SLOW AND FAST SURFACE ELECTROMAGNETIC WAVES IN PLANAR STRUCTURES CONTAINED LEFT-HANDED MATERIAL

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It was studied the properties of electromagnetic surface waves in left-handed material slab bounded by two semi-infinite nonmagnetic media with frequency dependent positive permittivity. It was assumed that all these media are isotropic and non-dissipative. It was shown that the phase velocities of waves are greater than the speed of light, and the group velocities are zero or negative.

PACS: 52.35g, 52.50.Dg

INTRODUCTION

A great amount of papers devoted to the studies of modern artificial materials (metamaterials or left-handed materials, LHM) were published after first publications [1, 2]. These papers were mainly devoted to the optical applications of LHM. But it is necessary to mention the studies carried out at the Argonne Wakefield Accelerator Facility; devoted to the investigation of the application of LHM to control the dispersion relation in a loaded waveguide [3, 4].

Very often the planar waveguide structures that contain LHM are embedded either by vacuum or by different ordinary dielectrics with constant permittivity [5, 6]. The presence of ordinary dielectrics leads to the narrowing of possible wavenumbers range for the eigen surface electromagnetic waves of such structures as compared with vacuum bounds.

In the present work it was considered the planar waveguide structure that consists of the left-handed metamererial slab bounded by two non-magnetic plasma-like media which permittivity depends on the wave frequency and may take the value in the range from 0 to 1.

1. TASK SETTINGS

Let us consider the eigen electromagnetic waves that propagate along the planar waveguide structure that contains isotropic LHM slab of thickness Δ. This LHM is characterized by effective permittivity and permeability which depend on the wave frequency as follow [2]:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \qquad (1)$$

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2}, \qquad (2)$$

here ω_p is effective plasma frequency, ω_0 is the characteristic frequency of LHM. In the further study we consider the LHM with $\omega_p / 2\pi = 10$ GHz, $\omega_0 / 2\pi = 4$ GHz and parameter $F = 0.56$ [5].

This LHM slab is bounded on the both sides by two different artificial non-dissipative non-magnetic ($\mu_{1,2} = 1$) materials with effective permittivities

$$\varepsilon_1(\omega) = 1 - \frac{\omega_{p1}^2}{\omega^2}, \qquad (3)$$

$$\varepsilon_2(\omega) = 1 - \frac{\omega_{p2}^2}{\omega^2}, \qquad (4)$$

here ω_{p1}, ω_{p2} is the effective plasma frequencies for corresponding media. In out study we consider that $\omega_{p1} / 2\pi = 2,4$ GHz and $\omega_{p2} / 2\pi = 3,2$ GHz. Such choice of parameters leads to the existence of the frequency range where $\varepsilon(\omega) < 0$ and $0 < \varepsilon_{1,2}(\omega) < 1$ simultaneously.

Let restrict our consideration by the surface electromagnetic wave that propagates along this structure. Besides, it is assumed that all wave components tend to zero far away from LHM and possess the dependence on time t and coordinate z in such form:

$$E, H \propto E(x), H(x) \exp[i(k_3 z - \omega t)], \qquad (5)$$

here x is the coordinate transversal to the wave propagation.

The system of Maxwell equations splits into two subsystems that describe the waves of H - type and E - type.

The wave of E -type possesses the dispersion relation of the following form:

$$\varepsilon(\omega)\,\kappa\left[h_2\varepsilon_1(\omega) + h_1\varepsilon_2(\omega)\right] + [h_1 h_2 \varepsilon(\omega)^2 + \varepsilon_1(\omega)\varepsilon_2(\omega)\kappa^2]\cdot\tanh(\kappa\,\Delta) = 0, \qquad (6)$$

here, $h_{1,2} = \sqrt{k_3^2 - \varepsilon_{1,2}(\omega)\mu_{1,2}k^2}$, $\kappa = \sqrt{k_3^2 - \varepsilon(\omega)\mu(\omega)k^2}$, $k = \omega / c$, c is the speed of light in vacuum.

