MODELLING OF SUPERRADIATION PROCESSES DRIVEN BY AN ULTRA-SHORT BUNCH OF CHARGED PARTICLES MOVING THROUGH A PLASMA

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The specific features of the superradiation processes excited by an ultra-short 1D monoenergetic bunch of charged particles, which moves through a plasma, are examined. When the size of the bunch is less then the wavelength of the excited plasma oscillations, the spontaneous bunch reshaping takes place. As a result, the bunch excites a wakefield with maximum possible amplitude, which twice as much the amplitude of the wakefield driven by an extended bunch with the same number of particles. The domains of maximum amplitude keep in time their location in the laboratory frame of reference.

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INTRODUCTION

The problem of obtaining the wakefields of highest possible amplitude driven by an electron bunch moving through a plasma had its origin from the analysis of the possibility to use the high-energy and high-currentshort electron beams – bunches, moving through a plasma, for ion acceleration [1, 2].Despite the fact that the bunches, which initial size is much smaller than the wavelength of the radiation, are unstable in a one-dimensional and three-dimensional cases (see, e.g. [3, 4]), at a certain reshaping of the particles density and velocity profile, one could hope for significant wakefield intensity, comparable with the wakefield, excited by particles, which are bundled in a very small spatial domain [5 - 8].If the goal is to ensure the stability of the bunch, it is preferable to use more extended beams, which further bunching allows to obtain the fields of high amplitude as inside the beam and behind it [9 - 12].

The wakefield behind the radiating particle moving with the velocity v can be written in the particle reference frame as a cosine curve $\cos[k\zeta] = \cos[k(x - vt)]$. In cold plasma, the group velocity of the Langmuir oscillations having a frequency ω_{pe} is equal to zero. So, after transfer to the laboratory frame of reference, the field amplitude in any point will change with time as $\cos[\omega_{pe} t] = \cos[kvt]$. It is important to note that this is one and the same field, but in different frames of reference. If the velocities of each particle are different, the cosine curves will have different values of the wave number $k = \omega_{pe} / v$. The total wakefield represents an interference pattern of the individual wakefields driven by particles composing the bunch. When the relative positions and velocities of particles are changed, the interference pattern is changed too. Wherein in the laboratory frame, the total wakefield is a sum of oscillations with different phases. There are exist the spatial and velocity configurations of particles, composing the bunch, which allow to minimize a phase spread of the wakefields driven by individual particles in separate, generally speaking, quite lengthy domains in the laboratory system of reference. It was proposed [13] to use a special profiling of the bunch over the velocity and density in order to achieve the wakefield of maximum amplitude, at least in some domains of the plasma space. It is clear that in the bunch reference frame, the domain with wakefield maximum amplitude moves in the opposite direction with approximately the same velocity [14].

Below, we will show that such profiling of the bunch inside its volume happens spontaneously. In addition, the domain with wakefield maximum amplitude appears, which in the bunch reference frame moves away from the bunch the mean velocity of the bunch, i.e. keeps the fixed position in the laboratory reference frame. Of particular interest are the regimes, when the highest wakefield amplitude can be achieved, that is found to be in the case of short bunches, which longitudinal size is smaller than the wavelength of the wakefield.

1. ASIMPLE MODEL OF SUPERRADIATION PROCESS DRIVEN BY COMPACT MOVING BUNCH OF CHARGED PARTICLES

It is important to note that if we consider the infinite periodic consequence of particles, as is often done, and then let the distance between them to infinity, the transition to the wake field of a single particle would be difficult. This is because of the field in the periodic system is present both in front of and behind the individual particles, but as known a single charge, moving in plasma, does not generate a radiation field in the direction of its movement (see, for example [3]).

Let present a charge density of an electron, moving with the velocity $v > 0$ as follows

$$\rho = -e \cdot \delta(-v \cdot t + x - s) = -e \cdot \delta(\xi - s) .$$

Here $\delta(x)$ is the Dirac delta function.

Then, we use the Poisson equation $\partial D / \partial x = 4\pi\rho$, which after the Fourier transformation takes the form

$$-ik\varepsilon(\omega,k) \cdot E(\omega,k) = 4\pi\rho(\omega,k) . \qquad (1)$$

The inverse Fourier transform of Eq. (1) gives

$$-i \int_{-\infty}^{\infty} \exp\{-ik\xi\} \cdot dk \cdot [k \cdot \varepsilon(k) \cdot E(k)] = = k_0 \left. \frac{\partial \varepsilon(k)}{\partial k} \right|_{k_0} \frac{\partial \mathrm{E}}{\partial \xi} \exp\{-ik_0 \xi\}. \qquad (2)$$

After substituting the charge density into Eq.(1) and taking into account Eq.(2) one can obtain the electric strength of the wakefield behind the charge, moving in positive direction

$$E = -4\pi e \cdot [k_0 \cdot \partial\varepsilon(k) / \partial k \,|_{k_0}]^{-1} \times \times \theta(s - \xi) \cdot \exp\{ik_0 (s - \xi)\} , \qquad (3)$$

where $\theta(x)$ is the Heaviside function.

