THE METHOD OF FUEL REARRANGEMENT CONTROL
CONSIDERING FUEL ELEMENT CLADDING DAMAGE AND BURNUP

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The method of LWR fuel element (FE) cladding damage calculation which allows us to reduce the safety factor five times, when estimating cladding durability according to the strength criteria, has been described. The criterion model of FE properties control efficiency and the probabilistic model of FE operating calculated parameters, have been developed. The method of VVER-1000 fuel rearrangement control which allows us to find rearrangement algorithms having the minimum values of maximum and average cladding damage, as well as the maximum uniformity of damage and burnup among all the FAs for the rearrangement algorithm, has been proposed.

INTRODUCTION

When considering Generation IV LWR designs, a great simultaneous increase of such parameters as core power density, fuel campaign duration and burnup should be marked out as one of the most important features of these promising projects. Though fuel element (FE) cladding integrity is the most important limiting factor when increasing these parameters, the source of FE cladding failures remains unknown in 20% of all cases [1].

In the open sources of information there have been no published data on the localization of FE cladding failure areas depending on FE loading conditions. Not taking into account fretting, the following cladding failure sources are most typical: pellet-cladding mechanical interaction (PCMI), especially at low burnups, and stress corrosion cracking (SCC); cladding corrosion at high burnups (>50 MWd/kg-U); cladding damage due to a joint influence of the creep and fatigue processes. The influence of PCMI and SCC on cladding durability is eliminated by implementing restrictions for maximum linear heat rate (LHR) and maximum LHR jumps in a FE. The influence of corrosion at high burn-ups is eliminated by optimization of the cladding material composition and production technology. Hence in order to control FE properties under normal operating conditions, the most significant demand is a correct calculation of FE cladding damage due to a joint influence of creep and fatigue, because this factor cannot be eliminated by existing methods.

According to the effective approach to VVER-1000 FE cladding damage \( \sigma(\tau) \) estimation, \( \sigma(\tau) \) is estimated using the strength criterion SC4 through the relative service life of cladding, where the damage components corresponding to stationary and variable modes are considered separately and summarized. When estimating \( \sigma(\tau) \) using SC4, the fatigue component of cladding deformation is dominant after 2 years of variable reactor loading [2].

The limitations of the effective approach are: disregard of the real order of cladding loading conditions when calculating \( \sigma(\tau) \); the limiting components of SC4 depend on reactor loading conditions including the power maneuvering method, the disposition of control rods in the active core and their movement amplitude, the fuel assembly (FA) rearrangement algorithm; in the open sources of information there are no published values of SC4 limiting components corresponding to any set of cladding operating conditions; the cladding operating conditions used for calculation of the SC4 limiting components do not correspond to constantly changing real conditions; the main role of creep in the process of \( \sigma(\tau) \) accumulation at loading frequencies \( \nu << 1 \) Hz [3] is not taken into account; uncertainty of \( \sigma(\tau) \) estimation using SC4 forces us to accept the unreasonably high value (10) of the safety factor for SC4 [2].

When modeling FA rearrangements in the core, the use of the Advanced VVER-1000 power control algorithm (A-algorithm) was supposed [2]. A core segment containing a sixth part of all the FAs (not considering the FA placed in the central core cell) and a sixth part of all the control rods used at reactor power maneuvering, was analyzed. The distribution of FAs in the core segment by campaign years was found using the known distribution of long-lived and stable fission products causing reactor slagging. Marking a FA cell number by the Arabic numeral and the 1st, 2nd, 3rd and 4th campaign year by the Roman numeral I, II, III and IV, respectively, the distribution of FAs in the core segment cells was found (fig. 1).

![Fig. 1. The model of FA rearrangements in the core segment](image)

When performing FA rearrangements in the core segment, the following two main approaches are possible [4]: 1) a 4th-year FA is placed in the central cell 82; 2) either a 1st-year FA or a 2nd-year FA is placed in the central cell 82. The last approach secures an optimal use of the fuel when ensuring the necessary campaign duration, hence cell 82 was not used in the model of typical FA rearrangements. 7 core cells were appointed for FAs...
of each campaign year, excepting 4th-year FAs which had only 6 appointed cells (fig. 1).

The model of change of FE properties based on the two-group neutron diffusion model takes into account the radial distribution of energy-release in a fuel pellet and the axial distribution of LHR in a FA, pellet cracking, release of gaseous fission products, the gas flow in the pellet-cladding gap, the cladding oxide layer width, PCMI depending on the following input data: FE design parameters, burnup, VVER-1000 operating parameters, VVER-1000 power change program, movements of control rods and rearrangements of FAs [5]. The cladding stress and strain distributions calculated with the help of the FEMAXI code [6] were used as input data for the model of FE cladding damage distribution.

