https://doi.org/10.46813/2023-148-186

INFLUENCE OF GENERATED PUMP ELECTRIC FIELD ON MULTIHARMONIC INTERACTION OF WAVES IN AMPLIFICATION SECTION OF SUPERHETERODYNE FEL

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In the cubic nonlinear approximation, we analyze the processes of generating the electric pump field by an electron beam and its effect on the amplification of space charge waves (SCW) in the amplification section of a parametric superheterodyne free-electron laser (FEL). We found that the generated pump electric field amplitude is within 21…33% of the amplitude of the external pump electric field. We showed that the generated pump electric field is in phase with the external one. It leads to an increase in the growth increments of SCW; therefore, the length of the SCW amplification section is reduced by 22%. We found out that this field does not destroy the mode of amplification of multi-harmonic SCW. Thus, the studied effect makes it possible to create FELs with smaller longitudinal dimensions and powerful electromagnetic waves with a complex multi-harmonic spectrum.

PACS: 41.60.Cr, 52.59.Ye

INTRODUCTION

Free-electron lasers (FELs) are powerful terahertz electromagnetic radiation sources. One of their main advantages is the convenient tuning of the radiation frequency in a wide range and creating powerful signals. However, it should be noted that these devices have certain disadvantages, mainly their large size and complex operation, which ultimately limits the scope of their use. That is why researchers have recently focused on improving the existing schemes of such devices and creating new ones [1 - 4]. The subject of this article is devoted to this common research direction.

In this paper, we analyzed physical processes in the amplification section of a parametric superheterodyne free-electron laser [5 - 6] to find optimal operating modes. The operation essence of superheterodyne FELs is to use an additional amplification mechanism along with the traditional mechanism for increasing the growth increments of the signal amplitudes. Two-stream, plasma-beam, parametric and other instabilities can be used as an additional wave amplification mechanism [7]. When two-stream instability is used, such devices are called superheterodyne two-stream FELs. Our work considers parametric FEL, which uses parametric instability as an additional wave amplification mechanism. Superheterodyne free electron lasers are attractive because they have very high growth rates of signal waves compared to traditional free electron lasers. They can also generate a powerful multi-harmonic signal with a broad frequency spectrum [6 - 9]. Such multi-harmonic signals can be used to investigate the properties of materials and other special applications [10].

In Fig. 1 we can see one of the possible schemes of such FEL [5]. An electron beam 1 passes along the Z axis through a non-resonant modulator 2, where the beam is modulated, and multi-harmonic space charge waves (SCWs) are formed. Then, the created multi-harmonic SCWs are amplified, passing through the amplifying section 3, where an undulator with a longitudinal periodically changing electric field is located. In section 4, the electron beam is accelerated. Section 5, which contains a transverse multi-harmonic H-ubitron magnetic field, transforms the amplified and accelerated slow SCW into a multi-harmonic electromagnetic wave. Thus, such FELs can form powerful electromagnetic signals.

One of the main elements of the FEL under study is the section for amplifying multi-harmonic SCWs (position 3, Fig. 1), in which an additional amplification mechanism is released. In the amplification section, we create conditions for a three-wave parametric resonance between the longitudinal periodically reversible pumping electric field and the electron beam fast and slow space charge waves. Due to the parametric instability, the slow SCW receives additional amplification that is then transferred to the electromagnetic signal wave.

Earlier in [6], we analyzed multi-harmonic interactions of longitudinal waves in the amplification section of parametric superheterodyne FELs. It is shown that multiple parametric resonances of various types are realized in the system under study. Due to the system's features under investigation, it was found that at the initial stage of wave interaction, the growth rates of space charge waves do not depend on the frequencies of the amplified waves. This property allows for the creation powerful multi-harmonic waves with a broad frequency spectrum.

