# https://doi.org/10.46813/2023-148-058 SLOW ELECTROMAGNETIC WAVES IN PLANAR THREE-COMPONENT WAVEGUIDE STRUCTURE WITH MU-NEGATIVE METAMATERIAL

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This work is devoted to study of the dispersive properties of the slow electromagnetic waves that propagate along the planar waveguide structure that consists of semi-bounded plasma region, metamaterial slab and semi-bounded region of ordinary dielectric. It is studied the case when all media are homogeneous and isotropic. The dispersion properties, the phase and group velocities, as well as the electromagnetic field spatial structure of the eigen TE modes are studied in the frequency range where the metamaterial possess negative permeability.

PACS: 52.35g, 52.50.Dg

#### **INTRODUCTION**

During few decades there are both theoretical and experimental researches of artificially created manmade materials called metamaterials. These metamaterials have combinations of electrodynamics parameters that are do not occur in the nature [1 - 5].

Often this metamaterial are called "left-handed material" for the reason that in the unbounded medium, the three vectors of the electric, magnetic fields, and wave vector of a plane waves form the left triple. This is true for the metamaterials, in which the both permittivity and permeability are negative.

The properties of surface electromagnetic waves in the metamaterials were studied [6 - 13]. Were usually studied intensively the so-called double negative metamaterials, in which the negatives were both permittivity and permeability.

But it's no doubt, that creation of material with only negative permeability is easier that for the double negative ones [14]. In recent papers [15 - 17] were studied surface electromagnetic waves in such mu-negative metamaterials.

The fields of application are very wide ranging from the photovoltaic device development, the sensing and detection, the particles accelerators and many others [18 -20].

In the present work, it has been studied slow electromagnetic waves that propagate in planar waveguide structures involving both mu-negative metamaterial and plasma.

#### 1. TASK SETTING

The considered electromagnetic waves propagate in the planar three-component waveguide structure that consists of semi-bounded plasma-like region, munegative metamaterial slab of thickness d and semibounded region of ordinary dielectric.

The plasma medium is characterized by permittivity  $\varepsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$  here  $\omega_p$  is an effective plasma frequency and permeability  $\mu_1 = 1$ . The metamaterial is character-

ized by permittivity  $\varepsilon = \varepsilon_0$  and permeability that depend on the wave frequency and commonly expressed with the help of experimentally obtained expressions [2]:

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2} \,. \tag{1}$$

Here  $\omega_0$  is the characteristic frequency of metamaterial,  $\omega_0 / 2\pi = 4$  GHz and F = 0.56 [14].

Such parameters ratio leads to existence the frequency region where  $\varepsilon_1(\omega) < 0$  and  $\mu(\omega) < 0$  simultaneously. On other side near metamaterial slab is located the semi-bounded conventional dielectric with constant dielectric permittivity  $\varepsilon_2$  and permittivity  $\mu_2 = 1$ .

Let's consider electromagnetic wave that propagates along this structure. It was assumed that wave disturbance tends to zero far away from metamaterial and the dependence of the wave components on time t and coordinate and z is expressed the following form:

$$E, H \propto E(x), H(x) \exp[i(k_3 z - \omega t)].$$
 (2)

Here x is coordinate rectangular to the wave propagation direction and to the metamaterial slab.

It's possible in this case the system of Maxwell equations to split on two subsystems. One of them described the waves of H-type and another – the waves of E-type.

The wave of *E*-type possesses the dispersion relation in the following form:

$$\varepsilon_0 (h_2 \varepsilon_1 + h_1 \varepsilon_2) \kappa \operatorname{Cosh}[d\kappa] + (h_1 h_2 \varepsilon_0^2 + \varepsilon_1 \varepsilon_2 \kappa^2) \operatorname{Sinh}[d\kappa] = 0.$$
(3)

Here  $h_1 = \sqrt{k_3^2 - \varepsilon_1 k^2}$ ,  $h_2 = \sqrt{k_3^2 - \varepsilon_2 k^2}$ ,  $\kappa = \sqrt{k_3^2 - \varepsilon_0 \mu(\omega) k^2}$ ,  $k = \omega/c$ , were *c* is the speed of light in vacuum.

