https://doi.org/10.46813/2021-133-091

STABILITY OF A VISCOUS INCOMPRESSIBLE CONDUCTING LIQUID LAYER OF A CYLINDRICAL SHAPE IN AN INHOMOGENEOUS TEMPERATURE FIELD AND A MAGNETIC FIELD OF A VACUUM ARC CURRENT THROUGH IT

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Convective mass transfer in a cylindrical viscous incompressible conductive fluid layer in an inhomogeneous temperature field and in the external magnetic field of the vacuum arc current through it is theoretically investigated in this work. For a horizontal layer of a viscous, incompressible, conducting liquid of a cylindrical shape, located in a temperature field inhomogeneous in height and in an external magnetic field of a vacuum arc current flowing through it, the original equations are written. These equations consist of linearized equations for small velocity perturbations, small deviations from the equilibrium values of temperature, pressure, and magnetic field strength. The considered boundary value problem is solved for the case of free boundaries. Comparison of the experimental data with theoretical calculations made it possible to determine the rotation velocity of the steel melt during vacuum arc melting.

PACS: 47.20.−k, 47.20.Bp, 65.20.−w, 65.20.Jk

INTRODUCTION

In some metallurgical plants liquid metal mixing is used to improve the quality of the final product and reduce the energy intensity of the production. High-quality mixing of a liquid metal can be provided by gas stirring or by electromagnetic mixing methods [1]. One of the electromagnetic methods of mixing liquid metal is realized in direct current arc furnaces (DCAF), where electric vortex flows (EVF) are used [2].

In the process of smelting in DCAF the metal can be conventionally represented as a horizontal layer of a viscous incompressible fluid with a vertical temperature gradient and a direct current distributed over its volume.

In turn, due to the temperature gradient the convective motion of a liquid viscous incompressible conductive metal with a current flowing through it is subject to the effect of magneto-hydro-dynamic forces, which can affect its convective motion and adjust the equilibrium conditions of its existence. The magnetic field transforms the direction of the convective flow of the conducting fluid into a transverse direction, and thus, in some cases, can have a certain effect on the convection process. This effect is due to the allowance for the magnetic field in the Rayleigh problem on the equilibrium of a horizontal fluid layer [3]. Taking into account the magnetic field leads to increasing the number of unknown variables and characteristic parameters in the Rayleigh convection equations. Therefore, in the characteristic equation of the convection problem, taking into account of magneto-hydro-dynamic forces can lead to the emergence of new solutions describing monotonically unstable or vibrational states of a heated from below horizontal layer of a viscous incompressible fluid.

For the first time, the existence of such solutions is indicated by the studies of W. Thompson for free boundaries [4] and S. Chandrasekhar for rigid and free boundaries [5, 6].

The simplest case of isothermal layer boundaries, when a constant magnetic field and gravity act in the same direction, was investigated in [5]. The dependence of the Rayleigh number on the Hartmann number characterized the magnetic field strength was determined analytically for the case of monotonically stable perturbations. The critical values of the Rayleigh numbers and the corresponding critical wave numbers for this case were calculated. The critical values of the Rayleigh numbers and the corresponding critical wave numbers for rigid bounding surfaces or for one free bounding surface and the other – rigid were investigated numerically. It is shown that in all cases the critical Rayleigh numbers increase with increasing magnetic field strength and at high magnetic fields cease to depend on the type of boundaries. It is also shown that the critical Rayleigh numbers are determined only by the vertical component of the external magnetic field.

In the case, when the applied magnetic field acts in the direction different from the direction of gravity, it is found that, when they are various-directional, the convection, which occurs at ultimate stability, has the shape of shafts elongated in directions parallel to the plane containing the vectors of the magnetic field and gravity [6].

The theoretical conclusions [4 - 6], that an increase in the magnetic field strength increases the stability of the fluid convective motion and leads to reduction in horizontal dimensions of convective cells, are confirmed in a series of Y. Nakagawa`s experimental works [7, 8].

These papers describe experiments on magnetic suppression of thermal convection in horizontal layers of mercury heated from below. A large cyclotron magnet of diameter 36 ½ inch adapted for hydromagnetic research was used in these experiments. Using layers of mercury of a depth of 3 to 6 cm and magnetic fields of a strength of 500 to 8000 G it was possible to determine the dependence of the critical Rayleigh number for instability onset on the non-dimensional parameter Q_1, where $Q_1 = \sigma H^2 h^2/\pi^2 \rho \nu ж$ H – field density; σ – electric conduction; ν – kinematic viscosity coefficient; ρ – density and h – layer depth. The parameter Q_1 varied in the range from 40 to 1.6×10^6.

