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CONVECTION OF A VISCOUS INCOMPRESSIBLE COOLANT IN A HORIZONTAL CYLINDRICAL PIPE HEATED FROM BELOW. AN ANALYTICAL SOLUTION

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To intensify heat transfer processes in nuclear power, it is proposed to use natural convection in horizontal heat exchange tubes. In the article, the problem of convection of a viscous, incompressible fluid in a horizontal cylinder heated from below is analyzed analytically. The proposed analytical method is based upon the use of basic functions that satisfy the initial equation but do not satisfy the boundary conditions. The problem with boundary conditions with a sufficient degree of accuracy is solved by dividing the area of the square described around the cylinder into a large number of smaller element cells. This separation has made it possible to obtain analytical expressions of critical Rayleigh numbers for different positions of current lines in a horizontal cylinder with a viscous, incompressible fluid.

INTRODUCTION

In nuclear energy, an important place is occupied by the problems of calculating heat and mass transfer, which are encountered when choosing the design of heat exchangers and boilers of nuclear reactors [1]. In existing heat exchange systems, the intensification of heat transfer processes is carried out by changing the shape of the surface of the heat exchanger or the speed of the coolant. The intensification of heat transfer improves the economic characteristics of heat exchangers (reduction in surface size, metal consumption, reduction in size and cost). However, the intensification of heat transfer processes can be carried out due to internal thermal convection. Under operating conditions in such systems, convective heat and mass transfer may occur due to the occurrence of convective flows (CT).

For the first time, this problem was considered analytically by Rayleigh [2]. Then this problem has been systematically studied by many researchers and the results of their investigations are quite fully described in monographs [3–6].

The research conducted to date shows that there exists a critical Rayleigh number Ra_c below which small disturbances of equal temperature and speed will decay. At the same time, the value of the critical Rayleigh number depends on the type of limit conditions for the speed of liquid on horizontal lines limiting planes layer. For instance, if the limiting planes are surfaces that do not create tangential stresses (free boundaries), then Ra_c = 657.511. If these limiting planes are rigid and isothermal (rigid boundaries), then Ra_c = 1707.762. For free boundaries, the critical number of Rayleigh is obtained analytically, and for rigid borders – using numerous methods. Recently, a different approach has been proposed in describing the appearance of a cellular structure in the development of convective heat and mass transfer. This approach is based upon the fact that the cellular structure for free and rigid boundary conditions that was observed by Henri Bernard [7] arises from a set of elementary cylindrical convective cells [8, 9]. The cylindrical nature of convective cells is conditioned by the nature of the occurrence of convection which is based on a cylindrical symmetric thermic [10]. The resulting separately cylindrical convective cells grow in number with increasing temperature and by coming into contact with one another form visually observed hexagonal spatial structures.

Convective instability is observed not only in horizontal layers of viscous, incompressible liquid heated from below, but also in horizontal round cylinders with the same type of heating. Such research has been carried out in a number of works [5, 11, 12].

In these works, convective instability studies were carried out in a long horizontal round cylinder filled with a viscous, incompressible liquid, which is heated from below. This problem is solved by approximate numerous methods, and the resulting solutions are often not identical, and sometimes contradict each other.

Investigation of the convective instability of a coolant in a long horizontal round cylinder heated from below and filled with a viscous incompressible liquid is an urgent task in calculating heat transfer in the cooling circuits of nuclear reactors.

In this paper, an analytical method for solving the problem of convective instability in a long horizontal round cylinder filled with a viscous, incompressible liquid heated from below is proposed.

SOURCE EQUATIONS

Consider the unlimited in the direction of the axis z cylindrical channel which radius is a and which is filled with a viscous incompressible liquid. The channel is in a heat-conducting array. Here, axis y is directed upwards and is opposite in the direction of the acceleration vector of the free fall $\vec{g}$. The temperature inside the cylinder line depends on the coordinate y: $T_0(y) = T_2 - \Delta T \cdot (y/a)$, where $\Delta T = T_2 - T_1 = \Theta > 0$, T_2 and T_1 – the temperature in the center of the cylinder and at the top point of the forming cylinder is appropriate. The coordinate system in which the cylinder is located is the left triplet (Fig. 1).