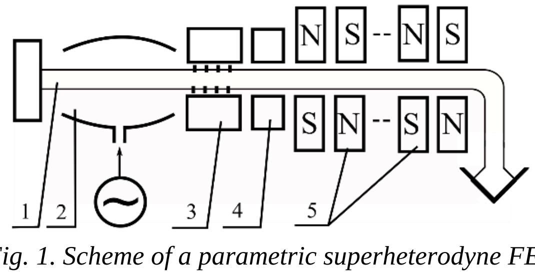

Fig. 1. Scheme of a parametric superheterodyne FEL with a longitudinal electrostatic undulator: 1 – electron beam; 2 – non-resonant modulator; 3 – amplification section of multi-harmonic SCWs; 4 – electron beam acceleration section; 5 – H-ubitron undulator

In the presented work, we describe the effect of a generated electric pump field and its influence on the processes of wave interaction in the amplification section of the FEL in the frame of the cubic nonlinear analysis. Analysis shows that the generated electric pump field has a significant effect on the processes of wave amplification. Due to this effect, the increments are increased by 21…33%, and device length can be reduced by 22%.

MODEL

Let us consider the physical processes occurring in the amplification section of a parametric superheterodyne free electron laser (see Fig. 1, position 3). The main structural element in this section is an electrostatic undulator, which forms a longitudinal, periodically reversible electric pump field. The electric strength vectors of this field are collinear to the Z axis and collinear to the relativistic electron beam velocity. Schemes of its implementation may be different [5], but the pump electric field created by such an undulator has the form:

$$\mathbf{E}_{20} = [E_{20}\exp(ip_{2,1}) + c.c.]\mathbf{e}_z , \qquad (1)$$

where E_{20} is the complex electric strength amplitude of the external undulator electric field, $p_{2,1} = -k_2 \cdot z$ is the first harmonic of its phase, $k_2 = 2\pi/\Lambda$ is its wave number, Λ is the undulation period, $\mathbf{e}_z$ is Z axis unit vector. Thus, we consider the external pump electric field of the undulator to be monochromatic.

A pre-modulated relativistic electron beam is fed in the amplification section, where the longitudinal slow (index α) and fast (index β) space charge waves propagate. In general, we consider these waves to be multi-harmonic, and so their electric fields have the form:

$$\mathbf{E}_\alpha = \sum_{m=1}^{N}\left[E_{\alpha,m}\exp\left(ip_{\alpha,m}\right) + c.c.\right]\mathbf{e}_z , \qquad (2)$$

$$\mathbf{E}_\beta = \sum_{m=1}^{N}\left[E_{\beta,m}\exp\left(ip_{\beta,m}\right) + c.c.\right]\mathbf{e}_z . \qquad (3)$$

In these formulas $p_{\alpha,m} = m\omega t - k_{\alpha,m}z$ and $p_{\beta,m} = m\omega t - k_{\beta,m}z$ are phases of the mth harmonics of the slow and fast SCW, $k_{\alpha,m}$ and $k_{\beta,m}$ are their wavenumbers, ω is the first harmonic frequency, $E_{\alpha,m}$ and $E_{\beta,m}$ are complex electric strength amplitudes of the mth harmonics of the SCWs.

Parameters of the system under study are adjusted so that a three-wave parametric resonance occurs here between slow and fast space charge waves and the periodically reversible pumping electric field. Due to this, parametric instability occurs in the volume of the electron beam. As a result, the slow and fast SCWs grow exponentially and provide additional amplification in the parametric superheterodyne FEL. The above condition of the three-wave parametric resonant interaction has the form:

$$p_{\alpha,m} - p_{\beta,m} = p_2 \text{ or } k_{\alpha,m} - k_{\beta,m} = k_2 . \qquad (4)$$

This condition imposes some requirements on the parameters of the electrostatic undulator. As is known, the wave numbers of the slow and fast space charge waves are defined as:

$$k_{\alpha,m} = m \cdot \omega/\upsilon_0 + \omega_p / \left(\gamma_0^{3/2}\upsilon_0\right),$$
$$k_{\beta,m} = m \cdot \omega/\upsilon_0 - \omega_p / \left(\gamma_0^{3/2}\upsilon_0\right), \qquad (5)$$

where ω_p is the Langmuir frequency, υ_0 is the constant (non-oscillating) term of the electron beam velocity, γ_0 is its Lorentz factor. Substituting (5) into (4), we can find that the conditions of parametric resonances (4) are satisfied if the undulation period of the pump electric field is equal to:

$$\Lambda = \pi\gamma_0^{3/2}\upsilon_0 / \omega_p . \qquad (6)$$

From (6), the undulation period does not depend on the harmonic number m. This feature means that the parametric resonance condition (4) is fulfilled for all mth harmonics. At the same time, only the first harmonic of the pump electric field is involved in all resonant processes. In other words, many three-wave resonant interactions between the mth harmonics of the fast and slow SCWs and the first harmonic of the pump electric field can be realized simultaneously. As is known, the slow SCW is characterized by negative energy and the fast one by positive energy [7]. Thus, fast and slow multi-harmonic SCWs amplify due to plural parametric wave resonances, which receive energy from the electron beam.

