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ANALYTICAL AND NUMERICAL SOLUTIONS OF THE PARABOLIC EQUATION ISHIMARU FOR THE ELECTROMAGNETIC FIELD, DESCRIBING THE TEMPORARY PROPERTIES OF THE OUTPUT VIDEO-PULSE HETEROGENEOUS NON-DISSIPATIVE MEDIUM

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The parabolic Ishimaru equation for the electromagnetic field coherence function, which describes the temporal properties of pulse at the output of inhomogeneous non-dissipative medium, is considered. The generalization of the approach used in the Ishimaru model to describe the temporal evolution of the envelope of the monochromatic electromagnetic video-pulse in homogeneous non-dissipative media is generalized to the case of non-homogeneous non-dissipative media. An attempt was made to take into account the influence of the inhomogeneity of the medium on the shape of the resulting pulse. An explicit expression for the Green's function of the problem is obtained and number of numerical experiments are carried out. The paper studies the invariant longitudinal (temporal) properties of the envelope of monochromatic electromagnetic pulses recorded after passing through flat layer of the scattering inhomogeneous medium, i.e. properties that remain unchanged when the environment parameters vary. Examples of numerical experiments are given.

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INTRODUCTION

The problem of the shape of a pulse propagating in an inhomogeneous and nonabsorbing medium of a scattering type will be posed and considered. In this work, an electromagnetic pulse in the form of δ-function with respect to the direction of propagation was used as the initial one. Thus, we are talking, in essence, about the Green's function of the problem under consideration. The pulses pass through layer of thickness L containing scattering centers with an arbitrary concentration profile $\rho(z)$ along the propagation axis z, while the scattering itself is considered to be small-angle [1, 2]. The following assumptions will be made in the review:

– the initial (starting wave) is flat;

– the characteristics of the environment, being variable in space, are unchanged in time; these characteristics will be assumed to be known;

– spatial characteristics of the medium are azimuthally symmetric with respect to the direction of radiation propagation;

– the considered radiation is monochromatic wave with an impulse envelope.

The third assumption means that the characteristics of the medium are changed by layers parallel to the front of the original plane wave. Although it is possible to consider the more general case, we restrict ourselves to this assumption below.

The paper considers the temporal properties of the envelope of monochromatic electromagnetic pulses recorded after passing through the flat layer of inhomogeneous scattering medium. In this case, special attention is paid to such properties that can be called invariant, i.e. on properties characteristic of all considered media. Thanks to the information about the invariant properties, it becomes possible to additionally check the conformity of the experimental data and their interpretation, which is not related to the specific parameters of the distribution environment. Such properties of the transfer function as positive definiteness, vanishing at the beginning of the pulse and on its periphery, the uniqueness of the maximum and the presence of only two inflection points, are qualitative features and are easily identified.

The problem of the Green's function of medium containing scattering centers can be classified as classical. Various physical and computational aspects of this problem have long been discussed [1 - 7]. The equations that describe the propagation of impulses are very complex, so obtaining sufficiently complete analytical solutions to the problem posed is a matter of the future. As a rule, publications give exact expressions for the first statistical moments of the pulse envelope or some expressions that are valid within the chosen approximations. The available exact analytical expressions are presented mainly in the form of path integrals and/or expansions into infinite series. In those works where the analytical solution of the problem is given in explicit form, it, as a rule, refers to simplified models.

The purpose of this work is to study the invariant temporal properties of the envelope of monochromatic electromagnetic pulses recorded after passing through flat layer of scattering inhomogeneous medium, i.e. properties that remain unchanged when the parameters of the medium vary. Particular attention is paid to the longitudinal (temporal) dimensions of the resulting pulse, which determine its resolving properties. Various numerical methods are often used to solve the initial equations.