In the plasma-like region ( $x \le 0$ ) the wave field components, normalized on the  $H_y(0)$ , can be written as:

$$\begin{cases} H_{y}(x) = e^{h_{1}x}, \\ E_{x}(x) = e^{h_{1}x}k_{3}/(\varepsilon_{1}k), \\ E_{z}(x) = i e^{h_{1}x}h_{1}/(\varepsilon_{1}k). \end{cases}$$
(4)

In the region of metamaterial slab  $(0 \le x \le d)$  the wave field components, normalized on the  $H_y(0)$ , can be written as:

$$\begin{cases} H_{y}(x) = C_{1E}e^{\kappa x} + C_{2E}e^{-\kappa x}, \\ E_{x}(x) = k_{3}\left(C_{1E}e^{\kappa x} + C_{2E}e^{-\kappa x}\right)/(k\varepsilon_{0}), \\ E_{z}(x) = i\kappa\left(C_{1E}e^{\kappa x} - C_{2E}e^{-\kappa x}\right)/(k\varepsilon_{0}). \end{cases}$$
(5)

Here  $C_{1E}$  and  $C_{2E}$  – are *E*-wave field constants.

In the dielectric region ( $x \ge d$ ) the wave field components, normalized on the  $H_y(0)$ , possess the form:

$$\begin{cases} H_{y}(x) = B_{\rm E}e^{-h_{2}x}, \\ E_{x}(x) = B_{\rm E}k_{3}e^{-h_{2}x} / (k \varepsilon_{2}), \\ E_{z}(x) = -i B_{\rm E}h_{2}e^{-h_{2}x} / (k \varepsilon_{2}). \end{cases}$$

$$(6)$$

Here  $B_E$  is *E*-wave field constant. Such constants are of the following form:

$$\begin{cases} B_{\rm E} = -2h_{\rm I} \varepsilon_2 \varepsilon_0 e^{(h_2 + \kappa)d} / \Psi_{\rm E}, \\ C_{\rm IE} = h_{\rm I} \varepsilon_0 (h_2 \varepsilon_0 - \varepsilon_2 \kappa) / (\kappa \Psi_{\rm E}), \\ C_{\rm 2E} = -h_{\rm I} \varepsilon_0 (h_2 \varepsilon_0 + \varepsilon_2 \kappa) e^{2\kappa d} / (\kappa \Psi_{\rm E}). \end{cases}$$
(7)

Here  $\Psi_{\rm E} = \varepsilon_1 \Big[ (1 + e^{2\kappa d}) h_2 \varepsilon_0 + (-1 + e^{2\kappa d}) \varepsilon_2 \kappa \Big].$ 

Similarly wave of *H*-type possesses the dispersion relation in the following form:

 $(h_1 + h_2)\kappa\,\mu(\omega)\operatorname{Cosh}[d\kappa] +$  $(\kappa^2 + h_1\,h_2\,\mu^2(\omega))\operatorname{Sinh}[d\kappa] = 0.$ (8)

In the plasma region ( $x \le 0$ ) the wave field components, normalized on the  $E_y(0)$ , can be written as:

$$\begin{cases} E_{y}(x) = e^{h_{1}x}, \\ H_{x}(x) = -e^{h_{1}x}k_{3}/k, \\ H_{z}(x) = -ie^{h_{1}x}h_{1}/k. \end{cases}$$
(9)

In the region of metamaterial slab  $(0 \le x \le d)$  the wave field components, normalized on the  $E_y(0)$ , can be written as:

$$\begin{cases} E_{y}(x) = C_{1H}e^{\kappa x} + C_{2H}e^{-\kappa x}, \\ H_{x}(x) = k_{3}(C_{1H}e^{\kappa x} + C_{2H}e^{-\kappa x})/(k \ \mu(\omega)), \quad (10) \\ H_{z}(x) = -i \kappa (C_{1H}e^{\kappa x} - C_{2H}e^{-\kappa x})/(k \ \mu(\omega)). \end{cases}$$

Here  $C_{1H}$  and  $C_{2H}$  – are *H*-wave field constants.