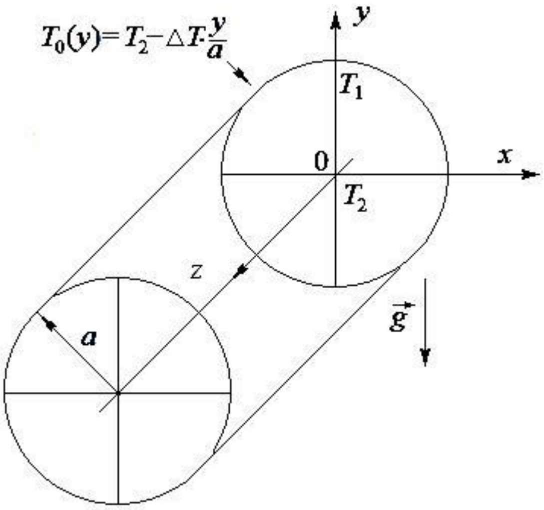


Fig. 1. Geometry of the cylinder location in a heat-conducting array

We will consider normal speed disturbances $\vec{v} = (v_x, v_y)$, pressure $p(x, y)$ and temperatures $T(x, y)$ independent of the coordinate z. The behavior of small stationary disturbances is described by a system of equations consisting of Navier-Stokes equations in the Boussinesque approximation, heat balance equalization, and continuity equalization [3, 4, 6]:

$$(\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{1}{\rho_0}\vec{\nabla}p + \nu\Delta\vec{v} + g\beta T\vec{e}_y; \quad (1)$$

$$(\vec{v} \cdot \vec{\nabla})T_0 = \chi\Delta T; \quad (2)$$

$$\vec{\nabla}\vec{v} = 0, \quad (3)$$

where ρ_0 – undisturbed liquid density; ν – coefficient of kinematic viscosity of the liquid; β – coefficient of temperature expansion of the liquid; χ – coefficient of temperature conductivity of the liquid; $\vec{e}_y -$ unit vector in the direction of the axis y.

For solid boundary conditions, the resented values must meet the conditions:

$$v_x(r = a) = v_y(r = a) = 0, \frac{dv_x(r=a)}{dx} = \frac{dv_y(r=a)}{dy} = 0, \quad (4)$$

where $r = \sqrt{x^2 + y^2}$.

In a heat-conducting massive, the distribution of the disturbed temperature of the massive T_m is described by the equation:

$$\Delta T_m = 0, \quad (5)$$

with conditions at the cylinder boundary $r = a$: $T = T_m$; $\tilde{\kappa}\frac{\partial T}{\partial r} = \frac{\partial T_m}{\partial r}$, where $\tilde{\kappa} -$ relative thermal conductivity of the liquid. At the same time, the amplitude of the disturbed temperature T_m fades when removed from the cavity.

To study stationary states, it is necessary to contain linear terms in the system (1)–(3).

But first, for the convenience of calculations, we give comparisons (1)–(5) to the dimensionless form. To do this, we will define the following measurement units. Distance unit – h which can be equal to the radius of the cavity a [12] or diameter, time – h^2/ν, speed – χ/h, pressure – $\rho_0\,\nu\chi/h^2$, temperatures – Θ.

In the specified units, when applied the operator to (1) $rotrot(\ldots)$ and using the projection of the resulting result on the axis x, we get the equations for the current function ψ and the perturbated temperature $\hat{T}$ in dimensionless form [5, 12]:

$$\Delta\Delta\hat{\psi} = Ra\frac{\partial\hat{T}}{\partial X}, \quad (6)$$

$$\Delta\hat{T} = -\frac{\partial\hat{\psi}}{\partial X}, \quad (7)$$

where $\Delta \ldots = \frac{\partial^2 \ldots}{\partial X^2} + \frac{\partial^2 \ldots}{\partial Y^2}$ – Laplace operator, $Ra = \frac{g\beta\Theta h^3}{\nu\chi}$ – Rayleigh number, $X = x/h$, $Y = y/h$, $\hat{\psi}(X, Y) = \psi/\chi$ – current function set by expressions $\hat{V}_x = h\,v_x/\chi = \frac{\partial\hat{\psi}}{\partial Y}$, $\hat{V}_y = h\,v_y/\chi = -\frac{\partial\hat{\psi}}{\partial X}$, $\hat{T} = T/\Theta$.

