# ${ }^{238} U$ FISSION NEAR THE THRESHOLD 

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Using the data on angular distribution of fission fragments, a threshold has been determined in a dipole fission channel with $J=1$ spin projection to the nucleus symmetry axis. It has been shown that the peak observed in the ${ }^{238} U$ fission cross section is determined by the contribution from the quadrupole excitation.
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## 1. INTRODUCTION

Studies of angular distribution of fission fragments provide important information on the properties of fission barriers for heavy nuclei as well as on quantum numbers of lower excitation states. In photofission, low-spin states are excited [1]. At small photon energies fission is determined mainly by the electric dipole ( $E 1$ ) excitation and by the much smaller contribution of the electric quadrupole $(E 2)$ excitation. These experiments provide information on the angular distribution of fission fragments. Using this information and the new technique, we have determined the fission barrier height of ${ }^{238} U$ in the dipole channel with spin projection on the nuclear symmetry axis $K=1$. Angular distributions of fission fragments from $E 1$ and $E 2$ of the excited states are well described by the equation (1).

$$
\begin{equation*}
W(\theta)=a+b \sin ^{2}(\theta)+c \sin ^{2}(2 \theta) \tag{1}
\end{equation*}
$$

Coefficients $a, b, c$ are determined by the contributions from 5 channels with quantum numbers $\left(J^{\pi}, K\right)=\left(1^{-}, 0\right),\left(1^{-}, 1\right),\left(2^{+}, 0\right),\left(2^{+}, 1\right),\left(2^{+}, 2\right)$, where $J$ and $\pi$ are spin and parity of the excited state of the nucleus, respectively, and $K$ is the projection of spin $J$ to the nucleus symmetry axis. $\theta$ is the angle between the photon beam direction and the direction of exiting fragments (these two directions determine the plane of the reaction). Coefficients $a$ and $b$ are connected with dipole and quadrupole contributions, and coefficient $c$ is connected with quadrupole contributions only. From the fit of expression (1) to the experimental data obtained three quantities, $a, b$, and $c$, so for the data analysis respectively employs three main fission channels, $\left(1^{-}, 0\right),\left(1^{-}, 1\right),\left(2^{+}, 0\right)$, while the contributions of channels $\left(2^{+}, 1\right),\left(2^{+}, 2\right)$ are ignored.

In references [2-7] linearly polarized photons have been used for the studies of photofission of heavy nuclei. In these experiments, the new independent quantity has been obtained, namely, the
$\Sigma$-asymmetry, which characterizes the analyzing capacity of the photonuclear reaction.

Theoretical formalism for the analysis of fission by polarized photons has been developed in $[2,3]$. Within this approach, quantity $\Sigma(\theta)$ is defined as

$$
\begin{equation*}
\Sigma(\theta)=\frac{1}{P_{\gamma}} \frac{W(\theta, \varphi=0)-W(\theta, \varphi=\pi / 2)}{W(\theta, \varphi=0)+W(\theta, \varphi=\pi / 2)} \tag{2}
\end{equation*}
$$

where $P_{\gamma}$ is the degree of photon beam polarization, $W(\theta, \varphi)$ is the angular distribution of fission fragments, $\varphi$ is the angle between the polarization vector of the photon and the plane of the reaction.

The angular distribution of fragments from the fission by linearly polarized photons is given for the total moment $g \leq 2$ by the following expression [3],

$$
\begin{align*}
& W(\theta, \varphi)=a+b \sin ^{2}(\theta)+c \sin ^{2}(2 \theta)+ \\
& \quad \omega P_{\gamma} \cos (2 \varphi)\left(d \sin ^{2}(\theta)-4 c \sin ^{4}(\theta)\right) \tag{3}
\end{align*}
$$

where $\omega=+1$ for the electric excitations and $\omega=-1$ for the magnetic excitations. The coefficient $d$ is connected with dipole and quadrupole contributions. In this case, expression for $\Sigma(\theta)$ has the form

$$
\begin{equation*}
\Sigma(\theta)=\omega \frac{d \sin ^{2}(\theta)-4 c \sin ^{4}(\theta)}{a+b \sin ^{2}(\theta)+c \sin ^{2}(2 \theta)} \tag{4}
\end{equation*}
$$

For $\theta=\pi / 2$

$$
\begin{equation*}
\Sigma\left(\theta=\frac{\pi}{2}\right)=\omega \frac{d-4 c}{a+b} \tag{5}
\end{equation*}
$$