STARTING POSITIONS

Below, the problem of the shape of pulse that propagates in inhomogeneous and nonabsorbing medium of scattering type will be posed and considered. Let us discuss the above assumptions and the restrictions arising from them on the range of applicability of the resulting expressions. In the linear approximation, the initial $I_{in}(t)$ and the pulse at the exit from the medium $I(t)$

are related by expression [1 - 3] and can be determined from $I_{in}(t)$ the form $I(t)$, if the Green's function $G(t-t')$ is defined:

$$I(t)=\int G(t-t')I_{in}(t')dt' . \qquad (1)$$

Let us proceed to the formulation of the initial equations. The desired Green's function is the Fourier transform of the two-frequency coherence function Γ:

$$G(t)=\frac{1}{2\pi}\int \Gamma(\omega_d)\exp(-i\omega_d t)d\omega_d . \qquad (2)$$

In turn, for the function in the considered physical situation of propagation of the plane electromagnetic wave in the diffusion-scattering non-absorbing medium, we have [1, 2]

$$\left(\frac{\partial}{\partial z}+ia\frac{\partial^2}{\partial \mathbf{r}^2}+b(z)\mathbf{r}^2\right)\Gamma(z,\mathbf{r};\omega_d)=0. \qquad (3)$$

Here $a=k_d/2k^2$; k is the wave number of propagating monochromatic wave; $k_d=\omega_d/c$; c is the speed of light;

$$b(z)=(4\alpha_p)^{-1}\rho(z)\sigma_s(z)k^2 , \qquad (4)$$

where $\rho(z)$ is the concentration of scattering centers; $\sigma_s(z)$ is the scattering cross section; α_p is the scattering angular parameter; the longitudinal propagation coordinate z and the transverse vector $\mathbf{r}$ are given as arguments of the coherence function Γ.

Eq. (3) is the partial differential equation of parabolic type with potential

$$U(z,\mathbf{r})=b(z)\mathbf{r}^2 . \qquad (5)$$

Its steepness depends on the coordinate z, and the initial condition $\Gamma(z=0,\mathbf{r})=1$, which provides the Green's function property $G(t-t')\big|_{z=0}=\delta(t-t')$.

Eq. (3) belongs to the class of Hill's equations and, in the general case, its analytical solutions for a potential $U(z,\mathbf{r})$ of an arbitrary form cannot be obtained. Therefore, it turns out to be essential that for the potential of the quadratic form (5) it is possible to construct an approximation solution for the Green's function $G(t)$.

EQUATIONS FOR THE COHERENCE FUNCTION

Consider a scattering layer whose thickness (along the axis z) is equal to L. Let's split the length L into N segments $\{\Delta_n\}$: $L=\sum_{n=1}^{N}\Delta_n$. Let the quantities $\rho(z)$ and $\sigma_s(z)$ be given. Let us choose segments $\Delta_n=z_n-z_{n-1}$ and denote $b_n=b(z_n)$, $\rho_n=\rho(z_n)$ and $\sigma_n=\sigma_s(z_n)$. Keeping in mind the case $N>>1$, let us replace in the potential $U(z,\mathbf{r})$ all the values lying inside each n^{th} segment by the value $U(z_n,\mathbf{r})$, where $z_n=\sum_{m=1}^{n}\Delta_m$ is the right boundary of the segment.

The potential $U_N(z,\mathbf{r})$ thus obtained will be used below in the approximation Eq. (7). Now let us approximate the solution of Eq. (3) by the function $\Gamma_N(z,\mathbf{r};\omega_d)$, which is the solution of the equation with piecewise-constant expression $U_N(z,\mathbf{r})$ for the potential $U(z,\mathbf{r})$

$$\left(\frac{\partial}{\partial z}+ia\frac{\partial^2}{\partial \mathbf{r}^2}+U_N(z,\mathbf{r})\right)\Gamma_N(z,\mathbf{r};\omega_d)=0, \qquad (6)$$

with condition $\Gamma_N(0,\mathbf{r};\omega_d)=1$. If a solution to Eq. (6) is found for Γ_N, then the desired coherence function will follow from it in the limit at $N\to\infty$. Eq. (6) is equivalent to the sequence of equations ($n=1,...,N$)