The conclusion [5] that the horizontal magnetic field does not affect the stability of equilibrium has been experimentally confirmed in [9]. In this work it has been also shown that under the action of a horizontal magnetic field the convection occurs in the form of rolls elongated along the field.

The conditions for vibrational-convective instability of a conducting medium in a magnetic field were determined in [10]. It was concluded that vibrational instability arises if the electrical conductivity σ and thermal diffusivity χ of the medium satisfy the condition $4\pi\sigma\chi/c^2 > 1$, and if the magnetic field strength is greater than a certain critical value $H > H_{cr}$.

The general theory of perturbation spectrum and convective stability of the mechanical equilibrium of a conductive fluid in a magnetic field was developed in [11 - 13], which were later described in details in [14].

In stated above works the convection was studied in a Cartesian coordinate system in rectilinear magnetic fields. The main conclusions are that the horizontal component of the external magnetic field strength does not affect the critical Rayleigh numbers, but orients the arising convective rolls in its direction. The vertical component of the external magnetic field strength leads to increasing the critical Rayleigh numbers by a magnitude proportional to its square. With increasing the magnetic field strength the stability of the fluid convective motion increases and the horizontal dimensions of the convective cells decrease. At high magnetic field strengths the critical Rayleigh numbers cease to depend on the type of boundaries.

However, rectilinear magnetic fields and the conclusions obtained while studying their effect on convection are applicable only in specially created experimental conditions, which reduces their practical value.

The aim of this work is to study the stability of a viscous incompressible conductive cylindrically shaped fluid layer in an inhomogeneous temperature field and in an external azimuthally symmetric magnetic field created by a vacuum-arc discharge current flowing through the fluid.

1. THE INITIAL EQUATIONS FOR A VISCOUS CONDUCTIVE INCOMPRESSIBLE FLUID LAYER IN AN INHOMOGENEOUS TEMPERATURE FIELD AND IN A CYLINDRICAL VOLUME LOCATED IN AN EXTERNAL MAGNETIC FIELD

The initial equations describing convection of a viscous conductive incompressible fluid layer in an inhomogeneous temperature field and in a cylindrical volume located in an external magnetic field will be written in general form. However, in the final notation, due to the symmetry of the problem, they will be presented in a cylindrical coordinate system.

Let us describe the basic data of the problem being solved.

Low-carbon steel melt (viscous conductive incompressible fluid) is located in a cylindrical volume of a radius R_c. The lower and upper boundaries of the liquid volume coincide with the planes $z = 0$ and $z = h$. The magnetic field in the fluid is created by a direct current flowing between the anode and cathode of the vacuum-arc installation $I = 0.8....1.2$ kA (Fig. 1 [15]) and is axially symmetric $\vec{H}_0(r) = H_0(r)\cdot\vec{e}_\varphi = h_0\cdot(r/R_c)\cdot\vec{e}_\varphi$ [16], where $\vec{e}_\varphi$ – azimuthal unit vector in a cylindrical coordinate system; r – distance from the axis of a cylindrical volume to a fluid element; h_0 – constant. The chosen dependence of the magnetic field on the radius inside the melt is model and reflects the fact that the melt can be considered as a cylindrical conductor with a current.

The temperature distribution inside the cylinder $T_0(z)$, similar to [3], is assumed to be set in such a way that the temperature of the lower boundary is higher than the temperature of the upper: $T_0(0)=T_2$, $T_0(h) = T_1$ ($T_2 > T_1$). In this case, we assume that in a state of equilibrium the temperature distribution is described by a linear function of the vertical coordinate z: $\vec{\nabla}T_0(z) = -\Theta h^{-1}\vec{e}_z$, where $\Theta = T_2 - T_1$ – temperature difference between the bottom and top planes; $\vec{e}_z$ – unit vector directed perpendicular to the fluid layer vertically upwards.

Let us write down the linearized equations for small velocity perturbations $\vec{\mathrm{v}}$ and small deviations from the equilibrium values of a temperature $T_0(z) + \tilde{T}$, pressure $p_0 + \tilde{p}$, and magnetic field strength $\vec{H}_0(r) + \tilde{\vec{H}}$. These equations describe variation of perturbed magnitudes in space and time, and with no of viscous dissipation and Joule heating of the fluid, they can be represented in the form [14, 17]:

$$\frac{\partial\vec{\mathrm{v}}}{\partial t} = -\frac{1}{\rho}\vec{\nabla}\left(\tilde{p} + \frac{\vec{H}_0\tilde{\vec{H}}}{4\pi}\right) + \nu\Delta\vec{\mathrm{v}} + \vec{g}\beta\tilde{T}\vec{e}_z + \frac{h_0}{4\pi\rho}\left(\frac{r}{R_c}\vec{e}_\varphi\vec{\nabla}\right)\tilde{\vec{H}}, \quad (1)$$