It is known from the analysis of angular distribution of fragments from fission by nonpolarized photons $\left(P_{\gamma}=0\right.$ in Eq. (3)) that the contribution from the quadrupole fission in the energy range from 5 to 10 MeV is small, i.e. the process is determined by the dipole excitation only. As shown in [3] the value of analyzing capacity $\Sigma(\theta)$ is determined in case of purely dipole transition $(\omega=+1, c=0)$ by coefficients $a$ and $b$ only, because $d=b$. This allows one to compare the result obtained in the experiment with polarized photons with the value of $\Sigma(\theta)$ computed on

[^0]the basis of the results of angular distribution measurements for nonpolarized photon beam. It can be seen from Eq.(3) that also in this case ( $P_{\gamma}=0, c=0$ ) the angular distribution is determined only by coefficients $a$ and $b$, and the expression for $\Sigma(\theta)$ takes the form
\[

$$
\begin{equation*}
\Sigma\left(\theta=\frac{\pi}{2}\right)=\frac{b}{a+b} \tag{6}
\end{equation*}
$$

\]

In paper [8] is shown, that the coefficients of angular distribution of fragments from fission are related to the cross-sections of dipole channels of fission as follows

$$
\begin{equation*}
\frac{b}{a}=\frac{\sigma_{\gamma, f}\left(1^{-}, 0\right)}{\sigma_{\gamma, f}\left(1^{-}, 1\right)}-\frac{1}{2} \tag{7}
\end{equation*}
$$

## 2. ANALYSIS OF EXPERIMENTAL DATA $[10,11]$

Putting expression (7) into (6) it was got

$$
\begin{equation*}
\Sigma\left(\theta=\frac{\pi}{2}\right)=\frac{2 \sigma_{\gamma, f}\left(1^{-}, 0\right)-\sigma_{\gamma, f}\left(1^{-}, 1\right)}{2 \sigma_{\gamma, f}\left(1^{-}, 0\right)+\sigma_{\gamma, f}\left(1^{-}, 1\right)} \tag{8}
\end{equation*}
$$

Thus the size of $\Sigma(\pi / 2)$ is determined both through the coefficients of $a, b$ and through the sections of $\sigma\left(1^{-}, 0\right), \sigma\left(1^{-}, 1\right)$. From a formula evidently, that if present contribution only cross-section of $\sigma\left(1^{-}, 0\right)$, then the size of $\Sigma(\pi / 2)$ is equal +1 , and if a contribution is cross-section of $\sigma\left(1^{-}, 1\right)$ only, then $\Sigma(\pi / 2)$ is equal -1 .

Using Eq.(6), the values of $\Sigma$-asymmetry for ${ }^{238} U$ nucleus have been obtained. Experimental values of angular distribution of fission fragments for ${ }^{238} U$ nucleus were measured in $[9,10]$, and numerical values of coefficients of $a$ and $b$ Eq.(1) were defined in [11]. Values of $\Sigma$-asymmetry in the photon energy range from 5.0 to 6.8 MeV are shown in Fig.1.


Fig.1. $\quad \Sigma$-asymmetry of ${ }^{238} U$ photofission as obtained from the data [10] (circles) and [11] (squares). In absence of error bars, these bars are smaller than the symbol size. The straight horizontal line shows the $\Sigma$-asymmetry value egual to 1 determined only by contribution of the cross-section in the channel $\left(1^{-}, 0\right)$. The dotted line represents $\Sigma=P / E_{\gamma}^{4}$. The dash-dotted line shows the crossing point of horizontal and dotted lines, which corresponds to the barrier height of ${ }^{238} U$ fission in the channel $\left(1^{-}, 1\right)$

From Fig. 1 it is visible that at small energy $E_{\gamma}$ size of $\Sigma$-asymmetry is close to +1 , that is $\Sigma$ asymmetry in this area $E_{\gamma}$ is defined only by fission through dipole $\left(1^{-}, 0\right)$ - the channel, and other channels of fission, within experiment errors, have no impact on $\Sigma$-asymmetry size. In the energy range from 5 to 6.09 MeV average size of experimental data of $\Sigma$-asymmetry ${ }^{238} U$ (see Fig.1) is equal to $0.9645 \pm 0.014533$. This value differs from unit for $3.55 \%$. Such difference can be caused by dispersion of experimental data and a contribution of fission at quadrupole excitation. This contribution doesn't exceed $3.55 \%$ and it, is generally shown at low energy in this range of energy. Therefore influence of this contribution on value $\Sigma$-asymmetry at increase in energy will be even less where to be shown, generally only a fission contribution via the dipole channel with a projection a back $K=0$ on an axis of symmetry of a nucleus.