In the dielectric region ( $x \ge d$ ) the wave field components, normalized on the  $E_y(0)$ , possess the form:

$$\begin{cases} E_{y}(x) = B_{H} e^{-h_{2}x}, \\ H_{x}(x) = k_{3} B_{H} e^{-h_{2}x} / k, \\ H_{z}(x) = i B_{H} h_{2} e^{-h_{2}x} / k. \end{cases}$$
(11)

Here  $B_{\rm H}$  is *H*-wave field constant. Such constants are of the following form:

$$\begin{cases} B_{\rm H} = -2h_{\rm I}\,\mu(\omega)\,\varepsilon_0 e^{(h_2+\kappa)d} / \Psi_{\rm H}, \\ C_{\rm IH} = h_{\rm I}\,\mu(\omega) [h_2\,\mu(\omega) - \kappa] / (\kappa \Psi_{\rm H}), \\ C_{\rm 2H} = -h_{\rm I}\,\mu(\omega) [h_2\,\mu(\omega) + \kappa] e^{2\kappa d} / (\kappa \Psi_{\rm H}). \end{cases}$$
  
Here  $\Psi_{\rm H} = \kappa [(-1+e^{2\kappa d})\kappa + (1+e^{2\kappa d})h_2\,\mu(\omega)].$ 

]

#### 2. MAIN RESULTS

The results of numerical study of dispersion equations for *E*- and *H*-waves are shown at Fig. 1. The following dimensionless parameters were used to characterize the wave and the structure: normalized metamaterial slab thickness  $\tilde{d} = \omega_0 d / c = 2.2$ , normalized plasma frequency of the plasma-like media  $\Omega_p = \omega_p / \omega_0 = 2.5$ ,  $\varepsilon_2 = 1$ , the normalized frequency  $\Omega = \omega / \omega_0$  and the normalized wave number  $\beta = k_3 c / \omega_0$ .

In general case the dispersion equations (3), (8) have 3 solutions, including two waves of *H*-type and one wave *E*-type. Curves marked by the numbers 1, 2 correspond to waves of *H*-type ( $H_1$ -wave and  $H_2$ -wave) and curve marked by the numbers 3 correspond to wave of *E*-type (*E*-wave). The area to the right of the dashed line corresponds to the slow waves of surface type.

Thus, according to the example of a waveguide structure with parameters corresponding to Fig. 1, it can be concluded that the  $H_1$ -wave has the lowest frequency; the dispersion of this wave is straight in the entire range of wave numbers. The  $H_2$ -wave has a higher frequency  $\Omega$  and in the region of small wavenumbers  $\beta$  the wave has a straight dispersion, but for the larger values the dispersion becomes reverse. In the region of the dispersion type changing the group velocity of the  $H_2$ -wave goes to zero. The *E*-wave of the structure possesses the highest frequency and wave numbers, the dispersion of this wave is straight in the entire range of wave numbers. The frequency ranges in which the eigenwaves of the structure exist do not intersect; these regions in Fig. 1 are shown by horizontal dashed lines.



Fig. 1. The dependence of the normalized frequency  $\Omega = \omega / \omega_0$  on the normalized wave number  $\beta = k_3 c / \omega_0$ for normalized metamaterial slab thickness  $\tilde{d} = \omega_0 d / c = 2.2$ , normalized plasma frequency  $\Omega_n = \omega_n / \omega_0 = 2.5$  and  $\varepsilon_2 = 1$ 

In this work, a planar structure with normalized metamaterial thickness  $\tilde{d} = \omega_0 \Delta / c = 2.2$  is investigated. Under such parameters, in the region of not very short waves, it is possible to use the model of a dispersion medium with such magnetic permeability  $\mu(\omega) < 0$  that depends on the frequency according to the law (1).

