THE CAPACITIVE COMPONENT OF DOUBLE LAYER CURRENT IN PLASMA

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The features of nonstationary double layers in the high-current pulsed discharges have been theoretically and experimentally investigated in this paper. The expression for the capacity of strong double layer in quasi-MHD approximation has been obtained and the area of its applicability has been indicated. The equation for the capacitive component of the double layer current has been derived in this paper. The dynamics of the double layer current capacitive component in the high-current pulsed discharge and the way to verify the calculations has been shown.

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INTRODUCTION

Recently, the interest in the study of space charge electric double layer in the plasma has renewed [1]. This is due to the ability of the double layer to input the pulse energy with a power density of 1…10 GW/cm^2 locally into the plasma. Such power levels give the opportunity to obtain intensive neutron fluxes from the plasma, the super-powerful directional radiation and to provide high-gradient effect on the solids surface of different nature for modification of the structural-phase state on different scale levels.

However, it should be noted that a double layer is a powerful dynamic system whose parameters are change at high speed. From electrical point of view double layer can be represented as an aggregate of a resistor and a capacitor (Fig. 1,b). When changing the voltage and current of the layer the capacitor charge also changes. This entails the appearance of the capacitive component in the discharge current, which must be taken into account when calculating the input power into the discharge.

CALCULATING

The value of the current capacitive component $i_C(t)$ is determined by the double layer charge changing $Q(t)$:

$$i_C(t) = C_{DL} \cdot \frac{dV_{DL}}{dt} + V_{DL} \cdot \frac{dC_{DL}}{dt}, \qquad (1)$$

which depends both on the layer capacity C_{DL} and its voltage V_{DL}. Therefore, at first we define the layer capacity.

<u>*The double layer capacity*</u>

Because the layer thickness is much smaller than its transverse dimensions, the double layer specific capacity C_{1DL} we find from the expression:

$$C_{1DL} = \frac{q_1}{V_{DL}} = \frac{1}{V_{DL}} \cdot \int_0^{l_{DL}/2} |\rho(z)| dz . \qquad (2)$$

The charge density distribution in the layer $\rho(z)$ depends on the charge distribution $q_\alpha n_\alpha(z)$ of 4th particles components [2] (see Fig. 1,a):

$$\rho(z) = q_e n_{be}(z) + q_{bi} n_{bi}(z) + q_e n_{re}(z) + q_{ri} n_{ri}(z), \qquad (3)$$

where *be* – the accelerated electrons, *bi* – the accelerated ions, *re* – the reflected electrons, *ri* – the reflected ions.

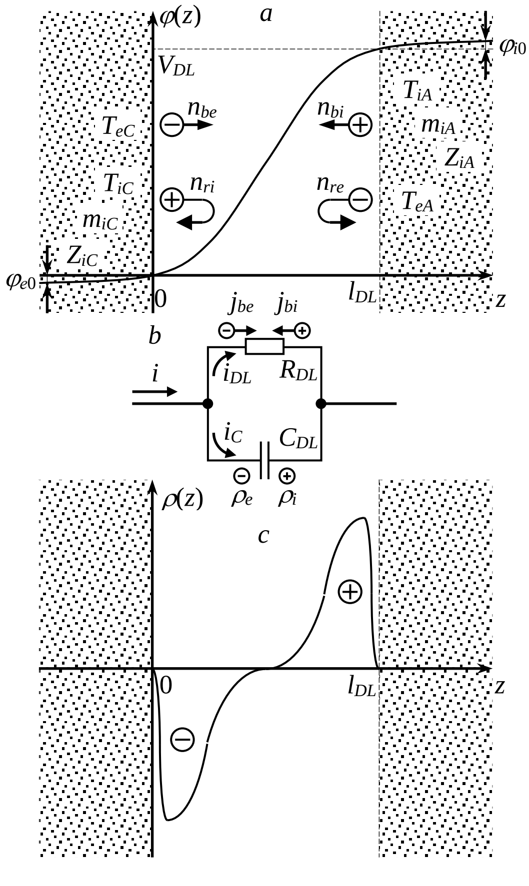

Fig. 1. The qualitative potential distribution (a) and the charge density (c) in the double layer, the equivalent electrical circuit of the double layer (b)