**THE METHOD OF FE CLADDING DAMAGE CALCULATION**

The evolution of deformations in a thin shell under thermomechanical conditions close to the conditions existing in the core was experimentally modelled in [3]. Using the stress-life diagram, analyzing the Zircaloy-4 metal structure and availability of the fatigue striations, it was found that when the loading frequency \( \nu \ll 1 \) Hz, creep governed the entire deformation process in the Zircaloy-4 cladding while cladding deformation due to fatigue was negligibly small [3].

For the first time, the publication [7] proposed to use creep energy theory (CET) [8] for calculation of FE cladding damage under VVER normal operating conditions, and so far to take into account creep as the main process of cladding failure. Based on CET, the criterion of cladding failure is written in the form:

\[
\omega(\tau) = A(\tau)/A_0 = 1 \quad A(\tau) = \int_0^\tau \sigma_e \cdot \dot{\varepsilon}_e \cdot d\tau ,
\]

where \( \omega(\tau) \) is cladding material failure parameter; \( A(\tau) \) is specific dispersion energy (SDE), J/m\(^3\); \( A_0 \) is the SDE at the moment \( \tau_0 \) that cladding material failure starts; \( \sigma_e(\tau) \), \( \dot{\varepsilon}_e(\tau) \) are, respectively, the equivalent stress (Pa) and rate of equivalent creep strain (s\(^{-1}\)) for the innermost cladding radial element having the maximum temperature.

The experimental results [3] showing the main role of creep in the process of cladding deformation failure when \( \nu \ll 1 \) Hz agree qualitatively with the experimental results [8] stating that, in a thin cladding, the dependencies \( A(\tau) \) for variable loading modes with \( \nu \ll 1 \) Hz are similar to \( A(\tau) \) for stationary loading modes and characterized by the same value of \( A_0 \).

The provisions of the FE cladding damage calculation method are: in order to operate FEs safely, it is obligatory to control FE cladding damage (failure parameter) accumulated under normal operating conditions and caused by a joint influence of creep and fatigue. As creep determines cladding deformation at stationary and variable (\( \nu \ll 1 \) Hz) modes, the calculation of cladding failure conditions must be based on the CET-method stating that creep and destruction processes in a cladding proceed simultaneously and influence each other. At any moment \( \tau \) the value of cladding failure is estimated from the SDE \( A(\tau) \) accumulated during creep process up to this moment. The limiting component \( A_0 \) of the cladding failure criterion does not depend on loading history but, rather, is a characteristic of the properties of the cladding material only. \( A_0 \) is found as \( A(\tau) \) at the moment \( \tau_0 \), when the following limiting condition for the innermost radial element of the studied cladding axial segment (AS) is satisfied:

\[
\lim(dA/d\tau)^{-1} \rightarrow 0 \quad \text{when} \quad \tau \rightarrow \tau_0 .
\]

The dependencies \( A(\tau) \) for Zircaloy-4 have been calculated for different operating modes of VVER-1000 and it was found that these calculation dependencies \( A(\tau) \) are quite similar to the experimental and calculation dependencies obtained in [8] for different alloys. Using Eq. (2) for Zircaloy-4, the calculated value of \( A_0 \) is 55 MJ/m\(^3\) – see fig. 2 (the symbol "Ne" means the core cell number).

![Fig. 2. The dependency \( A(\tau) \): (1) \#55 (N =100 %); (2) \#44 (daily cycle); (3) \#55-44-10-43-44-44 (daily cycle); (*) \#55 according to condition (3)](image)

When estimating \( A_0 \) using the established cladding strength criterion SC2

\[
\sigma_e(\tau_0) = \eta \cdot \sigma_0(\tau_0) ,
\]

where \( \sigma_e(\tau_0) \) and \( \sigma_0(\tau_0) \) are, respectively, the equivalent stress and yield stress (in Pa) for the innermost cladding radial element; and \( \eta \) is some factor, \( \eta \leq 1 \), it was found that \( A_0 = 30...40 \) MJ/m\(^3\) and \( A_0 \) differs for different cladding loading conditions: 37.12 (line 1), 34.44 (line 2) and 31.94 MJ/m\(^3\) (line 3), i.e. the calculated value of \( A_0 \) is not constant for a given material.

