CHILD-LANGMUIR LAW FOR CATHODE SHEATH OF GLOW DISCHARGE IN CO₂

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This work is devoted to the determination of the law that may be applicable to the description of the cathode sheath in CO₂. To this end three versions of the Child-Langmuir law have been considered – a collision free one (for the ions moving through a cathode sheath without collisions with gas molecules) as well as two collision-related versions – one for a constant mean free path of positive ions and one for a constant mobility of positive ions. The current-voltage characteristics and the cathode sheath thickness of the glow discharge in CO₂ have been simultaneously measured in the pressure range from 0.05 to 1 Torr with the discharge current values up to 80 mA. In the whole range of the discharge conditions we have studied the cathode sheath characteristics are found to obey correctly only to the Child-Langmuir law version with a constant ion mobility.

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INTRODUCTION

A direct current glow discharge in CO₂ is widely applied for pumping carbon dioxide gas discharge lasers [1]. Recently there has been observed the growing interest to the plasma conversion of greenhouse gases the carbon dioxide being the principal one (its presence in the Earth atmosphere is of fundamental importance in the ambient medium) to the species which are of interest for chemical industry or which may be used as a fuel for internal combustion engines [2, 3]. Besides, CO₂ is a component of different atmospheres enveloping the planets of the solar system as well as their satellites. The studies of CO₂ plasmas are also necessary because of their applications in plasma reactors of various types.

As the cathode sheath is the most important part of the direct current glow discharge [4-7], the optimization of plasma technologies and respective devices demands studying its characteristics in CO₂ and revealing which Child-Langmuir law version it obeys.

Child [8] and Langmuir [9] have presented the analytical models of a dc discharge in a flat gap between the anode and the cathode. The paper [8] has considered the motion of positive ions form the anode, whereas the paper [9] has dealt with the problem what secondary electrons leave the surface of the heated cathode due to thermoelectric emission and travel to the anode. The effect of the space charge on the charged particle motion has been taken into account. As a result both Child [8] and Langmuir [9] have established the law of the identical mathematical form:

\[ J = K_i \varepsilon_0 \left( \frac{2e}{M} \right)^{\frac{1}{2}} \frac{U^{3/2}}{d^2}, \]  

where \( K_i = 200/243 = 0.82 \), \( \varepsilon_0 \) is the vacuum dielectric permittivity, \( e \) is the elementary charge, \( M \) is the charged particle mass.

Thus the models [8, 9] have dealt only with a collision-free case when charged particles cross the sheath experiencing no collisions with gas molecules. However in many research and technological devices the gas pressure is sufficiently high for the charged particles to participate in frequent collisions with molecules. Therefore the papers of other researchers have reported the following two law versions (they are similarly also called the Child-Langmuir laws), one of which has assumed the ion motion with a constant mean free path \( \lambda_i \) [10, 11]:

\[ J = 1.68 \varepsilon_0 \left( \frac{2e}{M} \lambda_i \right)^{\frac{1}{2}} \frac{U^{3/2}}{d^{5/2}}, \]  

and another one has been obtained for the ion motion with a constant ion mobility \( \mu_i \) (not depending on velocity) [12, 13]:

\[ J = \frac{9}{8} \varepsilon_0 \mu_i \frac{U^2}{d^3}. \]  

Because of the strong electric field present in the cathode sheath one may assume that the law version (2) with the constant ion mean free path \( \lambda_i \) is the most appropriate for the description of the cathode sheath characteristics. The authors of paper [14] have claimed that in the glow discharge in argon the cathode sheath characteristics obey to the law version (2). But in molecular gases one may observe various situations. For example in N₂O [15] at the pressure below 0.3 Torr one observes the law version (3), at the pressure \( p \geq 0.75 \) Torr already the law version (2) is observed, but in the range from 0.3 to 0.75 Torr none of the law versions (1)-(3) cannot be employed for the description of the cathode sheath. In nitrogen [16, 17] at the pressure values \( p < 1 \) Torr and \( p > 1.5 \) Torr the law version (3) for the constant ion mobility \( \mu_i \) is met whereas within the range from 1 to 1.5 Torr none of the law versions (1)-(3) describe the cathode sheath. At the same time in hydrogen in the total range of pressure values from 0.05 to 2 Torr studied by the authors of paper [18] there holds only the law version (3) with the constant ion mobility \( \mu_i \).