Also, another exciting feature of the studied system should be noted. The relativistic electron beam moving through the pump field $\mathbf{E}_{20}$ is modulated by it, and as a result, the beam generates its own pump electric field (generated pump electric field). In general, we consider this field that consists of N harmonics and has the form:

$$\mathbf{E}_2^g = \sum_{m=1}^{N}\left[E_{2,m}^g\exp\left(ip_{2,m}\right) + c.c.\right]\mathbf{e}_z . \qquad (7)$$

In this equation $p_{2,m} = -m \cdot k_2 z$, $E_{2,m}^g$ is the complex electric strength amplitude of the mth harmonic of the generated pump field. As a result, the resulting electric field of phase p_2 has the form:

$$\mathbf{E}_2 = \mathbf{E}_{20} + \mathbf{E}_2^g = \sum_{m=1}^{N}\left[E_{2,m}\exp\left(ip_{2,m}\right) + c.c.\right]\mathbf{e}_z . \qquad (8)$$

As follows from (8), the resulting pump electric field, consisting of external and generated ones, is multi-harmonic. As we will show later in this work, the generated electric field significantly affects the processes of multi-harmonic SCW amplification.

Based on (5), we find that the dispersion dependences of the fast and slow SCWs are linear and shifted relative to each other by a constant value. This feature makes three-wave resonant interactions possible between the mth harmonics of the slow and fast SCWs (resonance condition (4)) and between harmonics with different numbers. As follows from (4) and (5), such plural parametric resonant interactions are possible:

$$\begin{aligned}
&k_{\alpha,n-m+l}\big|_{n-m+l>0} = k_{\beta,n} - k_{\beta,m} + k_{\alpha,l} ,\\
&k_{\alpha,n-m+l}\big|_{n-m+l>0} = k_{\alpha,n} - k_{\alpha,m} + k_{\alpha,l} ,\\
&k_{\alpha,n+m+l} = k_{\alpha,n} + k_{\beta,m} + k_{\alpha,l} ,\\
&k_{\beta,n-m+l}\big|_{n-m+l>0} = k_{\alpha,n} - k_{\alpha,m} + k_{\beta,l} ,\\
&k_{\beta,n-m+l}\big|_{n-m+l>0} = k_{\beta,n} - k_{\beta,m} + k_{\beta,l} ,\\
&k_{\beta,n+m+l} = k_{\beta,n} + k_{\alpha,m} + k_{\beta,k} .
\end{aligned} \qquad (9)$$

Here n, m, and l are integers.

Another resonant interaction type is realized in the studied system. As noted above, the electron beam generates a generated pump electric field $\mathbf{E}_2^g$ (7), the mth harmonic of which has the phase $p_{2,m} = -m \cdot k_2 z$. That is, the harmonic phases depend linearly on the harmonic number, and thus, multiple three-wave resonance interactions occur between the harmonics of the generated pump field:

$$p_{2,m1} = p_{2,m2} + p_{2,m3}, \tag{10}$$

where m_1, m_2 and m_3 are the numbers of harmonics of the generated pumping field. Using the relationship between phase and harmonic number $p_{2,m} = -m \cdot k_2 z$, condition (10) can be written as

$$m_1 = m_2 + m_3. \tag{11}$$

Condition (11) can be fulfilled in many ways. For example, 5 = 3+2, 5 = 4+1, 5 = 6–1, etc.

BASIC EQUATIONS

Let us analyze the amplification of the slow and fast multi-harmonic space charge waves in the amplification section, considering three-wave parametric resonances (4), (9), (11) and the generation of a generated pumping electric field $\mathbf{E}_2^g$ in the cubic nonlinear approximation. To do this, we use the hierarchical asymptotic approach [7], namely, one of the varieties of this approach – the method of averaged characteristics. In general, its main principle is to change variables in such a way as to separate the equations with fast oscillating variables (e.g., harmonic phases) and those with slowly varying variables (e.g., constant components of the beam's velocity and concentration, wave amplitudes). This allows us to obtain analytical expressions for fast-oscillating variables and, simultaneously, make differential equations for slowly changing variables that do not depend on fast-oscillating variables, are not stiff and can be easily solved by standard numerical methods.