Using Eq. (3), it is not possible to find a value of the factor \( \eta \) such that, for any alternative set of FE cladding normal operation parameters, the following conditions are satisfied:

\[
\sigma_e(\tau_0) = \eta \cdot \sigma_0(\tau_0) ; \eta = \text{idem} \; \omega \leq 1.
\]

The estimation of \( A_0 \) using Eq. (3) is more conservative than the estimation using the limiting condition (2), and does not satisfy the fundamental principle of CET, which states that \( A_0 \) does not depend on loading history and is a function only of the cladding material properties, hence it is reasonable to use the "conservative CET-estimation", i.e. to take into account both the principle of the independence of \( A_0 \) from loading history and the principle of conservative estimation of \( A_0 \), setting under normal VVER-1000 operation conditions \( A_0 \) to a constant value: \( A_0 = \text{const} = 30 \) MJ/m\(^3\). The safety
factor for this estimation is $K = 55/30 \approx 2$, and this value is 5 times smaller than the normative safety factor for SC4 ($K = 10$).

The main factors determining $\omega(\tau)$ were found for a combined cycle of VVER-1000 variable loading by means of calculating an averaged relative difference $\delta A_{i,\pm}(\tau)$ between the specific dispersion energy $A_{i,\pm}(\tau)$ for the set of parameters $\{X_{i,0}, X_{2,0}, \ldots, X_{i,\pm} \Delta X_{i,0}, \ldots, X_{i,\pm}\}$ and the specific dispersion energy $A_{\pm}(\tau)$ for the basic set of parameters $\{X_{1,0}, X_{2,0}, \ldots, X_{i,0}, \ldots, X_{n,0}\}$:

$$
\delta A_{i,\pm}(\tau) = \frac{|A_{i,+}(\tau) - A_{\pm}(\tau)| + |A_{i,-}(\tau) - A_{\pm}(\tau)|}{2 \cdot A_{\pm}(\tau) \cdot \Delta X_{i}},
$$

(5)

where $\tau$ is time (ef. days); $\Delta X_{i}$ is a variation of the $i$th varying parameter, %.

After a VVER-1000 has been operated for 5.48 eff. years, having calculated $\delta A_{i,\pm}(\tau)$ for the central AS of a medium-duty FE, the determining factors (DFs) with $\delta A_{i,\pm} > 2$ have been singled out (table 1).

<table>
<thead>
<tr>
<th>№</th>
<th>DF</th>
<th>Denotation</th>
<th>$\delta A_{i,\pm}$</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$q_{l,\max}$</td>
<td>FE maximum LHR</td>
<td>18.7</td>
<td>when $q_{l,\max}$ ↑, $\omega(\tau)$ ↑</td>
</tr>
<tr>
<td>2</td>
<td>$T_{in}$</td>
<td>VVER-1000 coolant inlet temperature</td>
<td>5.6</td>
<td>when $T_{in}$ ↑, $\omega(\tau)$ ↑</td>
</tr>
<tr>
<td>3</td>
<td>$d^\prime_{c}$</td>
<td>cladding outer diameter</td>
<td>4.19</td>
<td>when $d^\prime_{c}$ ↑, $\omega(\tau)$ ↓</td>
</tr>
<tr>
<td>4</td>
<td>$d_{p}$</td>
<td>pellet diameter</td>
<td>2.15</td>
<td>when $d_{p}$ ↑, $\omega(\tau)$ ↑</td>
</tr>
</tbody>
</table>

The FE maximum LHR $q_{l,\max}$ is the chief DF, and this fact is a scientific premise for control of FE properties by means of control of FA rearrangements. Having operated a VVER-1000 according to the combined cycle of variable loading for 4.32 eff. years, $A(\tau)$ in the central AS of a medium-duty FE of the serial VVER-1000 FA (TVS-A) increases from 15.6 to 37.69 MJ/m$^3$, if $q_{l,\max}$ increases from 248 to 298 W/cm [2].

Considering FA rearrangements in the core during a 4-year campaign and the daily VVER-1000 power maneuvering according to the Alternative algorithm, the amplitudes of LHR jumps in the axial segments of a medium-duty FE were calculated using the Reactor Simulator (RS) code [9], and it was obtained that the cladding failure parameter $\omega(\tau)$ was maximum in the axial segments located between the axial coordinates $z = 1.8$ and $2.7$ m [2, 10].

It was established that if $v << 1$ Hz and the reactor capacity factor (CF) $CF = \text{idem}$, then there was no decrease of $\tau_0$ when the loading frequency $v$ increased 2–4 times comparing with the case when $v = 1$ cycle/day. On the contrary, when $CF$ increased from 0.9 to 1, $\tau_0$ decreased greatly. Having $N = 100$ %, for a medium-duty FE of the FA located sequentially in the core cells № 55; 31; 69; 82, $q_{l,\max}$ equals to 236.8; 250.3; 171.9; 119.6 W/cm, respectively. Hence, from the FE claddinig durability point of view, the algorithm of FA rearrangements 55-31-55-55 is less favourable than the algorithm 55-31-69-82. As the algorithm of FA rearrangements characterized by a lower value of $A(\tau)$, at the same time is characterized by a lower value of fuel burnup, then it is reasonable to work out a method for control of FA rearrangement taking into account the balance between cladding damage and fuel burnup [2].