The Fig. 2 shows the dependence of the dielectric permittivity of the plasma-like medium  $\varepsilon_1(\omega)$  and the magnetic permeability of the metamaterial  $\mu(\omega)$  upon the frequency for the structure with the parameters  $\tilde{d} = \omega_0 \Delta / c = 2.2$ ,  $\Omega_p = 2$ ,  $\varepsilon_2 = 1$ . Vertical dashed lines show the bounds of the frequency intervals in which the eigenwaves of the considered planar wave-guide structure can propagate. In the frequency range, where  $\Omega < \Omega_p$ , it was obtained that  $\varepsilon_1(\Omega) < 0$  for all eigenwaves of the structure. The next necessary condition  $\mu(\Omega) < 0$  is fulfilled for  $H_1$ - and  $H_2$ -eigenwave of the considered structure.



Fig. 2. The dependence of the dielectric permittivity of the plasma-like medium  $\mu(\Omega)$  and the magnetic permeability of the metamaterial  $\mu(\Omega)$ on the normalized frequency  $\Omega$  for the structure

parameters  $\tilde{d} = \omega_0 d / c = 2.2$ ,  $\Omega_p = 2$ ,  $\varepsilon_2 = 1$ 

The paper presents the results of the study of the dispersion, phase and group speeds, spatial wave field structure of two eigenwaves of *H*-type, that propagate in the planar waveguide structure containing a metamaterial with a constant dielectric permittivity  $\varepsilon = \varepsilon_0 = 1$  in the case when  $\mu(\Omega) < 0$ .

Let us firstly presents the results of the study of  $H_1$ -wave properties. Let us determine the influence of the normalized plasma frequency  $\Omega_p = \omega_p / \omega_0$  of the plasma-like medium and the dielectric permittivity  $\varepsilon_2$  value of the medium that borders with metamaterial layer on the wave dispersion, phase and group speeds, and spatial wave field structure when  $\varepsilon_1(\omega) < 0$  and  $\mu(\omega) < 0$ .

The dependence of the normalized frequency  $\Omega$  for the  $H_1$ -wave upon the normalized wave number  $\beta$  for the waveguide structure with parameters  $\Omega_p = 2$  and  $\varepsilon_2 = 1$  is shown in the Fig. 3.



Fig. 3. The dependence of the normalized frequency  $\Omega = \omega / \omega_0$  on normalized wave number  $\beta$  for the thickness of metamaterial slab  $\tilde{d} = 2.2$ ,  $\Omega_p = 2$ ,  $\varepsilon_2 = 1$ 

The study have shown that the increase of the parameter  $\Omega_p$  value from 2 to 10 practically does not influence on the wave dispersion, on the frequency range where the wave exists, and on the phase  $V_{ph}(\Omega)$  and group  $V_{er}(\Omega)$  speeds that are shown in Fig. 4.



Fig. 4. The dependence of the normalized phase velocity  $\tilde{V}_{ph} = V_{ph} / c$  and the normalized group velocity

## $\tilde{V}_{gr} = V_{gr} / c$ of the $H_1$ -wave on the normalized frequency $\Omega$ for the structure parameters

$$d = 2.2$$
,  $\Omega_p = 2$ ,  $\varepsilon_2 = 1$ 

The maximum value of the group speed  $V_{gr}$  is reached in the region of low frequencies and the value does not exceeds the 0.3 of the speed of light in a vacuum and goes to zero as the frequency increases. The maximum value of the phase speed is reached in the region of low frequencies and is slightly less than the speed of light in a vacuum:  $V_{oh}(\Omega) \approx 0.97$ .