To solve the space charge wave amplification problem in the amplification section, we use a quasi-hydrodynamic model of a relativistic electron beam, i.e. the system of the quasi-hydrodynamic equation (for beam motion), the continuity, and Maxwell's equations. As a result of performing asymptotic integration procedures [7], we obtain a system of differential equations for the slow variables. They include the electric field strengths amplitudes of the mth harmonics of the slow and fast SCWs and the generated pump electric field, as well as non-oscillating components of the velocity and concentration of the relativistic electron beam:

$$C_{2,\alpha,m}\frac{d^2E_{\alpha,m}}{dz^2} + C_{1,\alpha,m}\frac{dE_{\alpha,m}}{dz} + D_{\alpha,m}E_{\alpha,m} = \\ = C_{3,\alpha,m}E_{\beta,m}E_2^* + F_{\alpha,m}(\mathbf{E}_\alpha, \mathbf{E}_\beta, \mathbf{E}_2), \tag{12}$$

$$C_{2,\beta,m}\frac{d^2E_{\beta,m}}{dz^2} + C_{1,\beta,m}\frac{dE_{\beta,m}}{dz} + D_{\beta,m}E_{\beta,m} = \\ = C_{3,\beta,m}E_{\alpha,m}E_2 + F_{\beta,m}(\mathbf{E}_\alpha, \mathbf{E}_\beta, \mathbf{E}_2), \tag{13}$$

$$C_{2,2,m}\frac{d^2E_{2,m}^g}{dz^2} + C_{1,2,m}\frac{dE_{2,m}^g}{dz} + D_{2,m}E_{2,m}^g = \\ = \left(C_0E_{20} + \sum_{n=1}^{N} C_{3,2,n}E_{\alpha,n}^*E_{\beta,n}\right)\delta_{1,m} + \\ + F_{2,m}(\mathbf{E}_\alpha, \mathbf{E}_\beta, \mathbf{E}_2), \tag{14}$$

$$\frac{d\upsilon_0}{dz} = F_\upsilon(\mathbf{E}_\alpha, \mathbf{E}_\beta, \mathbf{E}_2), \quad \frac{dn_0}{dz} = F_n(\mathbf{E}_\alpha, \mathbf{E}_\beta, \mathbf{E}_2). \tag{15}$$

In these equations the functions $F_{\alpha,m} = F_{\alpha,m}(\mathbf{E}_\alpha, \mathbf{E}_\beta, \mathbf{E}_2)$, $F_{\beta,m} = F_{\beta,m}(\mathbf{E}_\alpha, \mathbf{E}_\beta, \mathbf{E}_2)$, $F_{2,m} = F_{2,m}(\mathbf{E}_\alpha, \mathbf{E}_\beta, \mathbf{E}_2)$, $F_\upsilon(\mathbf{E}_\alpha, \mathbf{E}_\beta, \mathbf{E}_2)$, $F_n(\mathbf{E}_\alpha, \mathbf{E}_\beta, \mathbf{E}_2)$ depend on the amplitudes of multi-harmonic SCWs and provide cubic nonlinear terms that are related to the multiple parametric resonances that are described above. Note that these functions are cumbersome and are calculated by a recurrent procedure through the corresponding terms of the first and second approximations.

The coefficients of differential equations (12)-(15) have the following form.

$$D_{\chi,m} = -ik_{\chi,m}\left(1 - \frac{\omega_p^2}{(m\omega_\chi - k_{\chi,m}\upsilon_0)^2\gamma_0^3}\right), \tag{16}$$

$$C_{1,\chi,m} = \partial D_{\chi,m} / \partial(-ik_{\chi,m}),$$

$$C_{2,\chi,m} = \partial^2 D_{\chi,m} / \partial(-ik_{\chi,m})^2 / 2,$$

$$C_{3,\alpha,m} = \frac{k_{\alpha,m} \cdot \omega_p^2 e / m_e}{\Omega_{\alpha,m}\Omega_{\beta,m}k_2\upsilon_0\gamma_0^6} \times \\ \times \left(\frac{k_{\alpha,m}}{\Omega_{\alpha,m}} + \frac{k_{\beta,m}}{\Omega_{\beta,m}} - \frac{k_2}{k_2\upsilon_0} - \frac{3\upsilon_0\gamma_0^2}{c^2}\right),$$