$$\frac{\partial\tilde{T}}{\partial t} - \frac{\Theta}{h}\vec{\mathrm{v}}\vec{e}_z = \chi\Delta\tilde{T}, \quad (2)$$

$$\frac{\partial\tilde{\vec{H}}}{\partial t} - h_0\left(\frac{r}{R_c}\vec{e}_\varphi\vec{\nabla}\right)\tilde{\vec{\mathrm{v}}} = \frac{c^2}{4\pi\sigma}\Delta\tilde{\vec{H}}, \quad (3)$$

$$\mathrm{div}(\vec{\mathrm{v}}) = 0, \quad (4)$$

$$\mathrm{div}\left(\tilde{\vec{H}}\right) = 0, \quad (5)$$

where $\vec{\mathrm{v}}$ – small perturbations of the velocity of an elementary volume of a fluid in coordinate system rotated with a frequency $\vec{\Omega} = \Omega\cdot\vec{e}_z$; $\vec{r}$ – radius-vector of the fluid element; ρ – fluid density; $\vec{\nabla}$ – gradient operator; Δ – Laplace operator; $\vec{g}$ – gravitational acceleration directed against the axis z; χ – fluid temperature diffusivity coefficient; ν – fluid kinematic viscosity coefficient; β – fluid volumetric thermal expansion coefficient; c – light speed; σ – fluid electrical conductivity.

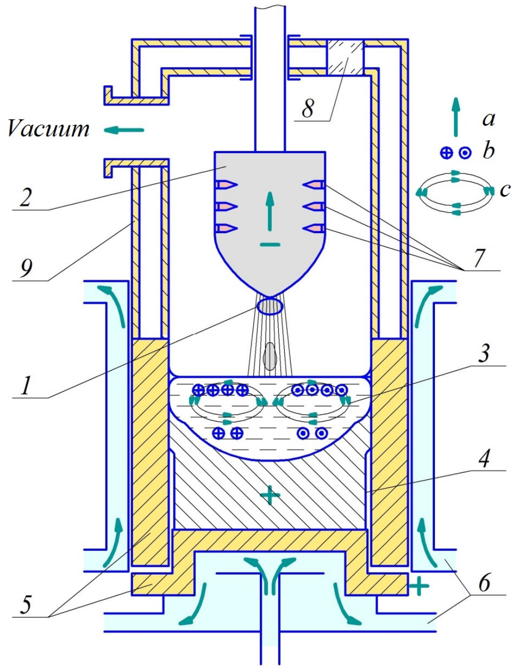

Fig. 1. Scheme for obtaining of oxide dispersion strengthened steels using vacuum arc remelting: 1 – corona; 2 – electrode (cathode) to be melted; 3 – liquid metal; 4 – ingot; 5 – copper crystallizer (anode); 6 – cooler (water); 7 – cavities with alloy additive; 8 – quartz glass window; a – direction of electric current; b – circular motion of metal in horizontal plane; c – motion of metal in vertical plane

For a liquid located in a solid massif with temperature diffusivity coefficient χ_m and electrical conductivity coefficient σ_m, the equations (1) - (5) should be supplemented with equations describing variation of temperature T_m and magnetic field strength $\vec{H}_m$ in the massif:

$$\frac{\partial T_m}{\partial t} = \chi_m \Delta T_m, \quad \frac{\partial \vec{H}_m}{\partial t} = \frac{c^2}{4\pi\sigma_m}\Delta \vec{H}_m. \qquad (6)$$

At the boundaries of the liquid and the massif the velocity vanishes, and the temperature and heat flux are continuous [14]:

$$\vec{v}\big|_{\substack{z=0,\\ z=h}} = 0, \left(T_0(z)+\tilde{T}\right)\Big|_{\substack{z=0,\\ z=h}} = T_m\big|_{\substack{z=0,\\ z=h}},$$

$$\kappa \frac{\partial T}{\partial n}\bigg|_{\substack{z=0,\\ z=h}} = \kappa_m \frac{\partial T_m}{\partial n}\bigg|_{\substack{z=0,\\ z=h}}, \vec{H}\big|_{\substack{z=0,\\ z=h}} = \vec{H}_m\big|_{\substack{z=0,\\ z=h}}, \qquad (7)$$

$$\sigma^{-1}\,\mathrm{rot}_t\left(\vec{H}\right)\Big|_{\substack{z=0,\\ z=h}} = \sigma_m^{-1}\,\mathrm{rot}_t\left(\vec{H}_m\right)\Big|_{\substack{z=0,\\ z=h}}$$

where κ and κ_m – temperature diffusivity coefficients of the fluid and the massif respectively; n – normal to the boundary; $\mathrm{rot}_t\left(\vec{A}\right)$ – tangential component of the operator $\mathrm{rot}\left(\vec{A}\right)$.