A similar dependence of the  $H_1$ -wave properties on the properties of the plasma-like medium becomes clear if one analyze the spatial distribution of the field of this wave for the structure with such set of parameters:  $\Omega_p = 2$  and  $\varepsilon_2 = 1$ . The results are shown in the Fig. 5,a for the point on the dispersion curve with the following values of the frequency  $\Omega = 1.102$  and the wave number  $\beta = 1.5$ .



with the parameters corresponding to Fig. 3

For the studied waveguide structure, the  $H_1$ -wave is mainly concentrated at the metamaterial-dielectric interface and is practically absent in the plasma-like medium. When the wave frequency increases up to  $\Omega = 1.168$ , the wave number increases to  $\beta = 4.0$ . At the same time, the normalized values of the wave field components increase and the field penetration depth into the metamaterial and ordinary dielectric decreases, as shown in the Fig. 5,b.



Fig. 6. The dependence of the normalized frequency  $\Omega$ on normalized wave number  $\beta$  for thickness of metamaterial slab  $\tilde{d} = 2.2$ ,  $\Omega_p = 2$ . The curve numbers 1, 2, 3 correspond the parameter

 $\varepsilon_2 = 1, 4, 10$  values

With such spatial distribution of the  $H_1$ -wave, its properties strongly depend on the dielectric permittivity  $\varepsilon_2$  of the dielectric layer that borders with the metamaterial. In the Fig. 6 it is shown the dispersion of  $H_1$ -wave for the waveguide structure with parameters  $\tilde{d} = 2.2$  and  $\Omega_p = 2$  for three values of dielectric permittivity  $\varepsilon_2 = 1.0; 4.0; 10$  (correspondent curves marked by the numbers 1, 2, 3).

The increase of the dielectric permittivity  $\varepsilon_2$  value of the dielectric layer leads to the decrease of the normalized frequency of the wave  $\Omega$  and to the decrease of the  $H_1$ -wave frequency range of existence.

More significant is the fact than under this  $\varepsilon_2$  value growth the increase of the lower limit of normalized wave numbers take place. This leads to a significant decrease in the phase speed of the  $H_1$ -wave (Fig. 7), the maximum value of which at  $\varepsilon_2 = 4$  decreases by almost at a half, as compared to wave phase speed when  $\varepsilon_2 = 1$  (see Fig. 4).



When the dielectric permittivity value  $\varepsilon_2$  increases from 1 up to  $\varepsilon_2 = 4$  the maximum value of the normalized group speed  $\tilde{V}_{gr}$  decreases from 0.28 to 0.23. The maximum value of the normalized phase speed  $\tilde{V}_{ph}(\Omega)$ with such  $\varepsilon_2$  value increase becomes almost twice smaller.

The influence of the dielectric permittivity  $\varepsilon_2$  of the dielectric region on the spatial distribution of the  $H_1$ -wave field was also studied. In the Fig. 8,a,b it is shown the spatial distribution of the  $H_1$ -wave field components for the fixed wave frequency value  $\Omega = 1.1$  in the waveguide structure with parameters  $\Omega_p = 2$  and  $\tilde{d} = 2.2$  for ordinary dielectrics with the following permeability values  $\varepsilon_2 = 7.0;10.0$ ; and in the Fig. 8,c,d – for dielectric with permeability values  $\varepsilon_2 = 7.0;10.0$ .



Fig. 8.a,b. Spatial distribution of wave field components of the  $H_1$ -wave, normalized by the  $E_y(0)$ ,

#### for $\varepsilon_2 = 1.0; 4.0$

The value of the dielectric permittivity  $\varepsilon_2$  has a weak influence on the wave field penetration depth into the metamaterial and the dielectric layers. At the same

ISSN 1562-6016. Problems of Atomic Science and Technology. 2023. № 6(148)

time, simultaneously with the  $\varepsilon_2$  value growth, the value of the  $H_x(x)$ ,  $E_y(x)$ ,  $H_z(x)$  wave field components at the metamaterial-dielectric interface, normalized by the electric field amplitude at the interface of the metamaterial with a plasma-like medium, increases significantly.



by the  $E_{v}(0)$ , for  $\varepsilon_{2} = 7.0; 10.0$ 

Next, let us consider the properties of the  $H_2$ -wave that propagates along such a structure.