$$\left(\frac{\partial}{\partial z}+ia\frac{\partial^2}{\partial \mathbf{r}^2}+U_n(z_n,\mathbf{r})\right)\Gamma_n(z,\mathbf{r};\omega_d)=0, \qquad (7)$$

whose solutions are determined by the initial condition $\Gamma_N(z=0,\mathbf{r};\omega_d)=1$ and the chain of boundary conditions $y_{n+1}(z_n)=y_n(z_n)$, where $y_n(z)$ is the function $\Gamma_N(z,\mathbf{r};\omega_d)$ on the n^{th} segment.

Let us turn to the solution of the expanded system of Eq. (7). For this purpose, consider an arbitrary n^{th} segment. In this section, the potential U_n depends on coordinate z_n as parameter $U_N(z,\mathbf{r})=b(z)\mathbf{r}^2\equiv b_n\mathbf{r}^2$, $b_n=(4\alpha_p)^{-1}\rho(z_n)\sigma_s(z_n)k^2$. Thus, each of Eq. (7) is the parabolic partial differential equation with piecewise constant potential. We will look for the solution of the n^{th} equation on the interval (z_{n-1},z_n) in the form

$$\Gamma_n(z,\mathbf{r};\omega_d)=\frac{1}{f(z)}\exp\left[g(z)\mathbf{r}^2\right]. \qquad (8)$$

This form of the desired solution is due to the parabolicity of the potential in $\mathbf{r}$. From (7) and (8) the following equations follow for the introduced functions $f(z)$ and $g(z)$:

$$-\frac{1}{f(z)}\frac{df(z)}{dz}+4iag(z)=0 , \qquad (9a)$$

$$\frac{dg(z)}{dz}+4iag^2(z)+b_n=0 . \qquad (9b)$$

Consider the second equation from system (9). Let $g_n=g(z_n)$, then for all n the solution of this equation with the initial condition $g(z_{n-1})=g_{n-1}$, at its right end has the form

$$g_n=\frac{g_{n-1}-(4ia/b_n)^{-1/2}\,\mathrm{tg}(\sqrt{4iab_n}\Delta_n)}{1+(4ia/b_n)^{1/2}g_{n-1}\,\mathrm{tg}(\sqrt{4iab_n}\Delta_n)} . \qquad (10)$$

The function $f(z)$ is found from (8) with the initial condition $f(z_{n-1})=f_{n-1}$, where f_{n-1} is the value of the function $f(z)$ at the right end of the previous $(n-1)^{\text{th}}$ section, and at its right end it has the form

$$f_n=\left[\cos\left(\sqrt{4iab_n}\Delta_n\right)+\sqrt{4ia/b_n}\,g_{n-1}\sin\left(\sqrt{4iab_n}\Delta_n\right)\right]f_{n-1}. \qquad (11)$$

So, the coherence function $\Gamma_N(z,\mathbf{r};\omega_d)$ for $z=z_N=L$ is equal to

$$\Gamma_N(z_N,\mathbf{r};\omega_d)=\frac{1}{f_N}\exp(g_N\mathbf{r}^2) , \qquad (12)$$

in this case, the sequences $\{f_n\}$ and $\{g_n\}$, $1<n\le N$, are determined from recurrent relations (10) and (11), as well as initial conditions $f_0(0)=1$ and $g_0(0)=0$. In particular, at the reception point, when $\mathbf{r}=0$ and $z=L$, we have

$$\Gamma_N(L,\mathbf{r};\omega_d)=\frac{1}{f_N(z_N)} . \qquad (13)$$

GREEN'S FUNCTION

Let $T_L = L/c$ the propagation time of the initial point of the pulse. Then the impulse response function (Green's function) is as follows

$$G_N(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\exp\left(-i\omega_d\left(t-T_L\right)\right)}{f_N\left(\omega,\omega_d\right)}d\omega_d \ , \qquad (14)$$

in this case, the dependence of the function $f_N = f_N(\omega,\omega_d)$ both on the difference frequency (integration variable) ω_d and on the frequency ω of the monochromatic pulse is indicated.