$$C_{3,\beta,m} = -k_{\beta,m}C_{3,\alpha,m} / k_{\alpha,m}, \quad C_0 = -ik_2\omega_p^2 / (\Omega_{2,1}^2\gamma_0^2),$$

$$\Omega_{\chi,m} = m\omega_\chi - k_{\chi,m}\upsilon_0,$$

where index «χ» indicates the wave type (α, β or 2), e and m_e are values of the electron charge and the electron mass, $\omega_2 = 0$, $k_{2,m} = mk_2$, c is the speed of light in vacuum; γ_0 is the constant term of the Lorentz factor; ω_p is the plasma frequency of the electron beam; $\delta_{1,m}$ is a Kronecker's symbol.

Please note that the slow and fast SCWs are eigenwaves of the electron beam and, therefore, $D_{\alpha,m}$ and $D_{\beta,m}$ are equal to zero. On the contrary, a periodically reversible pumping electric field is provided exclusively by an electrostatic undulator, i.e., it is not an eigenwave of the electron beam, and therefore $D_{2,m} \neq 0$.

ANALYSIS

Let us analyze the generated pump electric field generated by the electron beam. The dynamic of this field is described by the equation (14). If we consider only the terms of this equation that are linear in amplitude, we can obtain an analytical expression for the first harmonic of the generated pump electric field.

$$E_{2,1}^g = C_0E_{20} / D_{2,1}$$

$$\text{or} \quad E_{2,1}^g = E_{20} / \left(\left(k_2^2\upsilon_0^2\gamma_0^3 / \omega_p^2\right) - 1\right). \tag{17}$$

Certain conditions are imposed on the wave number k_2. To satisfy the conditions of three-wave parametric resonance (4), k_2, according to (6), must be equal to:

$$k_2 = 2\pi / \Lambda = 2 / (\gamma_0^{3/2}\upsilon_0 / \omega_p). \tag{18}$$

Substituting (18) into (17), we find that the amplitude of the first harmonic of the generated electric field in the amplification section $E_{2,1}^{g} = E_{20}/3$ is a third of the external pump electric field and amplifies it! It leads to a significant increase in the growth increment of the SCWs.

Qualitatively, this effect can be explained as follows. An external longitudinal periodically reversible electric field E_{20} is created by a system of electrodes, as shown in Fig. 2,a. The potential energy of the beam electrons has the form shown in Fig. 2,b. In this case, the electrons have the maximum potential energy near the negative electrodes (see points z_1, z_3 in Fig. 2,b) and the minimum potential energy near the positive electrodes (see points z_2, z_4 in Fig. 2,b). From the law of conservation of energy, the minimum kinetic energy of electrons is at points where the potential energy is maximum, namely, in the regions near the negative potential electrodes. Accordingly, the maximum kinetic energy of electrons is in the areas near the positive potential electrodes. Considering the continuity of the electron beam, we can state that in the regions of the negative potential, where the velocity of electrons decreases, their concentration increases. Accordingly, in the positive potential area, the electron's speed increases, and electron concentration decreases. Such a change in the electron concentration leads to the appearance of a generated pump electric field E_2^g, which increases the resulting pump electric field. As follows from the above reasoning, the generated pump electric field is increased:

• if the external pump electric field E_{20} is increased (it determines the potential energy level W_p);

• if the average concentration of the electron beam $\omega_p^2 \sim n_0$ is increased (the more electrons accumulate near the negative electrode);

• if the constant component of the electron beam velocity is decreased (because the relative deceleration determines the additional concentration of electrons).

The same follows from relation (18). In the case of transit motion $k_2^2 \upsilon_0^2 \gamma_0^3 / \omega_p^2 >> 1$ and from (18), we get $E_{2,1}^{g} \sim E_{20}\omega_p^2 / \left(\upsilon_0^2 \gamma_0^3\right)$.

Let us analyze the generated pump electric field dynamics in the framework of the cubic nonlinear approximation using the system of differential equations (12)-(16).