For *E*-wave the wave field components normalized by the $H_y(0)$ in the region of LHM can be written as:

$$H_y(x) = C_1 e^{\kappa x} + C_2 e^{-\kappa x},$$
$$E_x(x) = k_3\left(C_1 e^{\kappa x} + C_2 e^{-\kappa x}\right) / \left(k\,\varepsilon(\omega)\right), \qquad (7)$$
$$E_z(x) = i\kappa\left(C_1 e^{\kappa x} - C_2 e^{-\kappa x}\right) / \left(k\,\varepsilon(\omega)\right),$$

here C_1 and C_2 are wave field constants.

In first medium (3) the *E*-wave field components, normalized by $H_y(0)$, possess the form:

$$H_y(x) = e^{h_1 x},$$
$$E_x(x) = k_3 e^{h_1 x} / (k\,\varepsilon_1), \qquad (8)$$
$$E_z(x) = i h_1 e^{h_1 x} / (k\,\varepsilon_1).$$

In second plasma-like medium (4) the *E*-wave field components, normalized by $H_y(0)$, can be written as:

$$H_y(x) = A e^{-h_2 x},$$
$$E_x(x) = A k_3 e^{-h_2 x} / \left(k\,\varepsilon_2(\omega)\right), \qquad (9)$$
$$E_z(x) = -i\,A\,h_2 e^{-h_2 x} / \left(k\,\varepsilon_2(\omega)\right),$$

here A is wave field constant. These constants are of the following form:

$$A = -2\,h_1\,\varepsilon_2(\omega)\,\varepsilon(\omega)e^{(h_2+\kappa)\Delta}/(\varepsilon_1(\omega)\Psi_E),$$

$$C_1 = h_1\,\varepsilon(\omega)\left[h_2\varepsilon(\omega) - \varepsilon_2(\omega)\kappa\right]/(\varepsilon_1(\omega)\,\kappa\,\Psi_E), \quad (10)$$

$$C_2 = -h_1\,\varepsilon(\omega)\left[h_2\varepsilon(\omega) + \varepsilon_2(\omega)\kappa\right]e^{2\kappa\Delta}/(\varepsilon_1(\omega)\,\kappa\,\Psi_E),$$

$$\Psi_E = (e^{2\kappa\Delta}+1)\,\varepsilon(\omega)\,h_2 + (e^{2\kappa\Delta}-1)\,\varepsilon_2(\omega)\,\kappa\,.$$

The wave of *H*-type possesses the dispersion relation of the following form:

$$\mu(\omega)\,\kappa\,(h_2\mu_1 + h_1\mu_2) + [h_1h_2\mu(\omega)^2 + \mu_1\mu_2\kappa^2]\tanh(\kappa\,\Delta) = 0. \quad (11)$$

In the region of LHM slab the *H*-wave field components, normalized by the $E_y(0)$, can be written as:

$$E_y(x) = D_1e^{\kappa x} + D_2e^{-\kappa x},$$

$$H_x(x) = -k_3\left(D_1e^{\kappa x} + D_2e^{-\kappa x}\right)/(k\,\mu(\omega)), \quad (12)$$

$$H_z(x) = -i\kappa\left(D_1e^{\kappa x} - D_2e^{-\kappa x}\right)/(k\,\mu(\omega)),$$

here D_1 and D_2 are wave field constants.

In the first medium the *H*-wave field components, normalized by the $E_y(0)$, can be expressed as:

$$E_y(x) = e^{h_1x},$$

$$H_x(x) = -k_3e^{h_1x}/(k\,\mu_1), \quad (13)$$

$$H_z(x) = -ih_1e^{h_1x}/(k\,\mu_1).$$

In the second plasma-like half-space the *H*-wave field components, normalized by $E_y(0)$, can be written as:

$$E_y(x) = Be^{-h_2x},$$

$$H_x(x) = -Bk_3e^{-h_2x}/(k\,\mu_2), \quad (14)$$

$$H_z(x) = i\,B\,h_2e^{-h_2x}/(k\,\mu_2),$$

here B is wave field constant. These constants are of the following form:

$$B = -2\,h_1\,\mu_2\,\mu(\omega)e^{(h_2+\kappa)\Delta}/(\mu_1\Psi_H),$$

$$D_1 = h_1\,\mu(\omega)\,(h_2\mu(\omega) - \mu_2\kappa)/(\mu_1\,\kappa\,\Psi_H), \quad (15)$$

$$D_2 = -h_1\,\mu(\omega)\,(h_2\mu(\omega) + \mu_2\kappa)e^{2\kappa\Delta}/(\mu_1\,\kappa\,\Psi_H),$$

$$\Psi_H = (e^{2\kappa\Delta}+1)\,\mu(\omega)\,h_2 + (e^{2\kappa\Delta}-1)\,\mu_2\,\kappa\,.$$