In dimensionless variables the system of equations (1) - (5) takes the form:

$$\frac{\partial \vec{v}}{\partial t} = -\vec{\nabla}\left(\tilde{p} + \frac{r}{R_c} M \vec{e}_\varphi \tilde{\vec{H}}\right) + \Delta\vec{v} + R\tilde{T}\vec{e}_z + \qquad (8)$$

$$+M\left(\frac{r}{R_c}\vec{e}_\varphi \vec{\nabla}\right)\tilde{\vec{H}}, \quad P\frac{\partial \tilde{T}}{\partial t} - \vec{v}\vec{e}_z = \Delta\tilde{T}, \qquad (9)$$

$$P_M \frac{\partial \tilde{\vec{H}}}{\partial t} - M\left(\frac{r}{R_c}\vec{e}_\varphi \vec{\nabla}\right)\tilde{\vec{v}} = \Delta\tilde{\vec{H}}, \qquad (10)$$

$$\mathrm{div}\left(\vec{v}\right) = 0, \quad \mathrm{div}\left(\tilde{\vec{H}}\right) = 0, \qquad (11)$$

where

$$h,\ h^2/\nu,\ \chi/h,\ \Theta,\ \rho\nu\chi/h^2,\ (4\pi\chi/hc)\sqrt{\rho\nu\sigma}$$

are chosen as the units of measurement of distance, time, velocity, temperature, pressure and magnetic field strength. The equations (8) - (10) contain dimensionless variables: $R = g\beta h^3\Theta/(\nu\chi)$ – the Rayleigh number, $P = \nu/\chi$ and $P_M = 4\pi\sigma\nu c^{-2}$ – the Prandtl number and the magnetic Prandtl number, respectively, $M = \sqrt{\sigma}h_0 h/\left(c\sqrt{\rho\nu}\right)$ – the Hartmann number.

The amplitudes of dimensionless perturbations in fluid-bounding massifs are described by the equations:

$$P\frac{\partial \tilde{T}_m}{\partial t} = \hat{\chi}\Delta\tilde{T}_m, \quad P_M\frac{\partial \tilde{\vec{H}}_m}{\partial t} = \hat{\sigma}\Delta\tilde{\vec{H}}_m, \qquad (12)$$

where, $\hat{\chi} = \chi_m/\chi$, $\hat{\sigma} = \sigma/\sigma_m$.

In this case, the boundary conditions take the form:

$$\vec{v}\big|_{\substack{z=0,\\ z=1}} = 0, \tilde{T}\big|_{\substack{z=0,\\ z=1}} = \tilde{T}_m\big|_{\substack{z=0,\\ z=1}}, \ \kappa\frac{\partial \tilde{T}}{\partial n}\bigg|_{\substack{z=0,\\ z=1}} = \kappa_m \frac{\partial \tilde{T}_m}{\partial n}\bigg|_{\substack{z=0,\\ z=1}},$$

$$\tilde{\vec{H}}\Big|_{\substack{z=0,\\ z=1}} = \tilde{\vec{H}}_m\Big|_{\substack{z=0,\\ z=1}}, \ \mathrm{rot}_t\left(\tilde{\vec{H}}\right)\Big|_{\substack{z=0,\\ z=1}} = \hat{\sigma}\,\mathrm{rot}_t\left(\tilde{\vec{H}}_m\right)\Big|_{\substack{z=0,\\ z=1}}. \qquad (13)$$

The amplitudes of temperature and magnetic field strength perturbations determined from equations (12) should tend to zero at large distances from the fluid boundaries.

2. DEFINITION AND SOLUTION OF THE BOUNDARY VALUE PROBLEM ON STABILITY OF CONVECTIVE MOTION IN A LAYER OF A VISCOUS INCOMPRESSIBLE CONDUCTIVE FLUID OF A CYLINDRICAL SHAPE IN AN EXTERNAL MAGNETIC FIELD OF THE VACUUM ARC CURRENT FLOWING THROUGH IT

We assume that all perturbed variables of the system of equations (9) - (15) depend on time and azimuthal coordinate in the form:

$$\mathrm{v}_z(\vec{r},t) = \mathrm{v}(r,z)\cdot\exp(-\lambda t + im\varphi),$$
$$\tilde{T}(\vec{r},t) = \vartheta(r,z)\cdot\exp(-\lambda t + im\varphi), \qquad (14)$$
$$\tilde{H}_z(\vec{r},t) = \psi(r,z)\cdot\exp(-\lambda t + im\varphi),$$

where $\mathrm{v}(r,z), \vartheta(r,z), \psi(r,z)$ – amplitudes of perturbed velocity, temperature and magnetic field strength; $m = 0;1;2;...$ – azimuthal mode, which determines the dependence of solutions on the azimuthal angle φ.