We consider the amplification section with the following parameters. The plasma frequency of the electron beam is equal to $\omega_p = 3.0 \cdot 10^{11}$ s^{-1}, and the Lorentz factor is $\gamma = 3.5$. According to (6), the undulation period of the periodically reversible pump field equals $\Lambda = 2.0$ cm. Also, we consider the amplification of the slow space charge wave with ten harmonics, the frequency of the first of which is f_1=0.1 THz. As was found out earlier [6], the system under study is characterized by the same growth increments for different SCW harmonics. Therefore, we choose the same initial values of the amplitudes of all ten harmonics of the slow SCW. We take the initial amplitude values of the fast SCW harmonics to be equal to zero.

Fig. 3 shows the dependence of the first harmonic amplitude of the generated pump electric field strength normalized to the amplitude of the external pump electric field strength on the coordinate z of the amplification section. We see that in the interaction region, the amplitude of the generated electric pump field varies from 33% for the external pump field in the initial amplification region to 21% in the saturation region, which reduces the saturation length by about 22%. The decrease in the first harmonic amplitude of the generated pump electric field strength by 12% in the saturation region is due to the nonlinear terms in (14). Thus, the generated electric field of the pump is essential in the whole area of interaction waves.

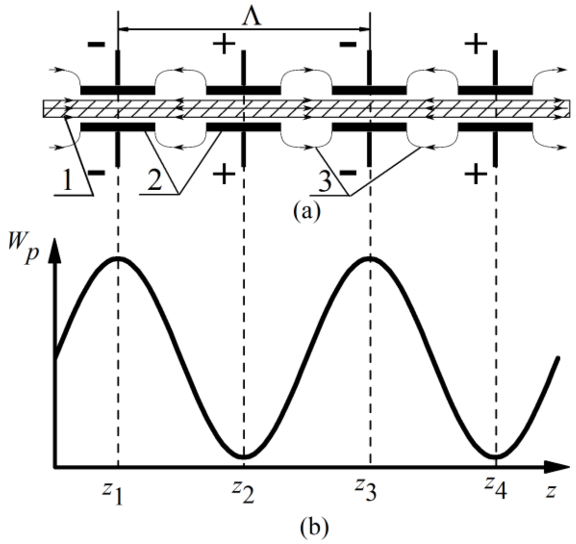

Fig. 2. Scheme of the amplification section for the SCWs: 1 – relativistic electron beam; 2 – electrostatic undulator's electrodes; 3 – electric lines of force of the external periodic reversible pump electric field (a); dependence of the potential energy of beam's electrons W_p in the external pump electric field E_{20} on the longitudinal coordinate z (b)

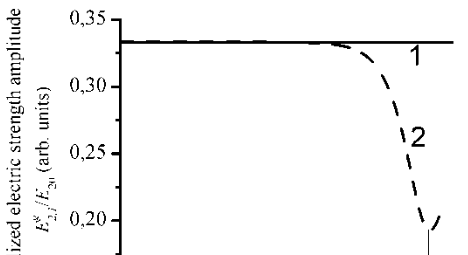

Fig. 3. Dependence of the first harmonic amplitude of the generated pump electric field strength, normalized to the external pump electric field strength amplitude, on the longitudinal coordinate z. Line 1 is built based on the linear approximation equation (14), and line 2 is made based on the nonlinear equation (14)

It should be noted that the higher harmonics of the generated pump electric field participate in plural three-wave parametric resonance interactions (10). Due to these interactions, they are generated by the system under study. We can draw some conclusions from analyzing the dynamics of the higher harmonics of the pump electric field. Firstly, their amplitude rapidly decreases. For example, the amplitude of the second harmonic is ~1% of the amplitude of the first, the amplitude of the third harmonic is 10^{-2} % of the first, and so on. Secondly, due to the parametric resonance conditions, the higher harmonics of the generated pump electric field can interact only with pump electric field harmonics. They do not interact with the harmonics of the space charge wave. Thus, we can state that the higher harmonics of the generated pump electric field have little effect on the amplification of the SCW.