From Fig. 1 it is also visible that with increase in energy $E_{\gamma}$ size of $\Sigma$-asymmetry decreases. It follows from Eq.(8) than $\Sigma$-asymmetry is positive and equal to +1 for the channel $\left(1^{-}, 0\right)$, and negative and equal to -1 for the channel $\left(1^{-}, 1\right)$. The values of $\Sigma$-asymmetry obtained from the experiments have positive values, i.e. they are mainly determined by the $\left(1^{-}, 0\right)$ fission channel. In the region where $\Sigma$-asymmetry is equal +1 , only the channel $\left(1^{-}, 0\right)$ contributes, i.e. $\Sigma$-asymmetry in it the $E_{\gamma}$ areas is defined only by fission through the dipole $\left(1^{-}, 0\right)$ channel, and other channels of fission, within errors, have no impact on the size $\Sigma$-asymmetry. The appearance of $\left(1^{-}, 1\right)$ channel contribution leads to the reduction of $\Sigma$-asymmetry value. This means that the energy, for which $\Sigma$-asymmetry becomes smaller than +1 , determines the height of ${ }^{238} U$ photofission barrier through the $\left(1^{-}, 1\right)$ channel. It can be seen from Fig. 1 that at energy above 6.1 MeV , the $\Sigma$ asymmetry becomes less than +1 .

To determine more precisely the energy height of the photofission barrier $E$ of the ${ }^{238} U$ nucleus, five curves

$$
\begin{array}{ll}
\Sigma=P / E_{\gamma}^{2}, & \Sigma=P / E_{\gamma}^{3}, \quad \Sigma=P / E_{\gamma}^{4} \\
\Sigma=P / E_{\gamma}^{5}, & \Sigma=P / E_{\gamma}^{6} \tag{9}
\end{array}
$$

where $P$ is an adjustable parameter and $E_{\gamma}$ is the incident photon energy, were least-squares fitted to the $\Sigma$-asymmetry values in the $\left(1^{-}, 1\right)$-channel. For this purpose we minimized the functional $\chi^{2}=$ $\sum \omega_{i}\left(\Sigma_{i}-\Sigma_{m, i}\right)^{2}$, where $\Sigma_{i}$ denotes the experimental $\Sigma$-asymmetry values in the energy range from 6.37 to 6.8 MeV , where these values are less than +1 (Table 1); $\Sigma_{m, i}$ denotes the $\Sigma$-asymmetry values calculated from curves (8); $\omega_{i}=1 /\left(\Delta \Sigma_{i}\right)^{2}$ is the statistical weight $\Sigma_{i}, \Delta \Sigma_{i}$ is the mean square error of $\Sigma_{i}$.

Table 1. These values of $E_{\gamma}$, $\Sigma$-asymmetry and $\Delta \Sigma$

| $E_{\gamma}, \mathrm{MeV}$ | $\Sigma$ | $\Delta \Sigma$ |
| :---: | :---: | :---: |
|  |  |  |
| 6.37 | 0.819 | 0.040 |
| 6.42 | 0.803 | 0.110 |
| 6.71 | 0.68 | 0.050 |
| 6.75 | 0.64 | 0.090 |
| 6.80 | 0.427 | 0.1 |

The fitting results for the five curves (9) are given in Table 2. It is apparent from Table 2, that the least $\chi^{2}$ value was obtained for the curve $\Sigma=P / E_{\gamma}^{4}$. Since for the ( $1^{-}, 0$-channel the $\Sigma$-asymmetry is positive and is equal to +1 , then the energy, at which the $\Sigma=P / E_{\gamma}^{4}$ curve with $P=1331 \pm 49$ is equal to +1 , determines the photofission barrier height of the ${ }^{238} U$ nucleus in the $\left(1^{-}, 1\right)$-channel. By making $P / E^{4}$ equal to 1 , we have obtained $E=P^{1 / 4}=$ $(6.04 \pm 0.06) \mathrm{MeV}$. This value represents the photofission barrier height of the ${ }^{238} U$ nucleus in the ( $1^{-}, 1$ )channel.