The wakefield, produced by a single particle, can be considered as result of its spontaneous radiation. The spontaneous fields, generated by individual particles composing the bunch (under condition of their uniform spatial distribution and in absence of other synchronization mechanisms) differ from one another by the phase, which, in general, can be randomly distributed, at least, at the initial moment. In other words, the spontaneous emission of n uniformly distributed non-phase-locked emitters is non-coherent. The intensity of the spontaneous radiation is proportional to the number of emitters, i.e. $\propto n$ .On each particle acts its own field and the field of the particles, which are ahead of it.

The field energy of the spontaneous radiation inanen closed volume growths with time linearly. In the open system (bunch), the field energy growth is restricted by energy flow outside the bunch. However, after a certain time, the grouping of emitters may result in the situation when the main impact on the particles will make the emission produced by the grouped particles. If the number of particles in the group is s, the intensity of the field, produced by this group is proportional to s^2. When $s \propto \sqrt{n}$, the field of the grouped particles will dominate in the process of self-modulation and wakefield formation behind the bunch. The growth of this coherent emission, which by now takes an appearance of the stimulated emission, happens exponentially. The part of coherent component in the total emission of the bunch increases, i.e. the phases of many individual emitters differ only slightly from each other. The change in the field energy per unit of time is proportional in this case to the squared number of synchronized oscillators. Note, that such synchronization takes place under the action of the emitted wave and is governed by it.

The total wakefield can be obtained after summation over all particles of the bunch

$$\mathrm{E}(\xi) = -\frac{2}{N}\sum_{\alpha}^{N} f_\alpha \cos[2\pi\, g_\alpha(\xi-\xi_\alpha)]\,\Theta(\xi_\alpha-\xi)\,. \quad (4)$$

The Eq. (4) should be supplemented with equation of particle motion

$$\frac{d\xi}{d\tau} = \nu\,,\ \frac{d\nu}{d\tau} = \mathrm{E}(\xi)\,, \quad (5)$$

where $2\pi\xi = K_0(z - V_0 t)$, $\nu = K_0(V - V_0)/2\pi\gamma_L$, $\gamma_L^2 = e^2 K_0 M/m_e$, $g = (1+\Delta\cdot\nu)^{-1}$, $\Delta = 2\pi\gamma_L/K_0 V_0$, $\tau = \gamma_L\, t$, and M is the total number of particles in the unit cross-section of the bunch, $\mathrm{E} = eK_0 E/2\pi m_e \gamma_L^2$, E is the electric field strength, f_α is a statistical weight of the modelling particle, e, m_e are the electron charge and mass, $K_0 = \omega_{pe}/V_0$ is the wakefield wave-number The Eqs. (4), (5) describe the nonlinear dynamics of a short one-dimensional electron bunch, which moves through a dense plasma in its rest frame. The conditions of applicability of the model and its comparison with more full description [15] were discussed in [3].

2. SIMULATION RESULTS

The program, which implements a mathematical model of the problem, was created using the technology JCUDA. JCUDA provides execution of Java-program code on the GPU (graphics processing unit (GPU)) written in the C programming language with inserts of code specific to technology CUDA (a brief description of this technology can be found in [16]).

It follows from the results of calculation, that for bunches, which longitudinal dimension is less than but still comparable with the wavelength of the produced wakefield, the phenomenon of bunch self-profiling appears. The wakefield amplitude E in this case reaches in some domain behind the bunch the values, which are significantly higher than the unit.

For bunches which length exceeds few wave lengths of the emitted waves, the wakefield amplitude is less or of the order of unity [3]. If all particles, which compose the bunch, are gathered into a point, than the amplitude of the wakefield in the considered representation (4)-(5) reaches the maximum value of 2.

At first, consider what happens to the bunch. It was found that it slows down as a whole, forming at this time the characteristic triangle distributions in the configuration and velocity spaces (see Figs. 1, 2).