We will determine the influence of the normalized plasma frequency of the plasma-like medium  $\Omega_p = \omega_p / \omega_0$  and the dielectric permittivity  $\varepsilon_2$  of the medium, that borders with the metamaterial region, on the dispersion, phase and group speeds, and  $H_2$ -wave wave field spatial structure.

For a planar structure with a normalized width of the metamaterial layer  $\tilde{d} = 2.2$ , the amplitude values of the  $H_2$ -wave components reach their maximum value at the plasma-like medium – metamaterial layer interface x = 0.



Fig. 9. Spatial distribution of wave field components the  $H_2$ -wave, normalized by the  $E_y(0)$ ,

for the structure with parameters  $\Omega_p = 2$  and  $\varepsilon_2 = 1$ 

The Fig. 9 presents the spatial distribution of the  $H_2$ -wave field components in the studied structure with  $\Omega_p = 2$  and  $\varepsilon_2 = 1$  for different values of the normal-

ized wave number  $\beta$ . The  $H_2$ -wave is localized at the interface between the metamaterial and the plasma-like medium. For the wave frequency value  $\Omega = 1.102$  the wave number is equal to  $\beta = 1.5$  (see Fig. 9,a). When the eigenwave frequency increases up to  $\Omega = 1.1825$ , the wavenumber increases to 4.0 (see Fig. 9,b). At the same time, the  $H_x$  and  $H_z$  components of the magnetic field of the  $H_2$ -wave, normalized by the  $E_y(0)$ , increased more than in 2.5 times. The field penetration depth into the both metamaterial and ordinary dielectric also decreases.

Such spatial wave field structure has as a consequence the fact that, unlike the  $H_1$ -wave, the properties of the  $H_2$ -wave depend on the normalized plasma frequency  $\Omega_p = \omega_p / \omega_0$ . The Fig. 10 presents the dispersion properties of the  $H_2$ -wave in the structure with a fixed value of  $\varepsilon_2 = 1$  at different  $\Omega_p = 2$ , 4, 10 values (correspondent curves marked by the numbers 1, 2, 3). When the normalized plasma frequency increases, the normalized frequency of the  $H_2$ -wave and the value of the minimum possible wave number  $\beta$  value also increases.



Fig. 10. The dependence of the normalized frequency  $\Omega$  of the  $H_2$ -wave on normalized wave number  $\beta$  for the structure with parameters  $\Omega_p = 2, 4, 10$  and  $\varepsilon_2 = 1$ 

The Fig. 11 presents the influence of the normalized plasma frequency  $\Omega_p$  on the phase and group speeds of the  $H_2$ -wave. The  $H_2$ -wave is the reversed because its phase and group velocities are opposite in sign.



Fig. 11. The dependence of the normalized phase velocity  $\tilde{V}_{ph}$  and the normalized group velocity  $\tilde{V}_{gr}$  of the  $H_2$ -wave on the normalized frequency  $\Omega$  for the structure with parameters that corresponds to Fig. 9

An increase in the normalized plasma frequency  $\Omega_p$  leads to an increase in the absolute value of the  $H_2$ -wave group speed  $\tilde{V}_{gr}$  and to the shift of the graph of  $\tilde{V}_{or}(\Omega)$  to the area of higher frequencies  $\Omega$ .

The Fig. 12 presents the influence of the dielectric permittivity value of  $\varepsilon_2$  the ordinary dielectric region that bounds the metamaterial slab at x = d, on the properties of  $H_2$ -wave that is localized at the interface of the plasma-like medium with metamaterial.