For accelerated electrons and ions the charge distribution is determined from the current continuity conditions:

$$q_e n_{be}(z) = \frac{j_{be}}{v_{be}(z)} = -\sqrt{\frac{m_e}{2e}} \cdot j_{be} \cdot \frac{1}{\sqrt{\varphi_{e0} + \varphi(z)}}, \qquad (4)$$

$$q_{bi}n_{bi}(z)=\frac{j_{bi}}{v_{bi}(z)}=j_{bi}\cdot\sqrt{\frac{m_{iA}}{2Z_{iA}e}}\cdot\frac{1}{\sqrt{V_{DL}+\varphi_{i0}-\varphi(z)}}, \quad (5)$$

where φ_{e0},φ_{i0} – potentials corresponding to the starting energy of the electrons $\varphi_{e0}=m_e v_{eC0}^2/2e$ and ions $\varphi_{i0}=m_{iA}v_{iA0}^2/2Z_{iA}e$ (Z_{iA} – the effective ions charge).

The volume charge density distribution of accelerated ions using a Langmuir ratio [3] $\alpha=\sqrt{m_{iA}/Z_{iA}m_e}\cdot j_{bi}/j_{be}$, can be represented as:

$$q_{bi}n_{bi}(z)=\sqrt{\frac{m_e}{2e}}\cdot j_{be}\cdot\frac{\alpha}{\sqrt{V_{DL}+\varphi_{i0}-\varphi(z)}}. \quad (6)$$

The volume charge density distribution of reflected electrons $q_e n_{re}(z)$ and ions $q_{ri}n_{ri}(z)$ in case of the Maxwell velocity particle distribution function obey the Boltzmann law, and for strong double layer is given by:

$$q_e n_{re}(z)\approx$$
$$\approx-\sqrt{\frac{m_e}{2e}}\cdot j_{be}\cdot\left(\frac{\alpha}{\sqrt{\varphi_{i0}}}-\frac{1}{\sqrt{V_{DL}+\varphi_{e0}}}\right)\cdot e^{-\frac{V_{DL}-\varphi(z)}{T_{eA}/e}}, \quad (7)$$

$$q_{ri}n_{ri}(z)\approx$$
$$\approx\sqrt{\frac{m_e}{2e}}\cdot j_{be}\cdot\left(\frac{1}{\sqrt{\phi_{e0}}}-\frac{\alpha}{\sqrt{V_{DL}+\phi_{i0}}}\right)\cdot e^{-\frac{\phi(z)}{T_{ic}/Z_{iC}e}}. \quad (8)$$

(For strong double layer $q_\alpha V_{DL}>>T_\alpha$). These expressions are obtained from the quasi-neutrality condition at the layer boundaries:

$$n_{re}\big|_{\varphi(z)=V_{DL}}+n_{be}\big|_{\varphi(z)=V_{DL}}\approx Z_{ia}n_{bi}\big|_{\varphi(z)=V_{DL}} \quad \text{and}$$
$$Z_{ic}n_{ri}\big|_{\varphi(z)=0}+Z_{ia}n_{bi}\big|_{\varphi(z)=0}\approx n_{be}\big|_{\varphi(z)=0}.$$

After the substitution of equations (4,6,7,8) into the expression (3) using dimensionless quantities $\phi(z)=\varphi(z)/V_{DL}$, $\varepsilon_{iC}=T_{iC}/Z_{iC}eV_{DL}$, $\varepsilon_{eA}=T_{eA}/eV_{DL}$, $\phi_{e0}=\varphi_{e0}/V_{DL}$, $\phi_{i0}=\varphi_{i0}/V_{DL}$ the charge density distribution in the strong double layer will look like:

$$\rho(\varphi)=\sqrt{\frac{m_e}{2e}}\cdot\frac{j_{be}}{\sqrt{V_{DL}}}\cdot\left[\frac{\alpha}{\sqrt{1+\varphi_{ai}-\varphi}}-\frac{1}{\sqrt{\varphi_{ce}+\varphi}}+\right.$$
$$+\left(1/\sqrt{\varphi_{ce}}-\alpha/\sqrt{1+\varphi_{ai}}\right)\cdot e^{-\varphi/\varepsilon_{ic}}-$$
$$\left.-\left(\alpha/\sqrt{\varphi_{ai}}-1/\sqrt{1+\varphi_{ce}}\right)\cdot e^{-(1-\varphi)/\varepsilon_{ea}}\right]. \quad (9)$$