Equations (6), (7) at $h = 2a$ received in [5, 12].

They can be reduced to the same level for the current function where the tilde sign is omitted below and further:

$$\Delta\Delta\Delta\psi = Ra\frac{\partial^2\psi}{\partial X^2}. \quad (8)$$

The boundary conditions (4) for the current function are taken as:

$$\psi(R = 1) = 0, \frac{d\psi(R=1)}{dx} = \frac{d\psi(R=1)}{dy} = 0, \quad (9)$$

where $R = \sqrt{X^2 + Y^2}$.

ANALYSIS OF PREVIOUS STUDIES OF STATIONARY CONVECTION IN HORIZONTAL CYLINDERS HEATED FROM BELOW

In works [10] the results of a study of the equation (8) with rigid boundaries are presented. Two different variable principles are used, each of which is equivalent to solving the problem for its own values. Based upon these approaches, two approximate methods for calculating the critical Rayleigh number have been developed. It is shown that the critical Rayleigh number of the horizontal circular cylinder for the first variant principle is 6552, and 6510 – for the second. These values are 3.8 times greater than the critical Rayleigh number for a horizontal layer with rigid boundaries, equal to 1707.8. These calculations show that the critical Rayleigh number for a horizontal cylinder is significantly lower than the values calculated in previous studies.

In the monograph [4] the results of a study of convective instability in a long horizontal circular cylinder are carried out. Comparisons are analyzed for small deviations of speed, pressure, and temperature from the initial state. In particular, in the case of a cylinder of infinite length, approximate solutions are obtained using the Galerkin method for current functions that do not depend on the axial coordinate of the cylinder. Based on the solution obtained, there were determined dependencies of the critical Rayleigh numbers from the coefficient of relative thermal conductivity of the liquid. As an example, we will present what we received in [5] the value of the critical Rayleigh number for concentric current lines (configuration a) with zero thermal conductivity of the liquid – $Ra = 408.2$. The following configurations are also calculated here b, c, d current lines.

Current line configurations are shown in Fig. 2. The direction of the arrows in the figure is shown conditionally and may have the opposite direction.

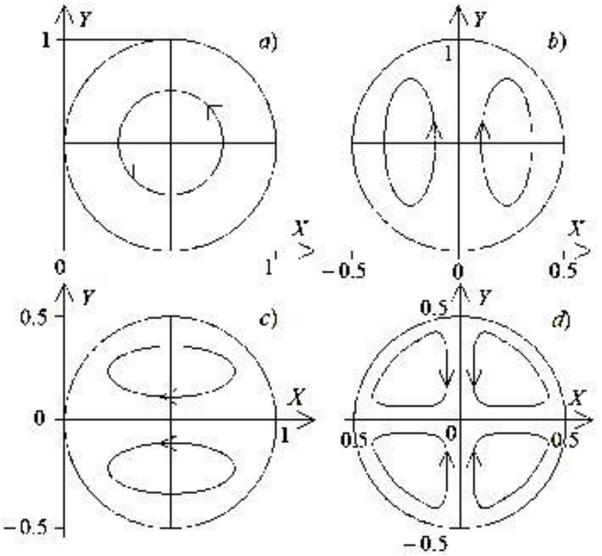


Fig. 2. Current line configurations in the Cartesian coordinate system

It is shown that critical Rayleigh numbers take the values of the order of magnitude: b – 720; c – 3000; d – 3200 if the coefficient of the relative thermal conductivity of the liquid is zero.

It should be noted that the critical Rayleigh numbers obtained by the Galerkin method are based on setting the model function of the current of a polynomial no higher than the sixth power of the coordinates and satisfying the boundary conditions on the cylinder surface. The question of the dependence of critical Rayleigh numbers on the type of model function remains open. Therefore, the applicability of numerous results obtained should be based on experimental data. However, such data is not available in scientific literature.

Below we present an analytical method for calculating critical Rayleigh numbers for the above current line configurations.