2. MAIN RESULTS

The numerical solutions of dispersion relations (6) and (11) for E- and H-waves are presented at the Fig. 1. Numbers of curves 1, 2 correspond to the waves of H-type and curves marked by the numbers 3, 4 correspond to the waves of E-type.

It is important to notice that the central metamaterial slab demonstrates left-handed properties ($\varepsilon(\omega) < 0$ and $\mu(\omega) < 0$ simultaneously) for the normalized frequency $1 < \Omega = \omega/\omega_0 < 1{,}5$. The letters on the Fig. 1a,b,c correspond to the curves $\xi = \Omega\sqrt{\varepsilon_2(\omega)}$, $\xi = \Omega\sqrt{\varepsilon_1(\omega)}$, $\xi = \Omega\sqrt{\varepsilon(\omega)\mu(\omega)}$, respectively. The waves of the surface type can exist in the area right to the lines marked by the letters *a, b* and higher of line marked by the letter *c*. The line *L* corresponds to the light in vacuum, i.e. $\xi = \Omega$.

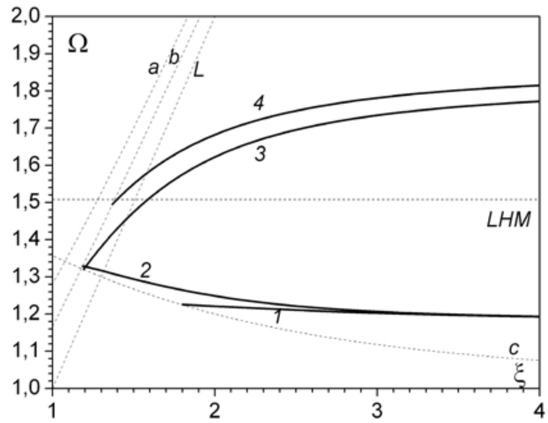

Fig. 1. The dependence of the normalized frequency $\Omega = \omega/\omega_0$ on the dimensionless wave number $\xi = k_3c/\omega_0$ at the thickness $\omega_0\Delta/c = 2$ for $H_{1,2}$-modes (lines 1, 2) and $E_{1,2}$ -modes (lines 3, 4)

Further we shall study the dependence of phase $V_{ph} = \omega/k_3$ and group $V_{gr} = d\omega/dk_3$ velocities on the problem's parameters. Now, the Fig. 1 shows that three modes can have phase velocity higher than light speed in vacuum.

Phase velocities of both $E_{1,2}$-modes (see curves 3, 4 of the Fig. 1) monotonously decreases with the increase of wave frequency. They do not change qualitatively its behavior with the increase of thickness of LHM layer (see Fig. 2). Phase velocity of H_2-mode (curve 2 of the Fig. 1) monotonously increases with the increase of wave frequency and also does not change qualitatively its behavior with increasing of thickness of LHM layer.

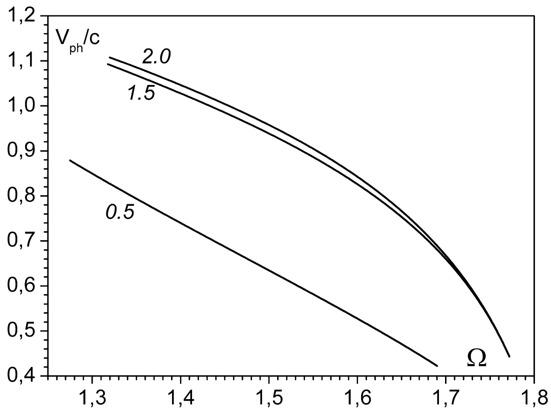

Fig. 2. The dependence of the phase velocity V_{ph}/c of E-wave (curve 3 on Fig. 1) on the frequency $\Omega = \omega/\omega_0$ for different LHM slab thickness $\omega_0\Delta/c = 0{,}5;1.5;2$