Thus, the specific capacity of the strong double layer (2) is given by:

$$C_{1DL}=\sqrt{m_e/2e}\cdot\left|I(\alpha,\varphi_{ai},\varphi_{ce},\varepsilon_{ic},\varepsilon_{ea})\right|\cdot\left|j_{be}\right|/\left|V_{DL}\right|^{3/2}, \quad (10)$$

where

$$I(\alpha,\phi_{ai},\phi_{ce},\varepsilon_{ic},\varepsilon_{ea})=$$
$$=\int_0^{0.5}\left[\alpha/\sqrt{1-\phi+\phi_{ai}}-1/\sqrt{\phi+\phi_{ce}}+\right.$$
$$+\left(1/\sqrt{\phi_{ce}}-\alpha/\sqrt{1+\phi_{ai}}\right)\cdot e^{-\phi/\varepsilon_{ic}}-$$
$$\left.-\left(\alpha/\sqrt{\phi_{ai}}-1/\sqrt{1+\phi_{ce}}\right)\cdot e^{-(1-\phi)/\varepsilon_{ea}}\right]d\phi, \quad (11)$$

whose solution has the form:

$$I(\alpha,\varepsilon_{ic},\varepsilon_{ea},\varphi_{ai},\varphi_{ce})=$$
$$=-2\alpha\left(\sqrt{0.5+\varphi_{ai}}-\sqrt{1+\varphi_{ai}}\right)-$$
$$-2\left(\sqrt{0.5+\varphi_{ce}}-\sqrt{\varphi_{ce}}\right)- \quad (12)$$
$$-\varepsilon_{ic}\left(1/\sqrt{\varphi_{ce}}-\alpha/\sqrt{1+\varphi_{ai}}\right)\cdot\left(e^{-1/2\varepsilon_{ic}}-1\right)-$$
$$-\varepsilon_{ea}\left(\alpha/\sqrt{\varphi_{ai}}-1/\sqrt{1+\varphi_{ce}}\right)\cdot\left(e^{-1/2\varepsilon_{ea}}-e^{-1/\varepsilon_{ea}}\right).$$

Integrating C_{1DL} on the layer surface, we obtain the double layer full capacity C_{DL}:

$$C_{DL}=\int_{S_{DL}}C_{1DL}ds=$$
$$=\int_{S_{DL}}\sqrt{\frac{m_e}{2e}}\cdot\frac{|j_{be}|}{|V_{DL}|^{3/2}}\cdot\left|I(\alpha,\varepsilon_{ic},\varepsilon_{ea},\varphi_{ai},\varphi_{ce})\right|ds=$$
$$=\sqrt{\frac{m_e}{2e}}\cdot\frac{\left|I(\varepsilon_{ic},\varepsilon_{ea},\varphi_{ai},\varphi_{ce})\right|}{|V_{DL}|^{3/2}}\cdot\int_{S_{DL}}|j_{be}|ds=$$
$$=\sqrt{\frac{m_e}{2e}}\cdot\left|I(\varepsilon_{ic},\varepsilon_{ea},\varphi_{ai},\varphi_{ce})\right|\cdot\frac{|i_{be}|}{|V_{DL}|^{3/2}}. \quad (13)$$

Here $|i_{be}|=\int_{S_{DL}}|j_{be}|ds$ – the electron beam current in the layer can be associated with the total current through the layer $|i_{DL}|$, which is the sum of electron i_{be} and ion i_{bi} currents. Taking into account the Langmuir ratio

$$|i_{DL}|=|i_{be}|+|i_{bi}|=|i_{be}|\cdot\left(1+\alpha\sqrt{Z_{iA}m_e/m_{iA}}\right), \quad (14)$$

from which:

$$|i_{be}|=|i_{DL}|\cdot\frac{1}{1+\alpha\sqrt{Z_{iA}m_e/m_{iA}}}. \quad (15)$$

Taking into account that $\alpha\sqrt{Z_{iA}m_e/m_{iA}}<<1$, then $|i_{be}|\approx|i_{DL}|$, and finally, the double layer capacity is given by:

$$C_{DL}\approx\sqrt{\frac{m_e}{2e}}\cdot\frac{|i_{DL}|}{|V_{DL}|^{3/2}}\cdot\left|I(\alpha,\varepsilon_{ic},\varepsilon_{ea},\phi_{ai},\phi_{ce})\right|. \quad (16)$$