From the recurrence relation (11) it follows

$$f_N = \prod_{n=1}^{N-1}\left[\cos\left(\sqrt{4iab_n}\Delta_n\right)+\sqrt{4ia/b_n}\,g_{n-1}\sin\left(\sqrt{4iab_n}\Delta_n\right)\right]. \qquad (15)$$

Assume that the function $b(z)$ is smooth. Then from (14) it follows $\varphi_{n+1} = \varphi_n - \sqrt{4ia/b_n}\Delta_n$, where $\mathrm{tg}(\varphi_n) = \sqrt{4ia/b_n}\,g_n$, which leads to the equality, $\phi_N = -\sum_{n=1}^{N}\sqrt{4iab_n}\Delta_n$, therefore

$$g_N = -\frac{1}{\sqrt{4ia/b_N}}\,\mathrm{tg}\left(\sum_{j=1}^{N}\sqrt{4iab_j}\Delta_j\right). \qquad (16)$$

Substituting this expression into relation (16), we successively find after simplifications

$$f_N = \cos\left(\sum_{n=1}^{N}\sqrt{4iab_n}\Delta_n\right). \qquad (17)$$

Since $ab_n = (\rho_n\sigma_n\omega_d)(8\alpha_p c)^{-1}$, where $\rho_n = \rho(z_n)$ and $\sigma_n = \sigma_s(z_n)$, then for the desired Green's function we obtain

$$G(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\exp\left(-i\omega_d(t-T_L)\right)}{\cos\left(\sqrt{i\omega_d}\sum_{n=1}^{N}\Delta_n\sqrt{(\rho_n\sigma_n)/(2\alpha_p c)}\right)}d\omega_d \ . \qquad (18)$$

In the expression $\tau^{1/2} = \sum_{n=1}^{N}\Delta_n\sqrt{(\rho_n\sigma_n)/(2\alpha_p c)}$, the sum in the limit $N\to\infty$ goes into the integral $\tau^{1/2} = (2\alpha_p c)^{-1/2}\int_0^L\sqrt{\rho(z)\sigma_s(z)}\,dz$, which gives $G(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\exp\left(-i\omega_d(t-T_L)\right)}{\cos\sqrt{i\omega_d\tau}}d\omega_d$. Using the replacement $\omega_d = is$, we finally obtain the desired expression for the Green's function, which formally coincides with the well-known solution of the Ishimaru equation [1] for homogeneous scattering media

$$G(t) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{\exp\left[s(t-T_L)\right]}{\mathrm{ch}\sqrt{s\tau}}ds\,. \qquad (19)$$

This expression depends on two parameters T_L and τ. The first of them is responsible for the arrival time of the pulse as whole and is equal to the ratio of the path length L to the speed of light c. The second describes the pulse broadening. Let us give one more representation for the Green's function

$$G(t) = \frac{1}{2\pi i\tau}\int_{-i\infty}^{i\infty}\frac{\exp\left[\eta(t-T_L)/\tau\right]}{\mathrm{ch}\sqrt{\eta}}d\eta\,, \qquad (20a)$$

$$T_L = L/c\,,\quad \tau = \frac{1}{2\alpha_p c}\left(\int_0^L\sqrt{\rho(z)\sigma(z)}dz\right)^2. \qquad (20b)$$

The obtained formulas (19), (20*a*), (20*b*), differ from the well-known result of Ishimaru [1] in that they are valid for inhomogeneous media.