This assumption allows obtaining from equations (8) - (12) with boundary conditions (13) the boundary value problem for determining the eigenvalues λ.

Proceeding similarly to [14], we substitute solutions of the form (14) into equations (8) - (12). Then we apply the operation $\mathrm{rot}(\mathrm{rot}(...))$ to equation (8), and take the projection of the resulting equation onto the axis *z*. As a result, we obtain an equation relating amplitudes of disturbances:

$$-\lambda\Delta\mathrm{v} = \Delta\Delta\mathrm{v} + R\Delta_\perp\vartheta + iM(m/R_c)\Delta\psi. \qquad (15)$$

As a result of the indicated substitution of solutions (14) into equations (10), it takes the form:

$$-\lambda P\vartheta = \Delta\vartheta + \mathrm{v}, \qquad (16)$$

and from the equation (10) we use only its projection onto the axis z:

$$-\lambda P_M\psi = \Delta\psi + iM\mathrm{v}m/R_c. \qquad (17)$$

The Laplace operator in the cylindrical coordinate system is used in equations (15) - (17):

$$\Delta... = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial...}{\partial r}\right) - \frac{m^2...}{r^2} + \frac{\partial^2...}{\partial z^2} = \Delta_\perp... + \frac{\partial^2...}{\partial z^2}.$$

Solutions of the system of equations (18) - (20) can be represented in the form:

$$\mathrm{v}(r,z) = AJ_m(k_m r)\sin(n\pi z),$$
$$\vartheta(r,z) = BJ_m(k_m r)\sin(n\pi z), \qquad (18)$$
$$\psi(r,z) = CJ_m(k_m r)\sin(n\pi z),$$

where $n = 1,2,3...$ – the integers determined the vertical mode of perturbation, $J_m(x)$ – the Bessel function of the first kind of *m*-th order and the constants A,B,C satisfy the system of equations:

$$K_m^2(K_m^2 - \lambda)A - Rk_m^2 B - iM\frac{m}{R_c}K_m^2 C = 0,$$
$$A + (\lambda P - K_m^2)B = 0, \qquad (19)$$
$$(\lambda P_M - K_m^2)C + iM\frac{m}{R_c}A = 0,$$

where $K_m^2 = k_m^2 + (n\pi)^2$.

For ideally heat-conducting boundaries and under the condition of infinite electrical conductivity the boundary conditions (13) are satisfied automatically.

In the case of stable solutions we assume $\lambda = 0$. Then from (19) follows the critical Rayleigh number:

$$R = k_m^{-2}\left((K_m^2)^3 + (Mm/R_c)^2(K_m^2)\right). \qquad (20)$$

Let us consider some corollaries followed from the expression (20).

With no magnetic field (the Hartmann number is zero $M = 0$) or for axially symmetric perturbations $m = 0$, it follows from (20) that the critical Rayleigh number corresponds to the critical Rayleigh number of a cylindrical convective cell with free boundary conditions with the wave number $k_0 = \sigma_{1,i}R_c^{-1}$, where $\sigma_{1,i}$ – *i*-th zero of the Bessel function of the first kind of the first order [18].

In the general case, as follows from (20), the expression for the critical Rayleigh numbers of a layer with free boundary conditions does not contain numbers P and P_M, but depends on the Hartmann number and the radial and vertical wave numbers.

The critical Rayleigh number has a minimum at

$$(k_m)_{\min} = \sqrt{A_+ + A_- - (n\pi)^2/2},$$

where $A_\pm = \sqrt[3]{-q/2 \pm \sqrt{Q}}$;

$$q = -2^{-2}(n\pi)^6\left(1 + 2(n\pi)^{-4}(Mm/R_c)^2\right);$$

$$Q = (n\pi)^{12}2^{-6}\left(\left(1 + 2(n\pi)^{-4}(Mm/R_c)^2\right)^2 - 1\right).$$

The minimum critical Rayleigh number is:

$$(R)_{\min} = \left(A_+ + A_- + (n\pi)^2/2\right)^2 + (Mm/R_c)^2.$$

3. USING A THEORETICAL MODEL TO DESCRIBE CONVECTION IN A CYLINDRICALLY SHAPED STEEL MELT LAYER IN AN EXTERNAL MAGNETIC FIELD OF A VACUUM ARC CURRENT FLOWING THROUGH IT