Fig. 2 shows the dependence of the reduced double layer capacity $C_{DL}/|i_{DL}|$ on the layer voltage drop $|V_{DL}|$ at zero particles temperatures on the layer boundaries ($\phi_{ai}=0$, $\phi_{ce}=0$, $\varepsilon_{ic}=0$, $\varepsilon_{ea}=0$). It is seen that when the voltage increasing the double layer capacity decreases. When the voltage on the layer ~ 100 *V* and the current ~ 33 *kA* the layer capacity is ~ 0.5 *μF*! But when the voltage ~ 1 *kV*, and the current has same value, the layer capacity is reduced to 15 *nF*.

Taking into account the particles temperature in the isothermal case reduced double layer capacity can be written as:

$$\frac{C_{DL}\cdot T_{ea}^{3/2}}{|i_{DL}|}\approx\sqrt{\frac{m_e}{2e}}\cdot\frac{\left|I(\alpha,\varepsilon_{ic},\varepsilon_{ea},\phi_{ai},\phi_{ce})\right|}{|eV_{DL}/T_{ea}|^{3/2}}. \quad (17)$$

Fig. 3 shows the dependence of specific double layer capacity $C_{DL}T_{ea}^{3/2}/|i_{DL}|$ on the relative layer potential drop $|eV_{DL}/T_{ea}|$ at $\phi_{ai}=\phi_{ce}$, $\varepsilon_{ic}=\varepsilon_{ea}$.

Nonmonotonic in the top left part of the graph associated with the violation of the expression applicability conditions (17), which is valid for the strong double layer ($eV_{DL} >> T_{ea}$), when the penetration of reflected particles on the opposite side of the layer not taken into account.

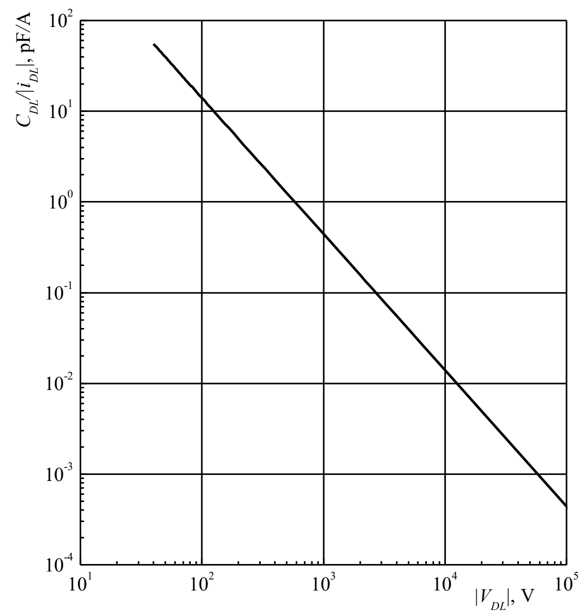

Fig. 2. The dependence of the reduced double layer capacity $C_{DL}/|i_{DL}|$ on the layer voltage drop $|V_{DL}|$ at $\phi_{ai} = 0$, $\phi_{ce} = 0$, $\varepsilon_{ic} = 0$, $\varepsilon_{ea} = 0$