PHYSICAL INTERPRETATION

An impulse with a given time dependence $I_{in}(t)$ will correspond to impulse at the exit from the medium, described by the expression

$$I(t) = \frac{1}{2\pi i}\int_{-\infty}^{\infty}I_{in}(t')\,dt'\int_{-i\infty}^{i\infty}\frac{\exp\left[s(t-t'-T_L)\right]}{\mathrm{ch}\sqrt{s\tau}}ds\,. \qquad (21)$$

To construct the asymptotic of the form $I(t)$, we consider Eq. (19) in which we set $I_{in}(t') = \delta(t')$, that is, consider the Green's function

$$I(t,L) = \frac{1}{\pi i}\int_C\exp\left(\lambda t-\sqrt{\lambda\tau}\right)\xi(\lambda)\,d\lambda\,, \qquad (22)$$

where $\xi(\lambda) = \left(1+\exp(-2\sqrt{\lambda\tau})\right)^{-1}$, that is, further, the Green's function will be counted from the moment of departure from the scattering layer. The main contribution to the integral (22) is due to the exponential kernel $\exp\left(\lambda t-\sqrt{\lambda\tau}\right)$. Let the relation hold for some variable $\sqrt{\lambda\tau} = \eta$. We use the variable η as the new integration variable and form the perfect square in the exponent

$$I(t,L) = \frac{2L^2}{\tau\pi i}\int_C\exp\left(L^2\eta^2\frac{t}{\tau}-L\eta\right)\xi'(\eta)\eta d\eta\,, \qquad (23)$$

where $\xi'(\eta)$ is the value of the derivative of the function $\xi'(\lambda)$ in new variables. In the framework of the approximate integration used, its estimate is $\xi'(\eta)\approx 1$, so we find

$$I(t,L) = \frac{2}{\tau\pi i}\int_C\exp\left(z^2\frac{t}{\tau}-z\right)zdz\,, \qquad (24)$$

then, after integration, we approximately obtain the asymptotic expression for the Green's function:

$$I(t,L) = \sqrt{\frac{\tau}{4\pi t^3}}\exp\left(-\frac{\tau}{4t}\right), \qquad (25)$$

which can also be used in the special case of narrow scattering layer. The found form reflects the integral effect of the trajectory formation in the process of multiple walks in the scattering layer of thickness *L*.

Expression (25) is normalized to 1, that is, it can be considered as distribution density, but it does not have statistical moments. To obtain them, we assume that the starting impulse reached the plane $z=0$ at the time $t=0$. Propagating in a scattering medium, it will leave the plane $z=L$, after which it will have the shape $I(t)$. Let us analyze the parameters of this pulse. We determine the values of its first three moments $\left\langle t^n\right\rangle_L = \int_0^\infty t^n I(t,L)dt$, $n=0,1,2$. In this case, we assume, without loss of generality, that the starting pulse has stored energy numerically equal to unity. The moments of interest to us will be counted from $T_L = L/c$:

$$\left\langle (t-T_L)^n\right\rangle_L = (-1)^n\frac{d^n}{ds^n}\frac{1}{\mathrm{ch}\sqrt{s\tau}}\bigg|_{s=0}. \qquad (26)$$

Taking into account the assumption about the unit energy of the starting pulse at $n=0$ for the zero moment, we find $\langle 1\rangle_L = 1$, which corresponds to the accepted model of radiation propagation without absorption. Next, from (21) we obtain

$$\langle t - T_L\rangle_L = \frac{1}{2}\tau + \langle t'\rangle_0, \qquad (27)$$

whence it follows that the moment of its arrival, averaged over the impulse, is equal $T_L + \tau/2$, taking into account the first moment $\langle t'\rangle_0$ of the starting impulse. Finally, at $n=2$, we find for the average pulse duration $\sqrt{D_L}$

$$D_L = \langle (t-T_L)^2\rangle_L = \frac{1}{6}\tau^2 + D_0, \qquad (28)$$

where $\sqrt{D_0}$ is the average duration of the initial pulse. Thus, the quantity $\tau^2/6$ is added to the dispersion D_0.