But H_1-mode (see curve 1 at Fig. 1) shows qualitatively different behavior (Fig. 3). If LHM slab thickness is rather small (see line marked by the number 0.5 at the Fig. 3) the *H*-wave decreases its phase velocity with the increase of wave frequency. When the LHM slab thickness increases its value (see lines marked by the numbers 1.5 and 2.0 at the Fig. 3) there is a qualitative transition of the behavior of phase velocity versus frequency: phase velocity of the H_1-mode increases with the increase of the wave frequency.

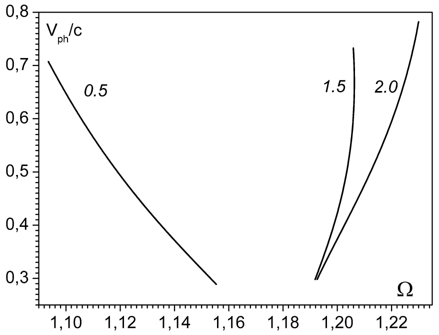

Fig. 3. The dependence of the phase velocity V_{ph}/c of H_1-wave (line 1 on Fig.1) on the frequency $\Omega = \omega/\omega_0$ for different LHM slab thickness $\omega_0\Delta/c = 0,5;1.5;2$

The carried out study shows that the considered surface waves can be either slow or fast depending on the choice of operating frequency.

It was shown that it is possible to change the phase velocity of the considered waves at fixed frequency by the variation of LHM slab thickness Δ. Such dependence is called the geometric dispersion and it is presented on Fig. 4.

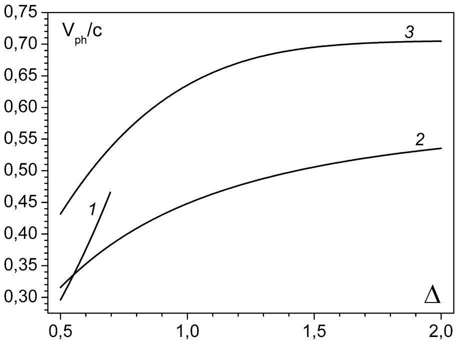

*Fig. 4. The dependence of the phase velocity V_{ph}/c on LHM slab thickness Δ for: **1** – H_1-wave (see curve 2 on Fig. 1, $\Omega = 1,64$); **2** – H_2-wave (see curve 3 on Fig. 1, $\Omega = 1,23$); **3** – E_1-wave (see curve 4 on Fig. 1, $\Omega = 1,154$)*

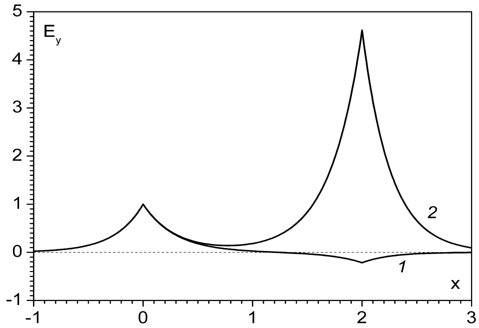

*Fig. 5. The spatial distribution of electric field wave amplitudes versus transversal coordinate x for different modes **1** – H_1-mode ($k_3 = 4,0$, $\Omega = 1,19258$) and **2** – H_2-mode ($k_3 = 4,0$; $\Omega = 1,193586$)*

The normalized phase velocity for E_2-wave linearly changes from 0.5 to 3.2 at the same parameters.

The dependence of electric wave field amplitude on the transversal coordinate x for the considered modes has symmetric or anti-symmetric character (for example Fig. 5).

When the wavelength of the surface wave increases the penetration depth of the considered waves increases sharply, and the wave becomes essentially a bulk wave (Fig. 6).