Equations (12)-(15) describe the amplification processes of multi-harmonic SCWs and the generation of pump electric field by electron beam, considering wave harmonics with cubic amplitude. Nevertheless, we can analyze the system's behavior in the small signal mode [6]. Let us present its solutions in the form $E_{\alpha,m}, E_{\beta,m} \sim \exp(\Gamma z)$ and find the growth increment Γ of the *m*th harmonics of the fast and slow SCWs

$$\Gamma = \frac{3\left| e E_{2,1} \right|}{4 m_e \gamma_0 \upsilon_0^2} . \qquad (19)$$

As follows from (19), the growth increment of the *m*th harmonics of the SCWs ***does not depend*** on the harmonic number *m*. This means that at the initial amplification stage, there is an exponential growth of wave amplitudes due to parametric instability, and the growth increment of all harmonics of the SCWs is the same. That is, the amplification of the multi-harmonic slow and fast SCWs occurs, and their amplitude spectrum is not changed. Moreover, to implement amplification of the multi-harmonic SCWs, it is sufficient to use only a monochromatic pump field. Suppose a multi-harmonic SCW is fed to the input of the amplification section, the harmonics of which have the same amplitude values at the input. In that case, the dynamics of the amplitudes of such harmonics at the initial stage will be the same. As a result, on the graph of the dependence of the harmonic amplitudes on the section length, different harmonic lines will be superimposed for one.

Let us analyze the dynamics of waves in the acceleration section, considering the changing generated electric pump field in the framework of the cubic nonlinear approximation using the system of differential equations (12)-(16). Fig. 4 presents the dependencies of the harmonic amplitudes of the slow SCW's electric field strength on the coordinate z. We consider two cases: 1) without the effect of generating a generated pumping electric field (lines 1 in Fig. 4) and with it (see lines 2 in Fig. 4).

As shown in Fig. 4, in the initial section with a length of approximately 80% of the saturation coordinate z_{st}, the lines of dependence of harmonic amplitudes on the coordinate are superimposed on each other. This is true for both cases with (2) and without (1) the effect of generating a generated pump electric field. Thus, we confirm the assumption that at the initial amplification stage of any multi-harmonic SCW, its amplitude spectrum is not changed within the framework of cubic nonlinear analysis.

As follows from Fig. 4, the generated pump electric field leads to an increase in the growth increment of the SCWs and, therefore, to a decrease in the saturation length z_{st} (by about 22.5% in our case). Thus, generating the generated electric pump field can essentially reduce the amplification section length.

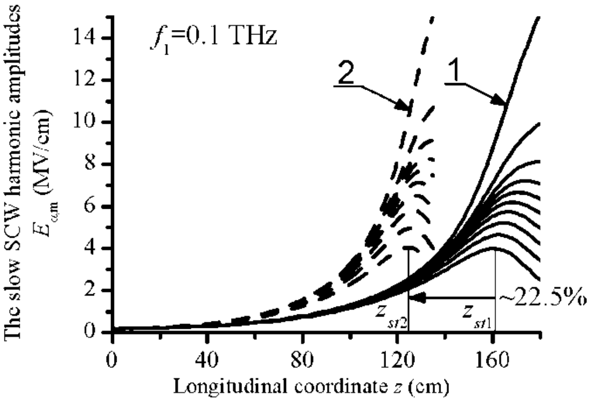

Fig. 4. Dependences of the harmonic amplitudes of the slow SCW's electric field strength on the coordinate z. Lines 1 shows this dependence without the effect of generating a generated pumping electric field, and lines 2 – with it. Each group has ten harmonics with the first frequency of 0.1 THz

Thus, we state that generating the generated electric pump field can essentially reduce the length of the amplification section.

Let us analyze the SCW dynamics for the case of higher wave frequencies compared to the case shown in Fig. 4. Fig. 5 presents the dependencies of the harmonic amplitudes of the slow SCW's electric field strength on the coordinate z. The SCW has ten harmonics with the first harmonic frequency of 0.32 THz. Let's compare the wavelength ranges in Figs. 4 and 5. We can notice that the first harmonic of this range corresponds to the fourth harmonic of the SCW in Fig. 4. The second harmonic of this range ($\lambda_{31,2} = 0.38$ mm) is close to the last harmonic of the SCW analyzed in Fig. 4. The effect of generating a generated pumping electric field in Fig. 5 is considered.