The pulse duration reflects variations in the concentration of the scattering medium, and if the initial pulse is the δ-function, $I_{in}(t') = \delta(t')$, then as the result of propagation in the scattering medium, its average duration will be

$$\sqrt{D_L} = \tau/\sqrt{6} = \frac{1}{\sqrt{24}\alpha_p c}\left(\int_0^L \sqrt{\rho(z)\sigma_s(z)}\,dz\right)^2. \qquad (29)$$

In the particular case of the medium homogeneous along the axis z, when $\rho(z)$ and $\sigma(z)$ are constant, we obtain $\sqrt{D_L} = (\rho\sigma_s L^2)(\sqrt{24}\alpha_p c)^{-1}$, i.e. here, the average pulse duration grows in proportion to the square of the flight distance L [1].

In view of the significant variety of options for the parameters of the scattering medium, we will carry out numerical simulation using relative values. As the working formula, we use the following

$$I(t,L) = \frac{1}{2\pi}\int_{-\infty}^{\infty} I_{in}(t')\,dt' \int_{-\infty}^{\infty} \frac{\exp[iu(t-t')]}{\mathrm{ch}\sqrt{iu\tau}}\,du, \qquad (30)$$

where L is the length of the path traveled by the front of the probing pulse to the moment t, and the pulse itself will be of the Gaussian form with dispersion δ:

$$I_{in}(t') = \frac{1}{\sqrt{2\pi\delta}}\exp\left(-\frac{t'^2}{2\delta^2}\right). \qquad (31)$$

As the effective variance δ, we choose

$$\delta = \left(\int_0^L N(z)\,dz\right)^2, \qquad (32)$$

where, in accordance with (20*b*), *N(z)* is the effective density of medium scattering centers.

To construct numerical examples, we choose the Gaussian distribution (31), which has a dispersion δ =0.075, as the shape of the probing pulse.

On Figs. 1-5 show examples of the formation of 5 scattered wave plumes at selected distances in the absence of scattering centers and in the case of their presence in the interval.

In the first case (Fig. 1), there is no scattering medium and the starting pulse propagates without changes as whole.

In the second case, various types of scattering layers are considered. On Figs. 2-5 shows the dynamics of the formation of temporary trails of the scattered wave, in which the tail is located in the peripheral time region *t*. The tail formed behind the wave front is the longer, the greater the effective dispersion τ, which in turn is determined by the effective density of medium scattering centers *N(z)*.

From the family of curves *I(t,L)* shown in Figs. 1-5, it can be seen that they have the Laguerre property [7], namely: the function *I(t,L)* is identically equal to zero at *t*=0 (fluctuation region), the function *I(t,L)* has one maximum and two inflection points (main region), the function *I(t,L)* has exponential asymptotic at *t* (peripheral region). Note that formulas (19) and (30) imply the possibility of considering different types of dependences of the concentration and scattering cross section on the current coordinate *z*.