**THE CRITERION MODEL**

When the criterion model (CM) of the efficiency $Eff$ of controlling the FE properties was worked out, the following basic principles were adopted: the goal of FE properties control under normal LWR operating condi-

- An increase of FE operating efficiency at the expense of simultaneous consideration of FE cladding failure parameters as well as economic and technological indicators of LWR operating efficiency; control of FE properties is carried out on the basis of requirements to the properties of FEs and to the whole active core, and on the basis of definition of the parameters to be controlled as well as definition of the determining factors; though the structure of the FE properties control efficiency criterion is the same for all control problems, the criterion components are not invariant.

The controlled parameters $c_i$ ($i = [1, n_c]$), $n_c$ is the number of controlled parameters) and the adjusted factors $d_j$ ($j = [1, n_c]$), $n_j$ is the number of adjusted DFs which determine the controlled parameters, are defined. Based on requirements to the properties of FEs and to the whole active core, the optimum values $c_i^{\text{opt}}$ and the maximum permissible values $c_i^{\lim}$ are defined for $c_i$, so that the following conditions for all permissible values of $c_i$ are satisfied:

$$
c_i^{\lim} \leq c_i \leq c_i^{\text{opt}} \quad \text{or} \quad c_i^{\text{opt}} \leq c_i \leq c_i^{\lim}
$$

(6)

After rewriting Eq. (6) in dimensionless form:

$$
c_i^{\lim*} \leq c_i^* \leq c_i^{\text{opt}*} \quad \text{or} \quad c_i^{\text{opt}*} \leq c_i^* \leq c_i^{\lim*} = 1.
$$

(7)

Generally, the maximum of efficiency $Eff$ is defined using a criterion having the following structure:

$$
\max \left\{ Eff = 1 - \frac{L}{L^{\lim}} \right\},
$$

(8)

$$
L = \sum_{j=0}^{n_j} (1 - c_{2j+1})^2 + \sum_{j=1}^{n_j} k_{1,j}(1 - c^*_{2j})^2;
$$


\[ L^{\lim} = \sum_{j=0}^{n_j} (1 - c_{2j+1}^{\lim*})^2 + \sum_{j=1}^{n_j} k_{1,j}(1 - c_{2j}^{\lim*})^2, \]

where $c_{2j+1}^{\lim*}$ and $c_{2j}^{\lim*}$ are dimensionless controlled parameters with odd (even) indices such that any variation of a dimensional controlled parameter $\Delta c_{2j+1} \ (\Delta c_{2j})$ yields...
a variation $\Delta E_{\text{ff}}$ being opposite in sign (equal in sign); $n_i$ ($n_j$) is the number of controlled parameters such that any variation of a controlled parameter yields a variation $\Delta E_{\text{ff}}$ being opposite in sign (equal in sign); $k_{ij}$ are weight factors taking into account a difference between $c_{l_{i+1}}^{\text{lim}}$ and $c_{l_{j+1}}^{\text{lim}}$ defined for the case $c_{l_{j+1}}^{\text{lim}} < c_{l_{i+1}}^{\text{lim}}$ as:

$$k_{ij} = \left[ \frac{1 - c_{l_{j+1}}^{\text{lim}}}{1 - c_{l_{i+1}}^{\text{lim}}} \right]^{\Delta}.$$ (9)

The physical meaning of Eq. (8) is that if $c_{l_{j+1}}^{\text{lim}} > c_{l_{i+1}}^{\text{lim}}$ ($c_{l_{j+1}}^{\text{lim}} < c_{l_{i+1}}^{\text{lim}}$) or $c_{l_{j+1}}^{\text{lim}} < c_{l_{j+1}}^{\text{lim}}$ ($c_{l_{j+1}}^{\text{lim}} < c_{l_{j+1}}^{\text{lim}}$), then this controlled parameter gives a negative contribution to the total efficiency $E_{\text{ff}}$. The advantage of one set of determining factors $d_j$ over another one is evaluated based on summation of the advantages given by the controlled parameters $c_i$.