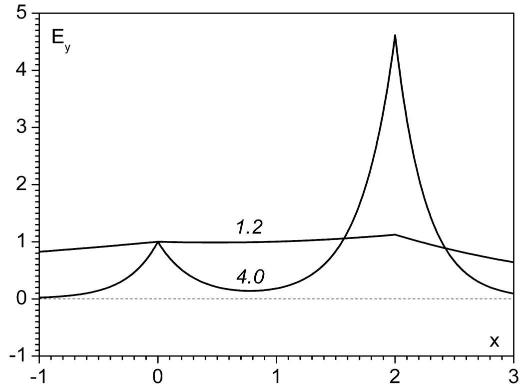

Fig. 6. The spatial distribution of electric field wave amplitudes versus transversal coordinate for H_2-mode for different wavenumber k_3 = 4.0 and k_3 = 1.2

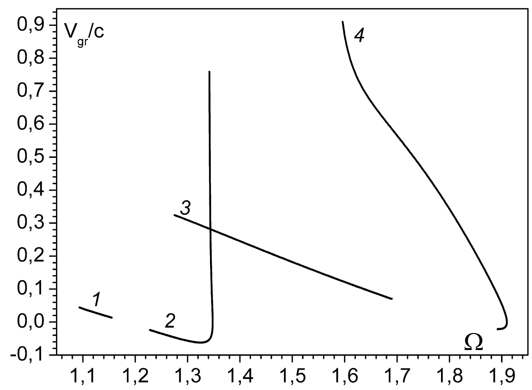

Fig. 7. The dependence of the normalized group velocity V_{gr}/c on of the frequency $\Omega = \omega/\omega_0$ for LHM slab thickness $\omega_0\Delta/c = 0.5$ (numbers of lines corresponds to the Fig. 1)

It is necessary to mention, that the group velocity value of the considered waves can be either zero, or small, or high (Fig. 7).

The Fig. 7 shows that the considered modes are forward and backward, slow and fast. It is shown that in the considered waveguide structure the *H*-modes with zero group velocity can exist. We supposed that such waves can be used in accelerating technology.

CONCLUSIONS

It was found the possibility of existence of the surface electromagnetic waves that propagate along the LHM planar slab that bounded by two semi-infinite plasma-like media.

It was found four modes (two *E*-modes and two *H*-modes) with different wave field structure. It was shown that three modes of this four can have phase velocity higher than light speed in vacuum.

It was also shown that waves considered possess both positive and negative frequency dispersion of the phase velocity.

It was obtained that one of the *H*-waves can be backward (phase and group velocity directed oppositely). The absolute value of the group velocity of *H*-waves can be quite low, down to zero.

It was shown that is possible to change the phase velocity of waves considered at fixed frequency by variation of LHN slab thickness.

The obtained results can be useful for the plasma electronics modeling and improvement of acceleration technology.

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Article received 23.06.2015

МЕДЛЕННЫЕ И БЫСТРЫЕ ПОВЕРХНОСТНЫЕ ЭЛЕКТРОМАГНИТНЫЕ ВОЛНЫ В ПЛОСКИХ СТРУКТУРАХ, СОДЕРЖАЩИХ ЛЕВОСТОРОННИЙ МАТЕРИАЛ

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Изучены свойства поверхностных электромагнитных волн, распространяющихся вдоль слоя левостороннего материала, ограниченного двумя полубесконечными немагнитными средами с положительными диэлектрическими проницаемостями, зависящими от частоты. Предполагалось, что все среды являются изотропны и в них отсутствуют потери. Показано, что фазовые скорости рассматриваемых волн могут быть больше скорости света, а групповые – нулевыми или отрицательными.

ПОВІЛЬНІ ТА ШВИДКІ ПОВЕРХНЕВІ ЕЛЕКТРОМАГНІТНІ ХВИЛІ В ПЛАСКИХ СТРУКТУРАХ, ЩО МІСТЯТЬ ЛІВОСТОРОННІЙ МАТЕРІАЛ

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Вивчені властивості поверхневих електромагнітних хвиль, що поширюються уздовж шару лівостороннього матеріалу, обмеженого двома напівнескінченими немагнітними середовищами з додатними діелектричними проникностями, що залежать від частоти. Вважалось, що всі середовища є ізотропними та в них відсутні втрати. Показано, що фазові швидкості розглянутих хвиль можуть бути більше швидкості світла, а групові – нульовими або від'ємними.