ANALYTICAL METHOD FOR CALCULATING THE CRITICAL RAYLEIGH NUMBER

The basic functions for describing current lines follow from the form of equation (8) and are proportional to the production of harmonic functions. Let's consider the production of the form $sin(k_X X) \cdot sin(k_Y Y)$, where k_X, k_Y – wave numbers of disturbances in the directions X, Y, accordingly. To determine a specific type k_X, k_Y, you need to select the location of the cylinder in the rectangular coordinate system X, Y. Let's consider the case when the cylinder is inscribed in a square on the side 1, and is in the first quadrant, as shown in Fig. 3.

With this geometric arrangement of the cylinder, the current function that satisfies equation (8) has the form:

$$\psi(X,Y) = A \cdot sin(n\pi X) \cdot sin(m\pi Y), \quad (10)$$

where n and m – integers denoting the modes of resentment, $0 \le X, Y \le 1$.

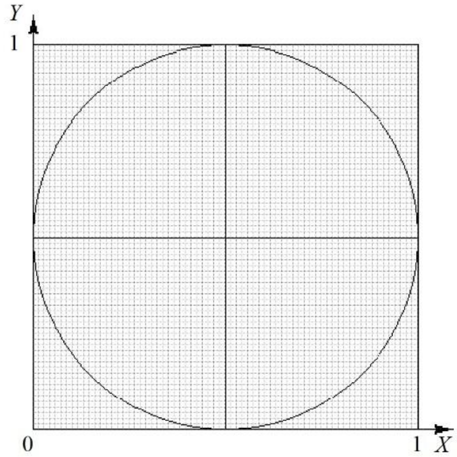


Fig. 3. Geometry of the cylinder location in the first quadrant of a rectangular coordinate system

From equation (8), the values of the Raleigh numbers follow:

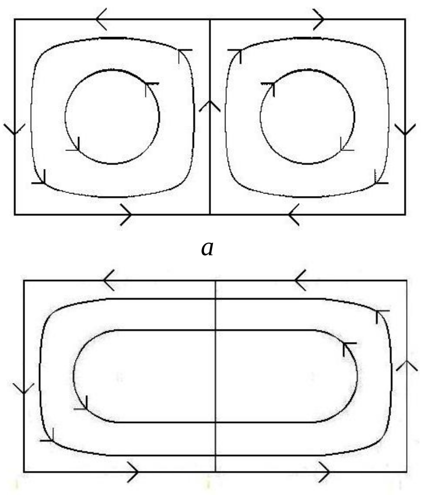
$$2 \cdot Ra_{nm} = \frac{((n\pi)^2 + (m\pi)^2)^3}{(n\pi)^2}. \quad (11)$$

The Fig. 2 in expression (10) is obtained by calculating the temperature difference at the radius of the cylinder, as it is formulated in the problem statement, to the temperature difference at the diameter of the cylinder.

It should be noted that solution (10) does not meet the boundary conditions (9). To satisfy them, let's do the following. Disassemble the side of the square of Fig. 3 for many cuts, so that $p \cdot \Delta a = 1$, where $p \gg 1$. This division covers the area of the square 1×1 element cells in the form of a square with an area $\Delta a \times \Delta a$ in the amount of p^2. In an elementary square with a side Δa current lines are determined from the equation: $A \cdot sin(\pi \hat{x}) \cdot sin(\pi \hat{y}) = 1$, where $A \le 1$, $\hat{x} = x/\Delta a$, $\hat{y} = y/\Delta a$. They are represented by concentric closed lines inscribed in a square. When close to one A, the shape of closed lines is close to the circle. When $A \ll 1$, reduced A the shape of closed lines tends to a square. The direction of movement of the liquid in this case is carried out along the current lines against the hour hand.

With this separation, the current line in the cylinder will consist of many current lines of element cells. At the same time, the velocity vectors of neighboring element cells will be formed at the contact boundary and form the resulting current. In one case, when the velocities at the contact boundary are unidirectional, they will add up and create a current at that boundary and in the cell, as a whole (see Figs. 1; 4,a). In another case, when the velocities are oppositely directed, the velocity at the contact boundary becomes zero, and the velocity vector inside two connected cells will have the form of concentric closed lines inscribed in a rectangle while maintaining the direction of the velocity vector of the element cell (see Fig. 4,b).

The above is schematically illustrated in Fig. 4.



a

b

Fig. 4. Formation of a vector field of fluid velocity in adjacent element cells

This principle of applying speeds in touching cells has been successfully used in [13] to describe the formation of Langmuir circulations in open water bodies.