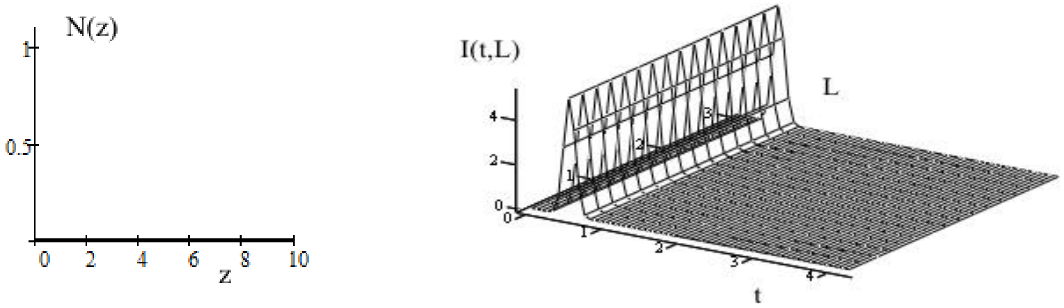

Fig. 1. Scattering zone profile and a family of time pulses I(t,L); N(z)=0

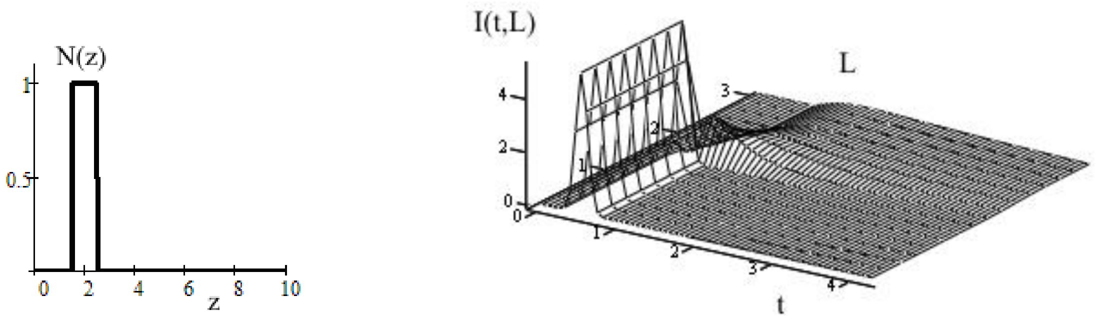

Fig. 2. Scattering zone profile and a family of time pulses I(t,L); N(z)=1.0 on the interval 1.5≤z≤2.5

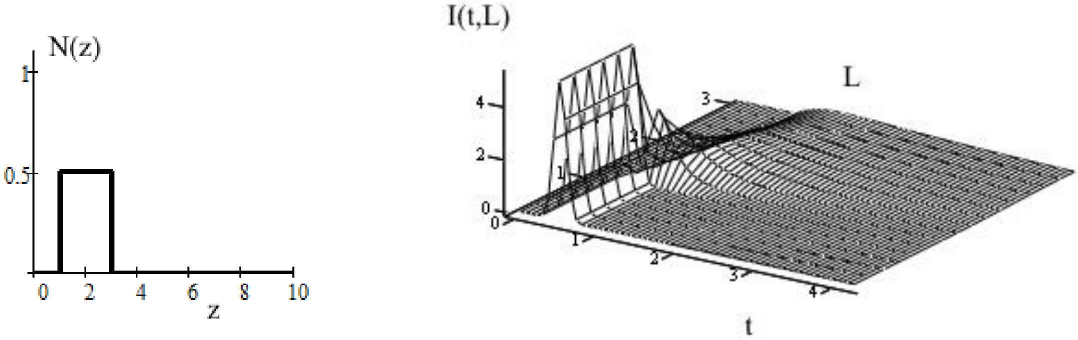

Fig. 3. Scattering zone profile and a family of time pulses I(t,L); N(z)=0.5 on the interval 1≤z≤3

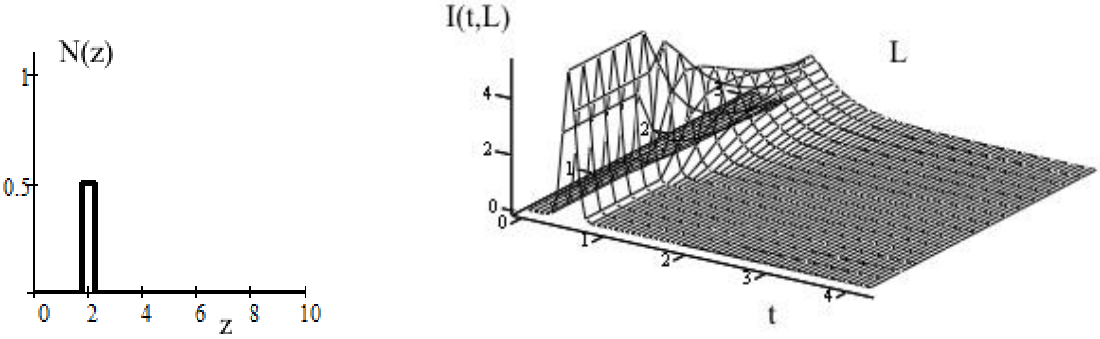

Fig. 4. Scattering zone profile and a family of time pulses I(t,L); N(z)=0.5 on the interval 1.75≤z≤2.25