Allowing for a magnetic field the critical Rayleigh number increases with increasing the magnetic field strength in the form of a term in (20) proportional to the square of the Hartmann number M value. However, for the steel melt at a temperature of 1300ºC, it has characteristic parameters: $\sigma \approx (6...7.7)\cdot 10^3$ s^{-1}, $\rho \approx 7.5...8$ g/cm^3 [19, 20]; $\nu \approx (8.57...10)\cdot 10^{-3}$ cm^2/s [20, 21]; $h \approx 2$ cm; $h_0 = 2I/cR_c \approx 10^2$ G. Whence the small value of the Hartmann number M follows:

$$M = \frac{h_0 h}{c}\sqrt{\frac{\sigma}{\rho\nu}} = \frac{10^2\cdot 2}{3\cdot 10^{10}}\sqrt{\frac{7\cdot 10^3}{7\cdot 10^{-2}}} \approx 2\cdot 10^{-6}. \qquad (21)$$

As follows from (21), the small value M is due to the low electrical conductivity of the molten steel.

Thus, it can be concluded that the critical Rayleigh number for the case of molten steel does not depend on the value of the magnetic field strength of the vacuum arc discharge.

To calculate the critical Rayleigh number (20), it is necessary to determine the value of the critical radial wave number k_m. For calculating it is necessary to specify the radial $\mathrm{v}_r(\vec{r},0)$ and azimuthal $\mathrm{v}_\varphi(\vec{r},0)$ velocities of the melt motion. These velocities can be determined from the fluid incompressibility equation (11) using the expression for the vertical velocity $\mathrm{v}_z(\vec{r},0)$. As a result, assuming

$$\mathrm{v}_\varphi(\vec{r},0) = A'k_m^{-1}J_m(x)\cos(n\pi z)\sin(m\varphi),$$

and using [22] we obtain expressions for the radial velocity of the melt:

$$\mathrm{v}_r(\vec{r},0) = -(xk_m)^{-1} An\pi u(x)\cos(n\pi z)\cos(m\varphi), \quad (22)$$

where $x = k_m r$, $b = A'k_m/An\pi$, $A, A'-$ constants, $u(x) = xJ_{m+1}(x) + b\cdot 2m\sum_{k=0}^{\infty} J_{m+2k+1}(x) + 2m\sum_{k=0}^{\infty} J_{m+2k+2}(x)$.

It follows from the expression (22) that the radial velocity is zero at the origin.

On condition that radial velocity is equal to zero at the cell boundary (at $x = k_1 R_c$), we determine the critical wave number for disturbance with the lowest azimuthal $m = 1$ and vertical $n = 1$ modes. Its value is found by a numerical method based on the condition that all projections of the perturbed velocities at the cell boundary are equal to zero. It follows, that at $b \approx -1.879$ the critical wave number is of the order $k_1 \approx 3.83/R_c$, where necessary to remember that R_c is measured in units of the melt layer thickness h.

Thus, for the experimental conditions $k_1 \approx 3.83 \cdot 2/9 = 0.75$ and the critical Rayleigh number is equal to $R = 675.196$, which corresponds to the critical Rayleigh number in the absence of a magnetic field for the same perturbation modes: $R(h_0 = 0) = 675.196$. It follows from given above assessments that the critical Rayleigh number in the presence of a magnetic field of a vacuum arc is easy attainable due to its equality to the minimum, which corresponds to the Rayleigh number without a magnetic field.

4. DESCRIPTION OF CONVECTIVE MOTION OF STEEL MELTS AT VACUUM ARC MELTING

4.1. THEORETICAL DESCRIPTION

Based on the theoretical results obtained above, we will analyze the azimuthal velocity of the melt, since it is an observed value in the experiment. In the laboratory coordinate system the azimuthal velocity of the melt motion at the upper boundary $z = 1$ for the modes $m = n = 1$ will be characterized by an expression of the form:

$$\mathrm{V}_{\varphi,1}(\vec{r},0) = \Omega r - A'J_1(k_1 r)\sin(\Omega t), \quad (23)$$

where the melt rotation is given by the dependence of azimuthal angle on time: $\varphi = \Omega t$.