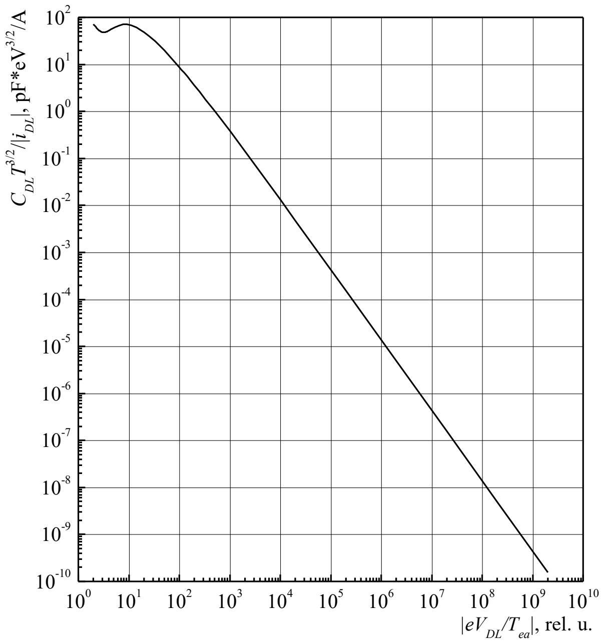

Fig. 3. The dependence of the specific double layer capacity $C_{DL}T_{ea}^{3/2}/i_{DL}$ on the relative layer potential drop eV_{DL}/T_{ea} at $\phi_{ai} = \phi_{ce}, \varepsilon_{ic} = \varepsilon_{ea}$

<u>The capacitive component of the current</u>

The differential of the double layer capacity in (1) can be obtained by differentiating the expression (16) by $|i_{DL}|$ and $|V_{DL}|$:

$$dC_{DL} = \frac{A}{|V_{DL}|^{3/2}} \cdot \left[d|i_{DL}| - \frac{3}{2} \cdot \frac{|i_{DL}| \cdot d|V_{DL}|}{|V_{DL}|} \right], \quad (18)$$

where $A = 1.687 \cdot 10^{-8} \cdot |I(\varepsilon_{ic}, \varepsilon_{ea}, \phi_{ai}, \phi_{ce})|$, F·V$^{3/2}$/A.

Substituting it in equation (1) we obtain the equation for current capacitive component $i_C(t)$:

$$i_C(t) \cdot dt = \frac{A}{|V_{DL}|^{3/2}} \cdot \left[|i_{DL}| \cdot dV_{DL} + V_{DL} \cdot d|i_{DL}| - \right.$$
$$\left. - \frac{3}{2} \cdot \frac{V_{DL}}{|V_{DL}|} \cdot |i_{DL}| \cdot d|V_{DL}| \right]. \quad (19)$$

Expressing the double layer current i_{DL} through the discharge current i and the capacitive current i_C taking into account that $|i_C| < |i|$ we can write $|i_{DL}| = |i - i_C| = |i| - |i_C|$. Substituting it in (19) and grouping terms, we can rewrite the expression for the current capacitive component in the form of:

$$\frac{d|i_C|}{dt} + \frac{1}{A} \cdot \frac{|V_{DL}|^{3/2}}{V_{DL}} \cdot i_C - \frac{1}{2V_{DL}} \cdot \frac{dV_{DL}}{dt} \cdot |i_C| =$$
$$= \frac{d|i|}{dt} - \frac{1}{2V_{DL}} \cdot \frac{dV_{DL}}{dt} \cdot |i|. \quad (20)$$

Here are two cases when the capacitive current takes a positive value ($i_C > 0$):

$$\frac{di_C}{dt} + \frac{1}{2V_{DL}} \cdot \left(\frac{2}{A} \cdot |V_{DL}|^{3/2} - \frac{dV_{DL}}{dt} \right) \cdot i_C =$$
$$= \frac{d|i|}{dt} - \frac{1}{2V_{DL}} \cdot \frac{dV_{DL}}{dt} \cdot |i|, \quad (21)$$

and negative value ($i_C < 0$):

$$\frac{di_C}{dt} - \frac{1}{2V_{DL}} \cdot \left(\frac{2}{A} \cdot |V_{DL}|^{3/2} - \frac{dV_{DL}}{dt} \right) \cdot i_C =$$
$$= -\frac{d|i|}{dt} + \frac{1}{2V_{DL}} \cdot \frac{dV_{DL}}{dt} \cdot |i|. \quad (22)$$

Introducing the notation

$$\kappa(t) = \frac{1}{2V_{DL}} \cdot \left(\frac{2}{A} \cdot |V_{DL}|^{3/2} - \frac{dV_{DL}}{dt} \right), \quad (23)$$

$$F(t) = \frac{d|i|}{dt} - \frac{1}{2V_{DL}} \cdot \frac{dV_{DL}}{dt} \cdot |i| \quad (24)$$

and solving the differential equations (21) and (22) we find the expression for the capacitive component of the double layer current at $i_C > 0$

$$i_C(t) = e^{-\int_0^t \kappa(\tau) d\tau} \cdot \int_0^t F(\tau) \cdot e^{\int_0^\tau \kappa(\xi) d\xi} d\tau, \quad (25)$$

and at $i_C < 0$:

$$i_C(t) = -e^{\int_0^t \kappa(\tau)d\tau} \cdot \int_0^t F(\tau)\cdot e^{-\int_0^\tau \kappa(\xi)d\xi} d\tau . \quad (26)$$

These equations are solved by numerical methods. Fig. 4 shows the dynamics of the double layer current capacitive component $i_C(t)$ (curve 1) and the active voltage $U_a(t)$ (curve 2) in the high-current pulsed discharge.