Fig. 12. The dependence of the normalized frequency  $\Omega$  of the  $H_2$ -wave on normalized wave number  $\beta$ 

for the waveguide structure with  $\Omega_p = 2$ 

for 
$$\varepsilon_2 = 1, 4, 10$$

The dispersion curve that corresponds to the  $\varepsilon_2 = 1$ the starts at the leftmost point on the blue part of the curve and ends at the rightmost brown point at  $\beta = 4.0$ . The curve that corresponds to the  $\varepsilon_2 = 4$  starts at the leftmost point on the green part of the curve and ends at  $\beta = 4.0$ . The curve that corresponds to  $\varepsilon_2 = 10$  starts at the leftmost point on the part of the brown curve and ends at  $\beta = 4.0$ .

With the increase of the  $\varepsilon_2$  value, the maximum value of the normalized frequency  $\Omega$  decreases, and the corresponding values of the normalized wave number increase, which leads to a significant decrease in the frequency range where the  $H_2$ -wave can exist.

#### CONCLUSIONS

It is shown the possibility of the existence of two surface electromagnetic eigen waves with Hpolarization that propagate along a planar waveguide structure consisting of a layer of homogeneous metamaterial whose permeability depends on frequency and is negative. The metamaterial is confined on the one side by a semi-bounded dispersion plasma-like medium, and on the other side by an semi-bounded ordinary dielectric.

It was obtained that the group and phase velocities of the  $H_1$ -wave, which has a lower frequency, coincide in the direction. It is shown that the phase and group velocities of the  $H_2$ -wave, which has a higher frequency, are directed opposite.

It was shown that the  $H_1$ -wave is localized near the interface of the metamaterial layer with a normal dielec-

tric region, and for the normalized thickness of the metamaterial  $\tilde{d} = 2.2$ , its properties practically do not depend on the characteristics of the plasma-like medium.

The increase of the dielectric permittivity value  $\varepsilon_2$ of the dielectric region leads to the decrease of the normalized wave frequency  $\Omega$  and the  $H_1$ -wave frequency range. At the same time, the lower limit of the normalized wave numbers increases significantly and the  $H_1$ -wave phase speed decreases significantly.

It is shown that the  $H_2$ -wave is localized near the interface between the metamaterial layer and plasmalike medium region and its properties depend both on the characteristics of the plasma-like medium and on the permeability of the dielectric medium. The increase in the normalized plasma frequency  $\Omega_p$  leads to the increase of the  $H_2$ -wave frequency interval of existence. At the same time, it is observed the increase of the absolute value of the wave group speed and a shift of the graph of  $\tilde{V}_{gr}(\Omega)$  to the region of higher frequencies. The increase of  $\Omega_p$  also leads to a shift of the phase velocity  $\tilde{V}_{ph}(\Omega)$  graph to the region of higher frequencies.

It is shown that the increase of the dielectric permittivity  $\varepsilon_2$  of the dielectric region leads to the decrease of the  $H_2$ -wave frequency range of existence and to the increase of the smallest possible value of the wave number.

The results obtained in the paper can be useful for modeling and creating of modern nano-devices based on the metamaterials.

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Article received 07.06.2023

### ПОВІЛЬНІ ЕЛЕКТРОМАГНІТНІ ХВИЛІ В ПЛАСКІЙ ТРИКОМПОНЕНТНІЙ ХВИЛЕВОДНІЙ СТРУКТУРІ З МЮ-НЕГАТИВНИМ МЕТАМАТЕРІАЛОМ

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Робота присвячена вивченню дисперсійних властивостей повільних електромагнітних хвиль, що розповсюджуються вздовж планарної хвилеводної структури, що складається з напівобмеженої області плазми, пластини метаматеріалу та напівобмеженої області звичайного діелектрика. Досліджується випадок, коли всі середовища однорідні та ізотропні. Досліджено дисперсійні властивості, фазову та групову швидкості, а також просторову структуру електромагнітного поля власних ТЕ-мод у діапазоні частот, де метаматеріал має негативну магнітну проникність.