At $\alpha = A'/\Omega R_c << 1$ the irregularity of the dimensionless azimuthal angular velocity φ is described by a small harmonic deviation from the equilibrium velocity:

$$w(\vec{r},0) = \mathrm{V}_{\varphi,1}/\Omega R_c = r/R_c + \alpha J_1(k_1 r)\sin(\Omega t). \quad (24)$$

Due to irregularity of the melt motion velocity, its hydrostatic pressure will change periodically in the vertical direction. This pressure deviation, according to the Bernoulli's law, can be related to the deviation of the upper boundary of the melt from $z = 1$. Therefore, there will be a section at the upper boundary of the melt that rises above the equilibrium level, and through the azimuthal angle π – a section with a level below the equilibrium level. The dependence of deviation of the melt upper boundary on time in dimensional variables can be represented as:

$$\Delta h = g^{-1} R_c^{-1} (\Omega R_c)^2\, r\alpha J_1(k_1 r)\sin(\Omega t). \quad (25)$$

Thus, the steel melt, where there is a section at the upper boundary that rises above the equilibrium level, will have a maximum area with a maximum brightness. A section with a level below the equilibrium one will have a smaller area of the same brightness. Moreover, these regions of brightness will rotate with angular velocity Ω.

The mentioned fact can be used to determine the velocity of the melt rotation in the magnetic field of a vacuum arc discharge.

4.2. EXPERIMENTAL OBSERVATION AND THEORETICAL DESCRIPTION

The process of steel vacuum arc remelting was recorded on a video through a window with darkened quartz glass 8 located on the upper flange of the vacuum chamber (see Fig. 1).

Snapshot of a steel melt with a melted electrode 1 oriented to the center of the melt 2 is presented in Fig. 2. Arrow 3 in the figure shows the direction of the melt rotation determined from the results of numerous observations of the remelting process through quartz glass with the most acceptable darkening.

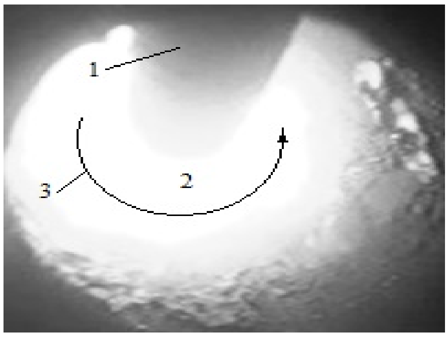

Fig. 2. Snapshot of the melt at vacuum arc remelting: 1 – electrode (cathode) to be melted; 2 – liquid metal; 3 – direction of melt rotation

Based on the observational data, the boundary of the maximum brightness of the melt surface was a circle with a radius harmonically varying in time (24), where it is necessary to assume $r = R_c$. The visible boundary of the brightness distribution corresponds to the azimuthal mode number $m = 1$ and is described by formula (25). Therefore, to determine the velocity of the melt rotation we will be guided by the dependence (25).

For this, the captured video sequence was divided into frames with an interval of 1 s and converted into black and white format. After digitizing the obtained images, curves bounded the area with maximum brightness were plotted. These curves were plotted so, that their scales and positions on the plane coincided. The resulting curves bounded the areas of maximum brightness are presented in Fig. 3. It can be seen that all curves intersect at points, which are marked in the figure by two almost diametrically spaced circles.

From Fig. 3 it follows that the melt makes a half revolution in 3 s, since the intersection points of the curves are shifted by π. Then the melt makes a complete revolution in 6 s.

Let us determine the velocity of melt rotation using the expression (25).

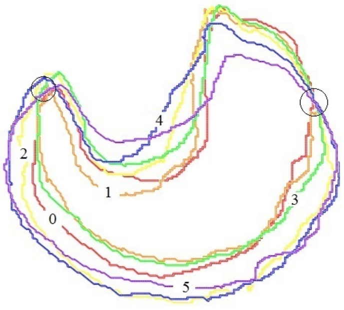

Fig. 3. Curves bounding the area of the greatest brightness in positions 0…5 *that correspond to the moments of time* $t = 0;3;6;9;12;15$ *s*

According to (25), the intersection of the curves in the area marked by the circle should be observed when $\sin(\Omega t_k) = 0$, where $t_k = 3k$ s ($k = 1;2;3;4...$).

This condition is satisfied when $\Omega t_k = \Omega k = l\pi$, where $l = 1;2;3;...$ At $l = k$, we getthat the period of the melt rotation is equal to: $T = 2$ s. This value of the melt rotation period coincides with the visually measured one. For $R_c = 0.09$ m the period of the melt rotation $T = 6$ s gives the following value for the equilibrium azimuthal velocity of the melt rotation: $v_{\varphi,1} = R_c\Omega = 9\,2\pi/6 = 9.42$ cm/s. If we assume, that the melt is put in motion by magnetohydrodynamic forces, then the characteristic velocity of the melt will be of the order of: $v_{\varphi,1} = H/\sqrt{4\pi\rho} \approx 10.7$ cm/s. It can be seen that these two velocity values practically coincide.

Thus, a comparison of the experimentally and theoretically obtained values of azimuthal velocities confirms the validity of the proposed convective model for describing the stability of the heated from below layer of a viscous incompressible conductive cylindrically shaped fluid in the magnetic field of the vacuum arc current flowing through it.