Table 2. The fitting results for the five curves (9)

| Eq.(8) | $P+\Delta P$ | $\chi^{2} /(n-1)$ | $E+\Delta E$ <br> $M e V$ |
| :---: | :---: | :---: | :---: |
| $P / E^{2}$ | $31.33 \pm 1.17$ | 2.04 | $5.6 \pm 0.10$ |
| $P / E^{3}$ | $204.5 \pm 7.6$ | 1.4 | $5.89 \pm 0.07$ |
| $P / E^{4}$ | $1331 \pm 49$ | 1.035 | $6.04 \pm 0.06$ |
| $P / E^{5}$ | $8659 \pm 322$ | 1.09 | $6.13 \pm 0.05$ |
| $P / E^{6}$ | $57083 \pm 2108$ | 1.1 | $6.21 \pm 0.04$ |

$\Sigma$-asymmetry in the data presented in Fig. 1 has been obtained in the conditions where it is not influenced by the contribution of fission via the quadrupole excitation. In the range of energy from 5 to 6 MeV of $\Sigma$-asymmetry it is close to +1 . It specifies that the contribution only through dipolar excitation is shown and within errors the contribution of other multipolarities isn't observed. In the range of energy from 5 to 6.04 MeV of $\Sigma$-asymmetry it is close to +1 . It specifies that the contribution only through dipole excitation is shown and within errors the contribution of other channels of fission isn't observed. However, the contribution of quadrupole fission becomes noticeable in this energy range. In reference [12] angular distribution have been obtained for ${ }^{238} U$ fission fragments at six energies $6.25,6.61,7.14,7.44,7.75,8.50 \mathrm{MeV}$ near the fission barrier. It has been stressed there that these distributions show the shift of the maximum towards $45^{\circ}$. Such a behavior of the angular distribution of fission fragments is very different from its behavior at higher energies. We have processed the
data of [13]. In paper [12], the experimental data were presented in the figures. In paper [11], expression (1) was made to fit to those data, and numerical values of the coefficients $a, b, c$ were determined. Using the numerical values of c from ref. [11], we have obtained the contribution of the term $V(\theta)=c \sin ^{2}(2 \theta)$ in Eq.(1), which is determined by the quadrupole fission contribution alone, for all six energies, at which the measurements in ref. [12] were performed. As an example, in Fig. 2 the experimental data of [12] are shown for the electron energy 7.14 MeV . Also shown in this figure is the contribution of the quadrupole term $V(\theta)$ calculated by us as well as the values of $W(\theta)$ with $V(\theta)$ subtracted.


Fig.2. Angular distribution of fragments $W(\theta)$ from ${ }^{238} U$ fission by electrons with the energy 7.14 MeV . Black squares are values $W(\theta)$ from data [13]. Black circles are contributions of the coefficient $V(\theta)=c \cdot \sin ^{2}(2 \theta)$

It can be seen from Fig. 2 that the peak on the angular distribution of fission fragments located around $45^{\circ}$ is not visible after the subtraction of the quadrupole contribution. For the rest five energies, peak is also not seen because the deviation from the smooth change of the angular distribution of fission fragments stays within the experimental error bars. Thus, it has been determined that the appearance of the peak is connected with the contribution of the quadrupole fission. It should be noted that the magnitude of the quadrupole contribution is not significant as it is seen from Fig.2. This may be the reason why this contribution does not have a significant influence on the behavior of $\Sigma$-asymmetry in Fig.1. Despite that the contribution of the quadrupole fission in this energy range has a noticeable influence on the angular distribution of fission fragments.

## 3. CONCLUSIONS

Using the values of energy dependence of $\Sigma$ asymmetry obtained from the data on angular distribution of fission fragments, the numerical value of ${ }^{238} U$ fission threshold in $\left(1^{-}, 1\right)$ dipole channel, ( $6.04 \pm 0.06$ ) MeV , has been determined. It has been determined that the peak, which is observed around $45^{\circ}$ on the angular distribution of ${ }^{238} U$ fragments
near the fission threshold [12], results from the contribution to the fission of the quadrupole excitation.

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## ДЕЛЕНИЯ ${ }^{238} U$ У ПОРОГА

## В. М. Хвастунов, В. И. Касилов. С. С. Кочетов, А. А. Хомич

Определен порог в дипольном канале деления с проекцией спина $J=1$ на ось симметрии ядра из данных углового распределения осколков деления. Показано, что пик, наблюдаемый в сечении деления ${ }^{238} U$ около порога, обусловлен вкладом в деление при квадрупольном возбуждении.

## ДІЛЕННЯ ${ }^{238} U$ БІЛЯ ПОРОГУ

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Знайдено поріг у дипольному каналі діления з проекцією спіна $J=1$ на вісь симетрії ядра із даних кутового розподілу осколків діления. Показано, що пік, який спостерігається в перерізі ділення ${ }^{238} U$ біля порогу, зумовлений вкладом у ділення при квадрупольному збудженні.


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