\omega_B}{\omega_P} \right)^4 - \frac{2\tau_{\perp}}{3} \left( \frac{\omega_B}{\omega_P} \right)^2 \right). \quad (27)$$

The value equal to the ratio of the second and the third term of (27) is

$$\xi = 2 \frac{T_{\parallel}}{T_{\perp}} \left( \frac{\omega_B}{\omega_P} \right)^2. \quad (28)$$

It is an experimentally proved fact that with increasing values of the external longitudinal magnetic field the effect of the "fast" electron cooling are observed [3, 4]. The effect is observed in strong external magnetic fields which locks the transverse motion of the electrons. So the energy exchange is possible only through the longitudinal component of the temperature which is several orders of magnitude low than transverse one. The cooling of the beam of heavy charged particles is several times better as result.

In early experiments term was only a fraction of a unit [9]. In modern installations, for example, project HESR (High Energy Storage Ring),  $\xi$  reaches values ( $\xi \approx 10$ ). The contribution of the longitudinal component of the thermal motion of electrons is much higher than the perpendicular one.

## 5. QUANTUM EFFECTS IN THE ELECTRON COOLING

In the electron gas in a magnetic field two types of quantum effects are possible: 1) if the temperature of the electron gas below the degeneracy temperature then all of the electron gas behaves as a quantum object; and 2) the energy of particles is characterized by the Landau levels due to the motion of particles in a magnetic field. If the transverse temperature of the electron gas is less than the distance between adjacent Landau levels then there will be quantum effects.

There are characteristic density of the electron gas  $N \approx 3 \cdot 10^7 cm^{-3}$ , plasma  $\omega_P \approx 2.9 \cdot 10^8 c^{-1}$  and cyclotron  $\omega_B \approx 3.5 \cdot 10^8 c^{-1}$  frequencies in the electron cooling. For such parameters the degeneracy temperature of the electron gas is

$$T_0 = \frac{\hbar^2}{2m} (3\pi^2 N)^{2/3} \sim 10^{-10} eV. \quad (29)$$

The temperature below the temperature of the electrons is assumed while technically impossible.

In the second case with the Landau levels the temperature of the electron  $T_0$  gas are imposed less stringent conditions

$$\frac{\hbar\omega_B}{T_{\perp}} = \frac{eB\hbar}{2mcT_{\perp}} \sim 10^{-5}. \quad (30)$$

So on devices such HESR quantum effects are not important. However, it should be noted that quantum electron cooling can be observed in the laboratory if we increase the magnetic field and decrease the transverse temperature by two orders of magnitude, that is technically possible.

In conclusion, the main advantage of the quantum approach to the description of electron cooling is avoiding of the disadvantages associated with the empirical determination of the Coulomb logarithm which is typical for the theory of binary collisions and for the plasma model. In quantum field theory the Coulomb logarithm is defined from the first principles.

The authors are grateful to Miroshnichenko V.I. for useful discussions

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## КВАНТОВО-ПОЛЕВЫЕ МЕТОДЫ В ЗАДАЧЕ ЭЛЕКТРОННОГО ОХЛАЖДЕНИЯ

*А.В. Хелемеля, Р.И. Холодов*

Предложено использовать квантово-полевые методы для описания процесса электронного охлаждения. Рассмотрено в рамках квантовой теории поля влияние магнитного поля при учете анизотропии температурного распределения электронного газа. Показано, что в присутствии достаточно сильного магнитного поля главную роль в электронном охлаждении тяжелых ионов играет продольная составляющая теплового движения электронов. Проведена оценка квантовых эффектов в электронном охлаждении.

## КВАНТОВО-ПОЛЬОВІ МЕТОДИ В ЗАДАЧІ ЕЛЕКТРОННОГО ОХОЛОДЖЕННЯ

*О.В. Хелемеля, Р.И. Холодов*

Запропоновано використовувати квантово-польові методи для опису процесу електронного охолодження. Розглянуто в рамках квантової теорії поля вплив магнітного поля з урахуванням анізотропії температурного розподілу електронного газу. Показано, що в присутності сильного магнітного поля головну роль в електронному охолодженні важких іонів відіграє повздовжня компонента теплового руху електронів. Проведено оцінку квантових ефектів в електронному охолодженні.