CONCLUSIONS

Electro-vortex flows (EVF), which arise as a result of interaction of magnetodynamic forces of a vacuum arc direct current with a conductive melt, are used for mixing liquid metal in DCAF. An installation for vacuum arc melting of steel is described in the paper. To study the melting process in such an installation, the molten metal is presented in the form of a horizontal layer of a viscous incompressible liquid with a vertical temperature gradient and a direct current distributed over its volume. Onset of convective motion of the molten metal, modified by the action of a magnetic field created by an electric current of a vacuum arc, was shown. In contrast to the well-known works on taking into account the effect of rectilinear magnetic fields on the convection of conductive fluids, we investigated in this work another model configuration of the magnetic field − an axially symmetric magnetic field corresponding to the magnetic field inside the conductor. The initial equations were written for such a heated from below layer of a viscous incompressible conductive cylindrical fluid located in an external magnetic field of the vacuum arc current flowing through it. They consist of linearized equations for small velocity perturbations, small deviations from the equilibrium values of temperature, pressure and magnetic field strength. The considered boundary value problem was solved for the case of free boundaries. The condition for the existence of stationary perturbations has been found, which is determined by the dependence of the Rayleigh number on the Hartmann number, radial and vertical wave numbers. The dependence of the equilibrium melt velocity on coordinates was described. The critical Rayleigh number and the corresponding radial wave number were determined. The obtained solutions were used to describe the experiment on vacuum arc melting of steel. Graphs of dependence of the equilibrium azimuthal velocity of the melt on a time are obtained for the experimentally observed first radial and vertical modes. Comparison of the experimental data with the theoretical calculations of this work made it possible to determine the velocity of steel melt rotation during vacuum arc melting, which turned out to be of the order of 9.42 cm/s.

Thus, the proposed model for describing the stability of the heated from below layer of the viscous incompressible conductive cylindrically shaped fluid in the external magnetic field of the vacuum arc current flowing through it can be used to study various aspects of such processes.

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Article received 14.04.2021

УСТОЙЧИВОСТЬ ВЯЗКОГО НЕСЖИМАЕМОГО ЖИДКОГО СЛОЯ ЦИЛИНДРИЧЕСКОЙ ФОРМЫ В НЕОДНОРОДНОМ ТЕМПЕРАТУРНОМ ПОЛЕ И МАГНИТНОМ ПОЛЕ ВАКУУМНОЙ ДУГИ, ПРОТЕКАЮЩЕЙ ЧЕРЕЗ ЖИДКОСТЬ

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Теоретически исследован конвективный массоперенос вязкой несжимаемой проводящей жидкости цилиндрической формы в неоднородном поле температуры и во внешнем магнитном поле протекающего по ней тока вакуумной дуги. Записаны исходные уравнения для горизонтального слоя вязкой, несжимаемой, проводящей жидкости цилиндрической формы, находящейся в неоднородном по высоте температурном поле и во внешнем магнитном поле протекающего по ней тока вакуумной дуги. Эти уравнения состоят из линеаризованных уравнений для малых возмущений скорости, малых отклонений от равновесных значений температуры, давления и напряженности магнитного поля. Рассматриваемая краевая задача решена для случая свободных границ. Сравнение экспериментальных данных с теоретическими расчетами позволило определить скорость вращения расплава стали при вакуумно-дуговой плавке.

СТІЙКІСТЬ ШАРУ В'ЯЗКОЇ НЕСТИСЛИВОЇ РІДИНИ ЦИЛІНДРИЧНОЇ ФОРМИ В НЕОДНОРІДНОМУ ТЕМПЕРАТУРНОМУ ПОЛІ І МАГНІТНОМУ ПОЛІ ВАКУУМНОЇ ДУГИ, ЩО ПРОТІКАЄ ЧЕРЕЗ РІДИНУ

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Теоретично досліджений конвективний масоперенос в'язкої нестисливої провідної рідини циліндричної форми в неоднорідному полі температури і в зовнішньому магнітному полі струму вакуумної дуги, що протікає по ній. Записані вихідні рівняння для горизонтального шару в'язкої, нестисливої, провідної рідини циліндричної форми, що знаходиться в неоднорідному по висоті температурному полі і в магнітному полі струму вакуумної дуги, що протікає по ній. Ці рівняння складаються з лінеаризованих рівнянь для малих збурень швидкості, малих відхилень від рівноважних значень температури, тиску і напруженості магнітного поля. Вже згадана крайова задача вирішена для випадку вільних меж. Порівняння експериментальних даних з теоретичними розрахунками дозволило визначити швидкість обертання розплаву сталі при вакуумно-дуговій плавці.