**THE METhOD OF CONTROL OF FA REARRANGEMENTS IN THE CORE**

The method of control of FA rearrangements in the core implies that FE cladding failure parameter $\omega(t)$ and fuel burnup $B(t)$ are the parameters to be controlled. To say more exactly, when considering the FAs used in the rearrangement algorithm $j$, the controlled parameters are the maximum value $\omega_j^{\text{max}}$ (among all the FAs) of the cladding failure parameter for the $j$th rearrangement algorithm and the average value $<\omega>$ of the cladding failure parameter for all the FAs used in the $j$th rearrangement algorithm, as well as the minimum value $B_j^{\text{min}}$ of fuel burnup (among all the FAs) for the $j$th rearrangement algorithm, while the FA rearrangement algorithm is the DF to be adjusted (fig. 3).

**Fig. 3. The method of fuel rearrangement control**

It was accepted that $A_0 = 30$ MJ/m$^3$. Using the model of VVER-1000 FA rearrangements during a 4-year campaign, taking into account the amplitude of the movement of control rods necessary to stabilize the axial offset at reactor power maneuvering according to the Alternative algorithm [2], the values of $\omega(1460$ days) and $B(1460$ days) in AS 6 were calculated for different FA rearrangement algorithms. 18 algorithms containing 126 rearrangements have been analysed, including 16 algorithms containing 112 rearrangements which were randomly chosen using the MATLAB function “rand”, while 2 algorithms (17 and 18) were used at Zaporizhzhya NPP Unit 5 during campaigns 22 and 23, respectively [4]. The values of $\omega(1460$ days) and $B(1460$ days) for algorithm 3 (random) and algorithm 18 (practical) are shown in table 2.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Rearrangement</th>
<th>$A$, MJ/m$^3$</th>
<th>$\omega(t)$, %</th>
<th>$B$, MW·day/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>9-19-21-8</td>
<td>2.25</td>
<td>7.51</td>
<td>62.5</td>
</tr>
<tr>
<td></td>
<td>5-41-68-43</td>
<td>1.39</td>
<td>4.64</td>
<td>60.5</td>
</tr>
<tr>
<td></td>
<td>55-22-10</td>
<td>2.17</td>
<td>7.22</td>
<td>54.7</td>
</tr>
<tr>
<td></td>
<td>13-11-20-6</td>
<td>1.42</td>
<td>4.74</td>
<td>56.8</td>
</tr>
<tr>
<td></td>
<td>3-30-54-1</td>
<td>1.39</td>
<td>4.62</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>4-32-18-42</td>
<td>1.72</td>
<td>5.74</td>
<td>62.7</td>
</tr>
<tr>
<td></td>
<td>2-31-12-29</td>
<td>1.98</td>
<td>6.59</td>
<td>63.9</td>
</tr>
<tr>
<td>18</td>
<td>2-99-21-6</td>
<td>2.22</td>
<td>5.17</td>
<td>54.9</td>
</tr>
<tr>
<td></td>
<td>3-41-68</td>
<td>1.13</td>
<td>3.93</td>
<td>48.8</td>
</tr>
<tr>
<td></td>
<td>4-11-29-18</td>
<td>1.16</td>
<td>3.86</td>
<td>60.8</td>
</tr>
<tr>
<td></td>
<td>5-19-20-1</td>
<td>1.45</td>
<td>4.83</td>
<td>54.6</td>
</tr>
<tr>
<td></td>
<td>9-32-12-42</td>
<td>2.59</td>
<td>8.62</td>
<td>67.9</td>
</tr>
<tr>
<td></td>
<td>13-30-10-43</td>
<td>2.55</td>
<td>8.5</td>
<td>67.7</td>
</tr>
<tr>
<td></td>
<td>55-31-54-8</td>
<td>1.98</td>
<td>6.61</td>
<td>61.4</td>
</tr>
</tbody>
</table>

Let’s introduce the conditions:

$$\omega^{\text{opt}} = \min \{\omega_j^{\text{max}}\};$$

$$<\omega>^{\text{opt}} = \min \{<\omega>_j\};$$

$$B^{\text{opt}} = \max \{B_j^{\text{min}}\}. $$ (10)

Let’s accept that $\omega^{\text{lim}}$, $<\omega>_j^{\text{lim}}$ and $B_j^{\text{lim}}$ are specified permissible limits for $\omega_j^{\text{max}}$, $<\omega>_j$ and $B_j^{\text{min}}$, respectively. Hence, the permissible values of $\omega_j^{\text{max}}$, $<\omega>_j$ and $B_j^{\text{min}}$ lie in the following ranges:

$$\omega^{\text{opt}} \leq \omega_j^{\text{max}} \leq \omega_j^{\text{lim}};$$

$$<\omega>^{\text{opt}} \leq <\omega>_j \leq <\omega>_j^{\text{lim}};$$

$$B_j^{\text{lim}} \leq B_j^{\text{min}} \leq B^{\text{opt}}. $$ (11)