This paper reports the current-voltage characteristics and cathode sheath thickness measured in CO₂. It has been...
obtained that in the total range of gas pressure and discharge current values studied one has to employ the law version (3) for the constant mobility $\mu_i$.

\[ I = C \frac{U^m}{d^n}, \quad (4) \]

where $I = IS$ is the discharge current, $S$ is the cathode area. Note that the exponents $m$ and $n$ in these law versions vary. In the collision free law version (1) they are $m = 1.5, n = 2$; in the law version with the constant mean free path (2) we have $m = 1.5, n = 2.5$, and in the version with the constant ion mobility (3) the exponents are equal to $m = 2, n = 3$. In this paper the discharge current $I$, the voltage drop across the sheath $U$ and the cathode sheath thickness $d$ have been measured simultaneously for different fixed values of the carbon dioxide pressure. With these data the dependence of the current $I$ on the $U^m/d^n$ ratio has been plotted. Note that the values of the $U^m/d^n$ ratio differ strongly for each of the $m, n$ pairs thus complicating the treatment of the (1)-(3) law versions. Therefore the $U^m/d^n$ values have been normalized over their average value for a given $m, n$ pair at the gas pressure fixed. If for one of the $m, n$ pairs the $I(U^m/d^n)$ dependence will be well described with a straight line drawn through the origin of coordinates then the Child-Langmuir version which corresponds to this pair is applicable for the description of the cathode sheath.

In Fig. 2 the dependences of the discharge current and cathode sheath thickness on the voltage across the electrodes are presented at the gas pressure of 0.05 Torr. As is clear from the figure, the increase in the voltage leads to the current growth, the CVC possesses a positive tilt and the discharge is burning only in the abnormal mode covering the total cathode surface. The maximum cathode sheath thickness is observed at the lowest current values. With the growth of the voltage across the electrodes and the current the cathode sheath thickness decreases and then becomes actually constant.

Fig. 2 also presents the dependence of the discharge current on the normalized values of the $U^m/d^n$ ratio. It is clear from the figure that in a broad range of discharge current values only the results for the $m = 2, n = 3$ pair fit a straight line drawn through the origin of coordinates, this pair corresponding to the law version (3) associated with the constant ion mobility in the cathode sheath. For two other $m, n$ pairs corresponding to the ion motion without collisions with gas molecules (1) as well as to the motion with constant ion mean free path (2) the results demonstrate poor agreement with a linear dependence especially in the range of high discharge current values.

One may clearly observe a normal mode at the discharge CVC (Fig. 3) at the CO$_2$ pressure of 1 Torr and the current below 11 mA what indicates a part of the negative (falling) CVC when the discharge occupied only a part of the cathode surface. After the discharge transition

**Fig. 1. Design of the discharge setup**

1. EXPERIMENTAL

We have employed the discharge chamber with the design show in Fig. 1. The glass discharge tube possessed the inner diameter of 56 mm. The CO$_2$ pressure has been measured with the capacitive manometer (Baratron) having the maximum measured pressure of 10 Torr. The experiments have been performed within the gas pressure range from 0.05 to 1 Torr.

The movable anode could be displaced along the discharge tube axis, its diameter comprising 55 mm. The discharge current studied did not exceed 80 mA.

Experiments have been performed in short tubes with the discharge consisting of only the cathode sheath and a small part of the negative glow. Therefore the voltage drop over the cathode sheath was approximately equal to the potential difference across the electrodes.

2. EXPERIMENTAL RESULTS

All three Child-Langmuir law versions (1)-(3) may be cast in the general form:

**Fig. 2. The discharge current $I$ and the cathode sheath thickness $d$ versus the voltage across the electrodes as well as the discharge current versus the $U^m/d^n$ ratio for the carbon dioxide pressure of 0.05 Torr**
to the abnormal mode when the discharge has covered the cathode completely, the current grows with the voltage across the electrodes increasing.

![Graph](attachment:graph.png)

**Fig. 3.** The discharge current $I$ and the cathode sheath thickness $d$ versus the voltage across the electrodes as well as the discharge current versus the $U^m/d^n$ ratio for the carbon dioxide pressure of $1$ Torr.