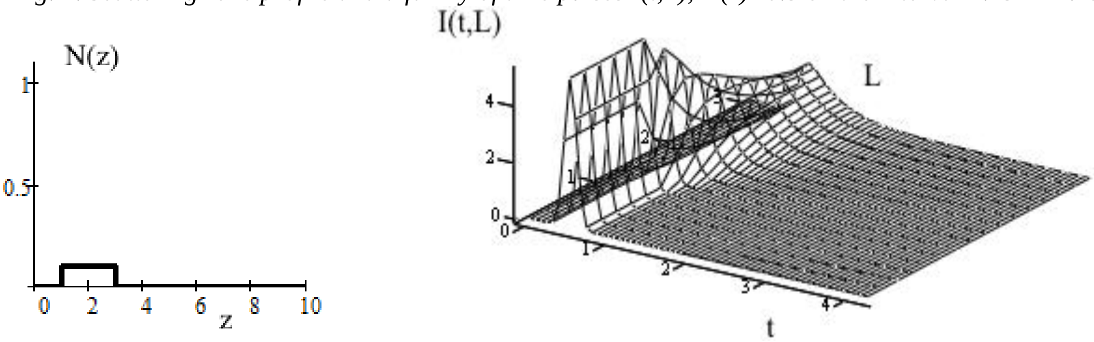

Fig. 5. Scattering zone profile and a family of time pulses I(t,L); N(z)=0.1 on the interval 1≤z≤3

CONCLUSIONS

Note that the proposed approximation approach in studying the processes that affect the temporal delay of electromagnetic pulses can be developed by taking into account the attenuation of radiation during its propagation in an inhomogeneous absorbing medium [2, 3]. In this case, the absorption term in the parabolic equation (3) will serve as the mathematical basis, while the coefficients in the equation may depend on the longitudinal coordinate z . We also note that the analysis of the evolution of the shape of a time pulse during its propagation makes it possible to judge the spatial distribution of the characteristics of the scattering medium along the propagation axis.

During the propagation of electromagnetic radiation, numerous breaks and branches of the trajectories of wave rays occur, which affects the length of their trajectories. The shape of these trajectories is determined by the distribution of scattering centers of the medium, and their length is random. Therefore, the distribution of the video pulse *I(t,L)* is the path integral average in the functional space of trajectories.

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АНАЛІТИЧНІ ТА ЧИСЕЛЬНІ РІШЕННЯ ПАРАБОЛІЧНОГО РІВНЯННЯ ІСІМАРУ ДЛЯ ФУНКЦІЇ КОГЕРЕНТНОСТІ ЕЛЕКТРОМАГНІТНОГО ПОЛЯ, ЩО ОПИСУЮТЬ ЧАСОВІ ВЛАСТИВОСТІ ВІДЕОІМПУЛЬСУ НА ВИХОДІ НЕОДНОРІДНОГО НЕДИСИПАТИВНОГО СЕРЕДОВИЩА

О.С. Мазманішвілі

Розглянуто параболічне рівняння Ісімару для функції когерентності електромагнітного поля, що описують часові властивості імпульсу на виході неоднорідного недисипативного середовища. Отримано узагальнення підходу, використаного в моделі Ісімару для опису часової еволюції монохроматичного електромагнітного відеоімпульсу, що огинає, в однорідних недисипативних середовищах на випадок неоднорідних недисипативних середовищ. Враховано вплив неоднорідності середовища на форму результуючого імпульсу. Отримано явний вираз функції Гріна задачі та проведено низку чисельних експериментів. Вивчено інваріантні часові властивості огинаючої монохроматичних електромагнітних імпульсів, що реєструються після проходження через плоский шар розсіюючого неоднорідного середовища, тобто властивостей, що залишаються незмінними при варіації параметрів середовища. Наведено приклади чисельних експериментів.