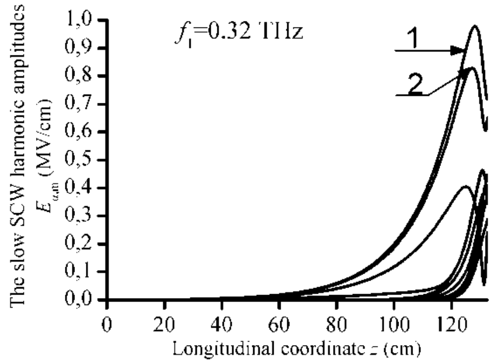

Fig. 5. Dependence of the harmonic amplitudes of the slow SCW's electric field strength on the coordinate z. The SCW has ten harmonics with the first harmonic frequency of 0.32 THz. Line 1 corresponds to the first harmonic, and line 2 corresponds to the second one

The dependence of the electric field strength harmonic amplitudes of the slow SCW on the coordinate is shown in Fig. 5. Comparing Figs. 5 and 4, we see that the first and second harmonics from Fig. 5 have the same growth increment as the 4th and 10th harmonics in Fig. 4, respectively. The other harmonics (3rd, 4th,..., 10th) have much smaller growth increments. At the same time, the value of these increments decreases with an increase in the harmonic number. The saturation levels of the first two harmonics are approximately two times higher than the saturation levels of the 3rd-10th harmonics. Too the saturation length is increased. Also, please note that the region where different harmonics of the same SCW have the same growth increment is much shorter than in the earlier example (see Fig. 4).

We can draw two important conclusions from the latter case. Firstly, amplifying the multi-harmonic SCW without distortion of its amplitude spectrum occurs only for specific system parameters. Secondly, this system can amplify the SCWs with a sufficiently high frequency in the submillimetre or even far-infrared range.

CONCLUSIONS

We analyze the processes of generating the generated electric pump field and its effect on the amplification of space charge waves in the amplification section of a parametric superheterodyne free-electron laser in the cubic nonlinear approximation.

We found out that the electron beam creates a strong generated electric pump field, the amplitude of which in the whole interaction area of the amplification section changes from 33 to 21% of the external electric pump field amplitude. The generated pump electrical field is in phase with the external one. This leads to an essential increase in the growth increments of the SCWs' harmonics, and as a result, the length of the amplification system is reduced by 22%.

We found out that the pump's generated electric field does not destroy the mode of amplification of multi-harmonic space charge waves. This is explained by the fact that to amplify multi-harmonic SCW waves with different frequencies, it is sufficient to have a monochromatic pump field, which increases due to the generated electric pump field. Thus, a generated electric pump field makes it possible to create FELs with smaller longitudinal dimensions and powerful electromagnetic waves with a complex multi-harmonic spectrum.

It is shown that the system under study can amplify the SCW of a sufficiently high frequency in the submillimetre and even far infrared ranges.

Thus, as a part of the cubic nonlinear analysis, we showed that the effect of generating the generated electric pump field significantly improves the amplification section characteristics. This allows us essentially to reduce the saturation lengths and amplification section, hence the dimensions of the entire device.

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Article received 08.10.2023

ВПЛИВ ГЕНЕРОВАНОГО ЕЛЕКТРИЧНОГО ПОЛЯ НАКАЧКИ НА МУЛЬТИГАРМОНІЧНУ ВЗАЄМОДІЮ ХВИЛЬ У СЕКЦІЇ ПІДСИЛЕННЯ СУПЕРГЕТЕРОДИННОГО ЛВЕ

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У кубічному нелінійному наближенні проаналізовано процеси генерації електронним пучком електричного поля накачки та його вплив на підсилення хвиль просторового заряду (ХПЗ) у секції підсилення параметричного супергетеродинного лазера на вільних електронах (ЛВЕ). Встановлено, що амплітуда генерованого електричного поля накачки знаходиться у межах 21…33% від амплітуди зовнішнього електричного поля накачки. Показано, що генероване електричне поле накачки знаходиться у фазі із зовнішнім полем накачки. Це призводить до збільшення інкремента зростання ХПЗ, а отже довжина секції підсилення ХПЗ зменшується на 22%. Виявлено, що таке поле не руйнує режим підсилення мультигармонічних ХПЗ. Досліджуваний ефект дає змогу створювати ЛВЕ з меншими поздовжніми розмірами та потужними електромагнітними хвилями зі складним мультигармонічним спектром.