As a result of this separation, it is possible to satisfy the boundary conditions (9) most accurately: the smaller the side of the element square, the more accurately the boundary conditions are fulfilled. This is explained by the fact that as the size of an element cell decreases, the number of corners located at the cylinder boundary increases where the function of the current line and its first vertical and horizontal coordinates are zero.

Using this division into element cells, you can calculate the Rayleigh number as accurately as possible. Let's find it.

Sequential transformation of scale fragmentation in expression (11) gives the following result:

$$2 \cdot \frac{g\beta\delta(p\Delta a)^4}{\nu\chi} = 2 \cdot Ra_{NM} = \frac{\left((Mp^2\pi)^2+(Np^2\pi)^2\right)^3}{(Np^2\pi)^2}, \quad (12)$$

where δ – temperature gradient which does not depend on the transformation of the division, $m = Mp^2$; $n = Np^2$; M, N – integers; $X(h) \to p^{-1}X(\Delta a)$ and $\cdot\, Y(h) \to p^{-1}Y(\Delta a)$ – transformations of coordinate fragmentation in expression (10).

From (12), we find the Rayleigh number for an elementary convective cell without specifying the location of the cylinder:

$$Ra_{M,N} = \frac{g\beta\delta(\Delta a)^4}{\nu\chi} = \frac{((Mp\pi)^2 + (Np\pi)^2)^3}{2(Np\pi)^2(p)^4} = \frac{((M\pi)^2 + (N\pi)^2)^3}{2(N\pi)^2} = \frac{\left((M)^2+(N)^2\right)^3}{2(N)^2}\pi^4. \quad (13)$$

When the cylinder is in the coordinates of Fig. 2,a the critical Rayleigh number is determined by the expression:

$$Ra_{1,1} = \frac{\left((1)^2+(1)^2\right)^3}{2(1)^2}\pi^4 \approx 4 \cdot 97.40909 = 389.6364. \quad (14)$$

This value differs by 4.5% from the value calculated by the approximate Galerkin method – 408.2.

Configuration of current lines in coordinates (see Fig. 2,b) turns out by reducing the length of the side of the element square along the axis X twice which is equivalent to a replacement N on $N/2$. The size of an elementary square by axis Y at the same time does not change which leaves the initial scale of the characteristic length of dimension unchanged and allows you to keep the number 2 in the expression (11). Then for the next integer $N = 3$ ($N = 2$ repeats the value (14) and, therefore, is not considered) we get the value of the Rayleigh critical number:

$$Ra_{1,3} = \frac{\left((1)^2+\left(\frac{3}{2}\right)^2\right)^3}{2\left(\frac{3}{2}\right)^2}\pi^4 = \frac{2\left(\frac{13}{4}\right)^3}{9}\pi^4 \approx 7.628 \cdot 97.40909 = 743.08253. \quad (15)$$

Configuration of current lines in coordinates (see Fig. 2,c) turns out by reducing the length of the side of the element square along the axis Y twice which is equivalent to a replacement M on $M/2$. The size of an element square by axis X at the same time does not change. When configuring current lines c the internal scale of the characteristic length of dimension changes from the diameter to the radius of the cylinder. This eliminates number 2 in expression (11). Then, for the Rayleigh critical number we get the value:

$$Ra_{3,1} = \frac{\left(\left(\frac{3}{2}\right)^2+(1)^2\right)^3}{(1)^2}\pi^4 = \frac{\left(\frac{13}{4}\right)^3}{1}\pi^4 \approx 34.328125 \cdot 97.40909 = 3\,343.87. \quad (16)$$

Similarly, you can get the Rayleigh critical number for configuring current lines Fig. 2,d:

$$Ra_{3,3} = \frac{\left(\left(\frac{3}{2}\right)^2+\left(\frac{3}{2}\right)^2\right)^3}{\left(\frac{3}{2}\right)^2}\pi^4 = (2)^3\left(\frac{3}{2}\right)^4\pi^4 \approx 45.5 \cdot 97.40909 = 4\,432.1. \quad (17)$$

In geometry *a*) when $M = N = 2$ the Rayleigh critical number has a value:

$$Ra_{2,2} = \frac{((2)^2 + (2)^2)^3}{2(2)^2}\pi^4 \approx 64 \cdot 97.40909 = 6\,234.18176, \quad (18)$$

which with an accuracy of 4.9 and 4.2%, responds to the results obtained in [11] the Rayleigh critical numbers of the first and second variant principles correspond accordingly.