Then we obtain
\[ o_j \leq \omega_j \leq \omega_j^{\text{max}} \leq 1; \]
\[ B_j^{\text{lim},*} < \omega_j < B_j^{\text{min},*} \leq 1, \]
where
\[ \omega_j^{\text{lim},*} = (1 - \omega_j^{\text{max}})/(1 - \omega_j^{\text{opt}}); \]
\[ \omega_j^{\text{max},*} = (1 - \omega_j^{\text{max}})/(1 - \omega_j^{\text{opt}}); \]
\[ < \omega_j^{\text{lim},*} = (1 - < \omega_j^{\text{max}})/(1 - < \omega_j^{\text{opt}}); \]
\[ < \omega_j^{\text{max},*} = (1 - < \omega_j^{\text{max}})/(1 - < \omega_j^{\text{opt}}); \]
\[ B_j^{\text{lim},*} = B_j^{\text{lim}}/B_j^{\text{opt}}; \]
\[ B_j^{\text{min},*} = B_j^{\text{min}}/B_j^{\text{opt}}. \]

In order not to use weight factors, the strict condition is set:
\[ o_j^{\text{lim},*} < < \omega_j^{\text{lim},*} = B_j^{\text{lim},*}. \]  (14)

Hence having some value of \( o_j^{\text{lim}} \), the corresponding values of \( < \omega_j^{\text{lim}} \) and \( B_j^{\text{lim}} \) are defined from the following equations:

**Table 3**

<table>
<thead>
<tr>
<th>( j )</th>
<th>( \omega_j^{\text{max},*} ), %</th>
<th>( &lt; \omega_j &gt; ), %</th>
<th>( B_j^{\text{min},*} ), MW·d/kg</th>
<th>( E_{\text{Eff} j} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8.84</td>
<td>5.86</td>
<td>47.6</td>
<td>-0.14</td>
</tr>
<tr>
<td>3</td>
<td>7.51</td>
<td>5.87</td>
<td>54.7</td>
<td>0.94</td>
</tr>
<tr>
<td>4</td>
<td>6.87</td>
<td>5.8</td>
<td>54.1</td>
<td>0.9</td>
</tr>
<tr>
<td>6</td>
<td>6.85</td>
<td>5.79</td>
<td>53.1</td>
<td>0.74</td>
</tr>
<tr>
<td>8</td>
<td>7.02</td>
<td>5.77</td>
<td>54.3</td>
<td>0.93</td>
</tr>
<tr>
<td>14</td>
<td>8.25</td>
<td>5.86</td>
<td>54.1</td>
<td>0.84</td>
</tr>
<tr>
<td>17</td>
<td>8.89</td>
<td>5.9</td>
<td>48.8</td>
<td>0.04</td>
</tr>
<tr>
<td>18</td>
<td>8.62</td>
<td>5.93</td>
<td>48.8</td>
<td>0.05</td>
</tr>
</tbody>
</table>

It can be seen that algorithms 3 and 8 are characterized by both high cladding durability and high burnup, hence all the corresponding dimensionless criterion components are high, so \( E_{\text{Eff} 3} \) and \( E_{\text{Eff} 8} \) are highest. Algorithms 17 and 18 have both cladding durability and burnup worse than the ones for algorithms 3 and 8, so \( E_{\text{Eff} 17} \) and \( E_{\text{Eff} 18} \) are close to 0. Algorithm 2 is characterized by cladding durability close to the same for algorithms 17 and 18, but burnup is considerably lower than the same for these algorithms, and as a result \( E_{\text{Eff} 2} < 0 \).

The goal of FA rearrangement control is achieved for algorithm 3.

Besides lowering of \( o_j^{\text{max}} \) and \( < \omega_j > \), as well as increasing of \( B_j^{\text{min},*} \), the physical meaning of increasing \( E_{\text{Eff}} \) is lowering of the variation intervals \( \Delta \omega_j \) and \( \Delta B_j \) within the algorithm (Table 4). This result decreases the probability of FE cladding failure and increases the economic efficiency of FE operation.