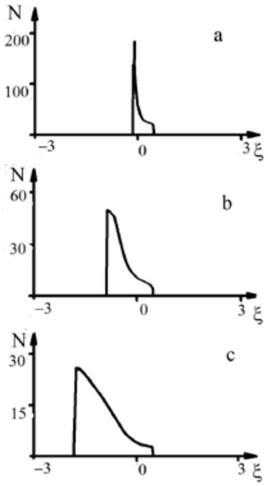

Fig. 1. The spatial distribution of bunch particles: $\tau = 1$ (a); $\tau = 2$ (b); $\tau = 3$ (c)

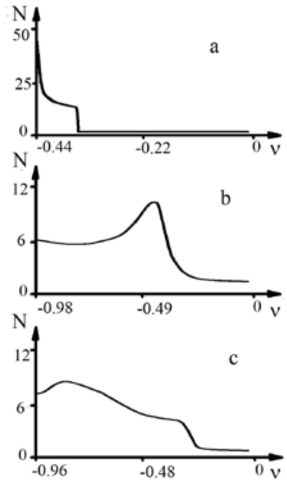

Fig. 2. The velocity distribution of bunch particles: $\tau = 1$ (a); $\tau = 2$ (b); $\tau = 3$ (c)

Namely the formation of triangular distributions similar to the initial profiling of the bunch leads to interference of wakefields driven by particles in a certain spatial domain, which moves away from the bunch in its rest frame with a mean velocity

$$<\frac{d\xi}{d\tau}> = \frac{K_0 V_0 t}{2\pi\gamma_L\, t} = \frac{1}{\Delta} \quad (6)$$

The maximum field domain moves with the velocity of the bunch in an opposite direction (Fig. 3). Note that the beam is on the right side of the figure. In the laboratory reference frame the domain with the maximum field remains motionless. Some distortion of the wakefield profile associated with a slight change of the bunch configuration during this time interval. This model describes qualitatively the dynamics of the bunch and emission produced by it, but allows to detect the effect of bunch self-profiling.

As a result of such self-profiling, the wakefield amplitude reaches the practically maximum possible value in the spatial domain, which dimension is more than an order of magnitude greater than the initial size of the bunch.

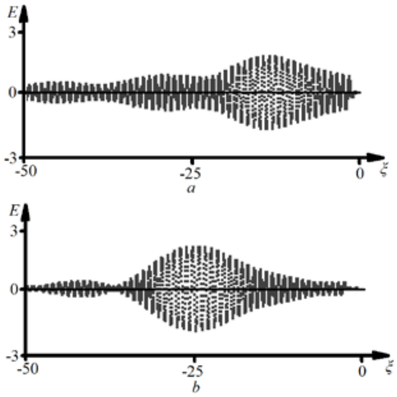

Fig. 3. The wakefield profile $E(\xi)$ *in the bunch rest frame of reference at the moments τ=2 (a); τ=3 (b)*

CONCLUSIONS

It is considered the model of one-dimensional ultra-short bunch of charged particles, moving through plasma. At the initial time, all particles are evenly distributed over the length L and have the same velocity. The case was examined, when the bunch length is significantly less then the wakefield wave length. As a result of the evolution of a short bunch, the profiling of its shape occurs and the amplitude of the wakefield in a fairly extended domain behind the bunch reaches the maximum possible value for a given number of particles of the bunch, which is twice as much the value of the wakefield generated by an extended bunch. The domain with maximum wakefield amplitude keep in time its location in the region of its formation in the bunch rest frame. The dimension of this domain is of the order of several tens of wavelengths.

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МОДЕЛИРОВАНИЕ ПРОЦЕССОВ СВЕРХИЗЛУЧЕНИЯ ДВИЖУЩЕГОСЯ В ПЛАЗМЕ УЛЬТРАКОРОТКОГО СГУСТКА ЗАРЯЖЕННЫХ ЧАСТИЦ

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Рассмотрены особенности режимов сверхизлучения коротких одномерных моноэнергетических сгустков заряженных частиц, распространяющихся в плазме. При размерах сгустка меньше длины волны излучаемых колебаний происходит самопроизвольное профилирование сгустка, в результате которого в обширной области за сгустком формируется кильватерный след с максимально возможной амплитудой поля, в два раза превышающей предельно достижимые амплитуды поля в случае протяженных сгустков с тем же числом частиц. Области максимума поля, возбуждаемые движущимися сгустками частиц в лабораторной системе отсчета, локализованы в месте их образования.

МОДЕЛЮВАННЯ ПРОЦЕСІВ СУПЕРВИПРОМІНЮВАННЯ УЛЬТРАКОРОТКОГО ЗГУСТКА ЗАРЯДЖЕНИХ ЧАСТИНОК, ЩО РУХАЄТЬСЯ У ПЛАЗМІ

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Розглянуто особливості процесів супервипромінювання коротких одномірних моноенергетичних згустків заряджених частинок, що рухаються у плазмі. При розмірах згустка менших за довжину хвилі, що збуджується, в великій області позаду згустка формується кільватерний слід з максимально можливою амплітудою коливань. При цьому відбувається профілювання згустка, що забезпечує його ефективне випромінювання. Області максимуму амплітуди коливань локалізовані там, де вони були сформовані згустками частинок, що рухаються в лабораторній системі відліку.