Thus, in this section, analytical expressions of the Rayleigh critical numbers for different positions of current lines in a horizontal, bottom-heated cylinder with a viscous, incompressible fluid are obtained by dividing into elementary squares the described circumference of the cross-section of the cylinder. The Rayleigh critical numbers calculated based on analytical expressions in some cases differ up to 5% from those found by other authors.

DEPENDENCE OF THE RAYLEIGH NUMBER ON THE COEFFICIENT OF RELATIVE THERMAL CONDUCTIVITY

In the monograph [5], using the Galerkin method, there was described the dependence of the Rayleigh critical numbers on the coefficient of relative thermal conductivity $\tilde{\kappa}$ for different configurations of convective currents. It is shown that with an increase in $\tilde{\kappa}$ the critical Rayleigh numbers decrease and, with sufficiently large thermal conductivities, tend to a constant value. The result obtained is not analytical. It predicts the quantitative behavior of the critical Rayleigh numbers, does not describe functional dependencies, and does not provide a physical explanation of the process. However, from numerous calculations it can be concluded that the temperature gradient δ and dynamic viscosity coefficient ν can decrease with an increase in the coefficient of relative thermal conductivity $\tilde{\kappa}$. We do not consider temperature conductivity dependence χ from the coefficient of relative thermal conductivity $\tilde{\kappa}$ because this corresponds to the manifestation of non-linear effects in the linear problem. Therefore, we assume that the following functional dependencies are valid:

$$\delta = \frac{\delta_0}{1+\alpha\tilde{\kappa}}, \nu = \frac{\nu_0}{1+\beta\tilde{\kappa}}, \tag{19}$$

where $\alpha,\ \beta$ – constants determined by multiple methods or from experimental data for various convective flow geometries.

As a result of substituting (19) into (13), we obtain an expression for the Rayleigh critical numbers of depending on the coefficient of relative thermal conductivity:

$$Ra_{M,N}(\tilde{\kappa}) = Ra^0_{M,N}(1+\alpha\tilde{\kappa})(1+\beta\tilde{\kappa})^{-1}, \tag{20}$$

where $Ra^0_{M,N} = \frac{g\beta\delta_0(\Delta a)^4}{\nu_0\chi}$ – the critical Rayleigh number at zero thermal conductivity coefficient.

The result of using expression (20) to describe numerous data [5] is presented in Fig. 5. The figure shows a fairly accurate description of the results obtained by numerous methods using the formula (20). This description allows for determining the constants with a sufficient degree of accuracy α, β for various geometries of convective flows. The values of these constants are shown in Table.

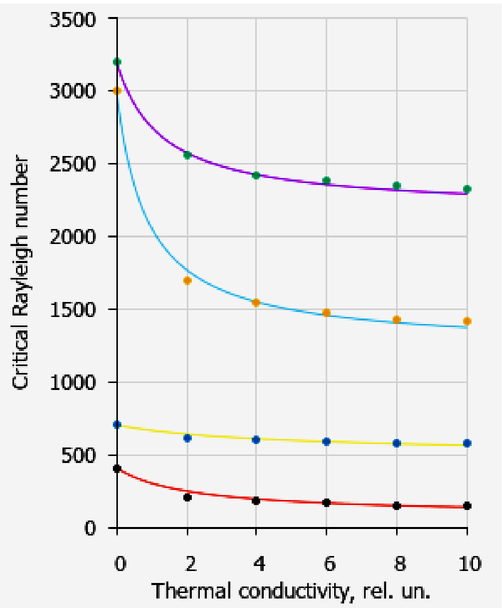


Fig. 5. Analytical dependences of the critical Rayleigh numbers on the coefficient of relative thermal conductivity (continuous lines) in comparison with numerically obtained (points) [5]

This section suggests considering the dependence on relative thermal conductivity $\tilde{\kappa}$ of the critical Rayleigh number of the horizontal cylinder with a viscous incompressible fluid heated from below.