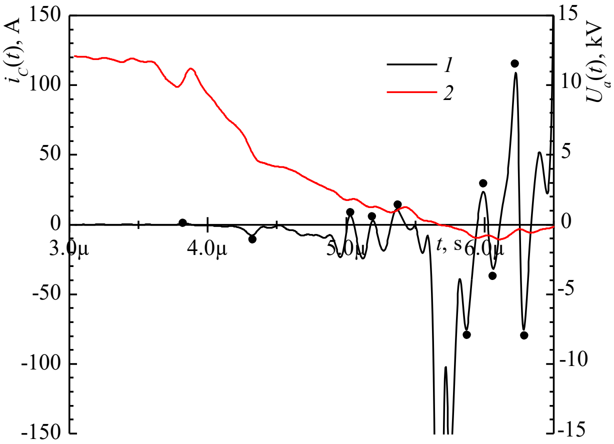

Fig. 4. The dynamics of the double layer current capacitive component $i_C(t)$ (1) and the active voltage $U_a(t)$ (2) of the high-current pulsed discharge

It is seen that the current capacitive component has the form of bursts with positive and negative polarity. It means that charge-discharge of the double layer capacity occurs constantly. When the voltage decreases the capacitive current value increases. This occurs due to the fact that the double layer capacity increases, that requires bigger current for its recharging.

The accuracy of calculations was determined by the verification points, which were calculated from phase measurements by means of expression $i_C \approx \omega \cdot C_{DL}(i, V_{DL}) \cdot \Delta V_{DL}$, where ω – the cyclic frequency of the discharge current "fine structure" at the given point, V_{DL} – the double layer potential drop which was assumed equal to the active discharge voltage $U_a(t)$, a layer capacity C_{DL} is calculated for the discharge current values i and $U_a(t)$. The figure shows that the verification points well agree with the calculated curve of the current capacitive component.

CONCLUSIONS

The expression for the capacity of space charge electric double layer in the plasma in quasi-MHD approximation has been obtained in this paper. The applicability area of the found expression has been indicated. The estimates have shown that in pulsed discharges the capacity double layer can reach considerable values comparable to the capacity of the supply capacitor bank. It has been shown that when the layer voltage increases its capacity decreases. So by increasing the voltage drops from 10^2 to 10^5 *V*, the specific layer capacity decreases in $3 \cdot 10^4$ times.

Taking into account the expression for the double layer capacity, the integral-differential equation for the capacitive component of the double layer current has been obtained in this paper. The numerical calculations have shown a good agreement between the calculated curve of the current capacitive component and the experimentally obtained verification points.

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ЁМКОСТНАЯ СОСТАВЛЯЮЩАЯ ТОКА ДВОЙНОГО СЛОЯ В ПЛАЗМЕ

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Теоретически и экспериментально исследуются особенности нестационарных двойных слоёв в сильноточных импульсных разрядах. В квази-МГД-приближении получено выражение для ёмкости сильного двойного слоя и указана область его применимости. Получено уравнение для ёмкостной составляющей тока двойного слоя. Показана динамика ёмкостной составляющей тока двойного слоя в сильноточном импульсном разряде и приведен способ верификации расчётов.

ЄМНІСНА СКЛАДОВА СТРУМУ ПОДВІЙНОГО ШАРУ В ПЛАЗМІ

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Теоретично та експериментально досліджуються особливості нестаціонарних подвійних шарів у сильнострумових імпульсних розрядах. У квазі-МГД-наближенні отримано вираз для ємності сильного подвійного шару та вказана область його застосування. Отримано рівняння для ємнісної складової струму подвійного шару. Показана динаміка ємнісної складової струму подвійного шару в сильнострумовому імпульсному розряді та приведено спосіб верифікації розрахунків.