**Table 4**

<table>
<thead>
<tr>
<th>( j )</th>
<th>( E_{\text{Eff}} )</th>
<th>( &lt; \omega_j &gt; ), %</th>
<th>( \Delta \omega_j ), %</th>
<th>( &lt;B_j &gt; ), MW·d/kg</th>
<th>( \Delta B_j ), MW·d/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
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<td>4.757</td>
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**THE ROBUST MODEL**

When modeling the FE operating parameters, the following assumptions of the probabilistic model are established: 1) the value of the \( j \)th DF \( d_j \) calculated on the basis of the model of changing FE properties is the mean of the corresponding random variable \( d_j^{\text{rand}} \), i.e.:
\[ d_j = < d_j^{\text{rand}} >. \]  (17)

2) the controlled parameters \( c_i \) are calculated at \( [<d_i^{\text{rand}}>, -\Delta d_i] \) and \( [<d_i^{\text{rand}}>, +\Delta d_i] \), \( \Delta d_i \) is the variation interval for \( d_i^{\text{rand}} \) (\( n_j = 1 \)); 3) using the three-sigma rule for normally distributed data, the means \( <c_i> \) and standard deviations \( \sigma(c_i) \) are found; 4) based on \( <c_i> \) and \( \sigma(c_i) \), having used the Monte-Carlo Sampling (MCS) method, the corresponding samples \( E_{\text{Eff}}[c_i] \) are calculated, and so far the means \( <E_{\text{Eff}} \{c_i\}> \) and standard
deviations $\sigma(\text{Eff } \{c_i\})$ for different sets of DFs are found; 5) for different sets of DFs, the efficiency curve is constructed in the coordinates $\{\sigma(\text{Eff } \{c_i\}) < \text{Eff } \{c_i\} >\}$, and the best sets of DFs are chosen.

The estimation of variation intervals $\Delta_i$ was performed for the most important DFs: $q_{l,\text{max}}$ and $T_{\text{in}}$. The accuracy of $q_{l,\text{max}}$ calculation using the RS code is near 5 % [9]. Taking into account the correctness of measuring and regulating, the possible deviation (from the nominal value) of the VVER-1000 capacity $N$ and inlet coolant temperature $T_{\text{in}}$ is near 4 % and less than 1 %, respectively [11]. As the uncertainty of knowledge for $q_{l,\text{max}}$ is 5 times greater than the uncertainty of knowledge for $T_{\text{in}}$, and the parameter $\Delta_i$ for $q_{l,\text{max}}$ is more than 3 times greater than $\Delta_i$ for $T_{\text{in}}$, the robust model takes into account the uncertainty of knowledge for $q_{l,\text{max}}$ only, while leaving the uncertainty of knowledge for $T_{\text{in}}$ out of account is compensated by a conservative value of the variation interval for $q_{l,\text{max}}$. The calculated maximum LHR in FA $j$ $q_{l,j,\text{max}}$ is the mean of some random variable $q_{l,j,\text{max}}^\text{rand}$, i.e.:

$$ q_{l,j,\text{max}} = q_{l,j,\text{max}}^\text{rand}, $$

The cladding failure parameter $\omega$ and burnup $B$ in the most strained AS 6 are calculated for the rearrangements of the algorithms 3, 4, 6, 8 and 14 having the maximum values of $\text{Eff}$ at $<q_{l,n,\text{max}}^\text{rand}>-10\%$ and $<q_{l,n,\text{max}}^\text{rand}>+10\%$, where $n$ is core cell number for the corresponding campaign year, e.g., for algorithm 3 and rearrangement 9-19-21-8: $n = 9, 19, 21$ and $8$ for 1st, 2nd, 3rd and 4th year, respectively. Hence, the use of the deterministic criterion (16) allows us to reduce the number of analysed algorithms $N_{\text{alg}}$ from 18 to 5. In the robust case there are 2 random variables ($\omega_{j,k}$ and $B_{j,k}^\text{rand}$) for each pair of algorithm $j$ and rearrangement $k$.

The following relations are true:

$$ \omega_j^\text{max} = \max\{\omega_j^\text{rand}\}, \ <\omega_j> = \{\omega_j^\text{rand}\}, \ B_j^\text{min} = \min\{B_j^\text{rand}\}, \text{where } j = 1, \ldots, N_{\text{alg}}, \ k = 1, \ldots, 7. $$

We have the total number of input random variables $2 \cdot N_{\text{alg}} \cdot 7 = 70$, that is 35 FA rearrangements are described by 70 random variables.