Constants α, β for various geometries of convective flows inside the cylinder

Geometry	Constants			
	a)	b)	c)	d)
α	0.0941	0.157	0.471	0.534
β	0.471	0.22	1.147	0.785

This dependence is conditioned the fact that temperature gradient δ and dynamic viscosity coefficient ν decrease with increasing coefficient of relative thermal conductivity $\tilde{\kappa}$. We do not consider temperature conductivity dependence χ from the coefficient of relative thermal conductivity $\tilde{\kappa}$ because this corresponds to the manifestation of non-linear effects in the linear problem.

This proposal is confirmed by numerous comparisons with the results of numerous calculations given in [5]. Comparison of the results of numerous calculations with an analytical representation shows their full compliance.

For analytically obtained dependences of the critical Rayleigh numbers on the coefficient of relative thermal conductivity there were determined constants $\alpha,\ \beta$ which are shown in the table for various geometries of convective flows inside the cylinder.

Thus, the proposed analytical method for calculating the dependence of critical numbers on thermal conductivity adequately describes the results of numerous studies of this problem.

CONCLUSIONS

This paper offers an analytical method for solving the problem of convective instability in a long horizontal round cylinder filled with a viscous, incompressible liquid heated from below. The method is based upon the use of basic functions that satisfy the initial comparison. However, basic functions do not meet the boundary conditions. The problem with boundary conditions can be solved in sufficient approximation if the area described around the cylinder of the square on side 1 is divided into element cells which are squares of a smaller area on side B p times smaller than the sides of the described square. As a result of this separation, the current line in the cylinder will consist of many current lines of element cells. At the same time, the velocity vectors of the neighboring elementary cells will be formed at the contact boundary and form the resulting current. In one case, when the velocities at the contact boundary are unidirectional, they will add up and create a current at that boundary and in the cell, as a whole (see Figs. 1; 4,a). In another case, when the velocities are oppositely directed, the velocity at the boundary of the contact of the element cells becomes zero, and the velocity vector inside the two combined cells will have the form of concentric closed lines inscribed in a rectangle while maintaining the direction of the velocity vector of the element cell (see Fig. 4,b).

As a result of breaking down what is described around the cylinder of the square, it is possible to meet the boundary conditions most accurately: the smaller the side of the element square is, the more accurately the boundary conditions are met. This is explained by the fact that as the size of an element cell decreases, the number of corners located at the cylinder boundary increases where the function of the current line and its first vertical and horizontal coordinates are zero.

Using the division of the square described around the cylinder into element cells, analytical expressions of the critical Rayleigh numbers are obtained for different positions of current lines in a horizontal cylinder with a viscous, incompressible fluid heated from below. The critical Rayleigh numbers calculated on the basis of analytical expressions in some cases differ up to 5% from those found by other authors.

The proposed analytical method for calculating the dependence of critical numbers on thermal conductivity adequately describes the results of numerous studies by other authors.

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КОНВЕКЦІЯ В'ЯЗКОГО НЕСТИСЛИВОГО ТЕПЛОНОСІЯ В ГОРИЗОНТАЛЬНІЙ ЦИЛІНДРИЧНІЙ ТРУБІ, ЩО ПІДІГРІВАЄТЬСЯ ЗНИЗУ. АНАЛІТИЧНИЙ РОЗВ'ЯЗОК

О.Л. Андрєєва, К.В. Абеленцева, В.І. Ткаченко

Для інтенсифікації процесів теплообміну в ядерній енергетиці запропоновано використовувати природну конвекцію у теплообмінних горизонтальних трубах. У статті аналітично досліджено завдання про конвекцію в'язкої, нестисливої рідини в горизонтальному циліндрі, що підігрівається знизу. Пропонований аналітичний метод заснований на використанні базових функцій, що задовольняють вихідному рівнянню, але не задовольняють граничним умовам. Задача з граничними умовами з достатньою мірою точності вирішується шляхом поділу площі квадрата, описаного навколо циліндра на велику кількість дрібніших осередків-елементів. Цей поділ дозволив отримати аналітичні вирази критичних чисел Релея для різних положень ліній струму в горизонтальному циліндрі з в'язкою нестисливою рідиною. Запропоновано аналітичний вираз залежності критичних чисел Релея від теплопровідності, який з високим ступенем точності описує результати чисельних досліджень інших авторів.