![Graph](attachment:graph2.png)

**Fig. 4.** The emission spectrum of the cathode glow (2 mm from the cathode surface) at the CO$_2$ pressure of $0.5$ Torr and the discharge current of $5$ mA.

In this case the expansion of the discharge over the cathode is accompanied, first, by a decrease of the voltage across the electrodes (from $390$ V before the discharge extinction at $374$ V when it experiences a transition from the normal mode to the abnormal one). Second, the cathode sheath thickness does not remain constant but also decreases. Despite the falling pattern of the $I(U)$ and $d(U)$ dependences the normal mode is depicted on the $I(U^m/d^n)$ graph by practically vertical sections of the curves for different $m$, $n$ pairs. After the discharge transition to the abnormal mode the increase of the voltage across the electrodes is accompanied by the current increase and the cathode sheath thickness decrease. Only the results for the $m = 2$, $n = 3$ pair fit well the straight line drawn through the origin of coordinates indicating the applicability of the law version (3) with the constant ion mobility for the description of the cathode sheath characteristics.

It has been obtained as a result that in the total range of discharge the cathode sheath characteristics obey only the Child-Langmuir law version with the constant ion mobility (3).

This phenomenon may be associated with a considerable conversion of CO$_2$ molecules in the cathode sheath and the negative glow. This is supported by the optical spectra of radiation (Fig. 4). It has been demonstrated that near the cathode surface the radiation line of atomic oxygen OI 777 nm is the most intense one (in the range from 400 to 1000 nm). Its intensity exceeds ten times the line intensities of CO, CO$_2$ and their ions but on moving away from the cathode it decreases fast. In the negative glow the line intensities of different atoms, molecules and ions are comparable in magnitude (Fig. 5).

![Graph](attachment:graph3.png)

**Fig. 5.** The axial distribution of emission line intensities at the CO$_2$ pressure of $0.05$ Torr, the discharge current of $10$ mA and the distance between the electrodes of $50$ mm.

**CONCLUSIONS**

In this paper the CVC and cathode sheath thickness of the glow discharge in CO$_2$ have been measured simultaneously in a wide range of pressure and discharge current values. The paper has been aimed to find out which of the Child-Langmuir law versions may be applied for the description of the cathode sheath in CO$_2$. It has been obtained that in the wide range of discharge conditions studied the characteristics of the cathode sheath obey only the Child-Langmuir law version associated with the constant ion mobility (3). This phenomenon may be caused by a substantial conversion of carbon dioxide molecules.
В результате получено, что во всём исследованном нами диапазоне розрядных условий характеристики катодного слоя корректно описываются законом Чайлд-Ленгмюра с постоянной подвижностью ионов.

**ЗАКОН ЧАЙЛЬДА-ЛЕНГМЮРА ДЛЯ КАТОДНОГО СЛОЯ ТЛЕЮЩЕГО РАЗРЯДА В CO2**

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Данная работа посвящена определению закона, который может применяться для описания катодного слоя в CO2. Выполнен анализ трёх законов Чайлд-Ленгмюра – бестолкновительного (для движения ионов через катодный слой без столкновений с молекулами газа), а также двух столкновительных – с постоянной и переменной подвижностью ионов. Для этого были одновременно измерены ионные характеристики с постоянной подвижностью ионов.

**ЗАКОН ЧАЙЛЬДА-ЛЕНГМЮРА ДЛЯ КАТОДНОГО ШАРУ ТЛЮЧОГО РОЗРЯДУ В CO2**

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Цей роботой присвячена визначенню закону, який може застосовуватися для опису катодного шару в CO2. Виконано аналіз трьох законів Чайлд-Ленгмюра – для руху іонів крізь катодний шар без згінень з молекулами газу, а також законів з постійною рухливістю вільного пробігу і рухливістю позитивних іонів. Для цього були одноразово виміряні вольт-амперні характеристики і товщина катодного шару тлючого розряду в CO2 в діапазоні тиску від 0,05 до 1 Торр і розрядних струмів до 80 мА. У результаті отримано, що в усьому дослідженному нами діапазоні розрядних умов характеристики катодного шару коректно описываються тільки законом Чайлд-Ленгмюра з постійною рухливістю іонів.

**REFERENCES**


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