For $k = 1, \ldots, 7$ and $j=3, 4, 6, 8, 14$, using the three-sigma rule (assuming normal distribution), the corresponding means $<\omega_{j,k}^\text{rand}>$, $<\omega_{j,k}^\text{max}>$ and standard deviations $\sigma(\omega_{j,k}^\text{rand})$, $\sigma(\omega_{j,k}^\text{max})$ of the random variables $\omega_{j,k}^\text{rand}$, $B_{j,k}^\text{rand}$ are calculated. For instance, algorithm 3 — (9-19-21-8 + 5-41-68-43 + 55-22-10 + 13-11-20-6 + 3-30-54-1 + 4-32-18-42 + 2-31-12-29) is described by the following random values $\tau_{j,k,p}$, where $p=1$ denotes $\omega_{j,k}^\text{rand}$ and $p=2$ denotes $B_{j,k}^\text{rand}$:

$$ \begin{align*}
\tau_{3,1,1} &= \omega_{9,19,21-8}^\text{rand}; \ldots \tau_{3,1,7} = \omega_{2-31-12-29}^\text{rand}; \\
\tau_{3,2,1} &= B_{9,19-21-8}^\text{rand}; \ldots \tau_{3,2,7} = B_{2-31-12-29}^\text{rand}.
\end{align*} $$

For rearrangement 9-19-21-8 of algorithm 3, $\tau_{3,1,1}$ and $\tau_{3,2,1}$ are random values described by $\{\omega_{9,19,21-8}^\text{rand} > \sigma(\omega_{9,19,21-8}^\text{rand})\}$ and $\{B_{9,19-21-8}^\text{rand} > \sigma(B_{9,19-21-8}^\text{rand})\}$, respectively.

As we have a great number of random variables, MCS methods are most computationally attractive [10].

A set of normally distributed random variables $\tau_{j,p,k}$ is obtained substituting the means and standard deviations of $\omega_{j,k}^\text{rand}$ and $B_{j,k}^\text{rand}$ into the MATLAB function “normrnd”, and the efficiency of algorithm $j$ is found using Eq. (16) in the form:

$$ \text{max}\{\text{Eff}_j = f(\theta_{j,1,1}, \ldots, \theta_{j,1,7})\}, $$

where

$$ \theta_{j,1,1} = \max(\tau_{j,1,1}, \ldots, \tau_{j,1,7}); \ \theta_{j,1,2} = \langle\tau_{j,1,1}, \ldots, \tau_{j,1,7}\rangle; \ \theta_{j,2,1} = \min(\tau_{j,2,1}, \ldots, \tau_{j,2,7}). $$

For the case of uncertain conditions, $\omega^\text{opt}, <\omega>^\text{opt}, B^\text{opt}$ and $L^\text{lim}$ can not be set as for the deterministic case. It should be noted that if $N_{\text{alg}}$ increases, then $\omega^\text{opt}$ decreases. On the contrary, when the number of core cells used for optimization increases, $\omega^\text{opt}$ increases also.

Using 100 samples of MCS method, for $\omega^\text{lim}=13\%$, the trade-off between $<\text{Eff}_j>$ and $\sigma(\text{Eff}_j)$ for the most effective 5 FA transposition algorithms ($A_0=30$ MJ/m³), and for 8 random algorithms in the case of the simplest robust control of rearrangements taking into account only 2 core cells appointed for each year ($A_0=40$ MJ/m³), is shown in fig. 4.

**Fig. 4.** The trade-off between $<\text{Eff}_j>$ and $\sigma(\text{Eff}_j)$: (numeral) algorithm number; (pentagon) random algorithm of the simplest robust control

Algorithm 3 had the largest efficiency in the deterministic case, while in the robust case algorithm 8 is most efficient (fig. 4). This can be explained by the fact that in the deterministic case $\omega^\text{max} - \omega^\text{max} = 0.5\%$. As the dependence of SDE on LHR is nonlinear and SDE depends greatly on LHR (FA rearrangement history), this deterministic difference $\omega^\text{max} - \omega^\text{max} = 0.5\%$ turned
to be sufficient to obtain $<\text{Eff}_3^\text{g}> < <\text{Eff}_8^\text{g}>$ in the robust case. In addition, $\sigma(\text{Eff}_3^\text{g}) > \sigma(\text{Eff}_8^\text{g})$ and thus there is no trade-off between these two options. Algorithm 8 dominates all the other options having both the highest $<\text{Eff}>$ and the smallest $\sigma(\text{Eff})$. Hence the goal of FA rearrangement robust control is achieved for the case of algorithm 8.

**CONCLUSIONS**

The proposed LWR FE cladding failure parameter calculation method based on CET allows us to reduce the safety factor 5 times, when estimating the cladding durability according to the group of strength criteria.

The criterion model of efficiency of FE properties control taking into account the balance between safety and economic efficiency of operation of FEs, has been developed. The probabilistic model of FE operating parameters taking into account robust FE operating conditions and ensuring the minimum dimension of the vector of random variables determining the FE operating conditions, has been developed.

The developed method of FA rearrangement control allows us to find the rearrangement algorithms having the minimum values of maximum and average cladding failure parameter, as well as the maximum uniformity of cladding damage and fuel burnup among all the FAs for the rearrangement algorithm.

**REFERENCES**


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