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EXCITATION OF WAKE SURFACE PLASMON-PHONON OSCILLATIONS BY A RELATIVISTIC ELECTRON BUNCH IN A POLAR SEMICONDUCTOR WAVEGUIDE

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The process of surface wakefields excitation by the relativistic electron bunch in a polar semiconductor waveguide with a vacuum channel for electron bunch propagation is studied. The spectra and spatio-temporal structure of the excited surface wakefield in such system are investigated. It is shown that the excited full surface wakefield in the terahertz and infrared frequency ranges consists of the fields of LF and HF hybrid plasmon-phonon oscillations. The amplitudes of them are determined.

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INTRODUCTION

The use of solid-state and, in particular, semiconductor plasma for the development of wakefield acceleration methods [1 - 3] has a number of advantages over gas plasma. The solid-state plasma has a high degree of uniformity and stationarity. Besides, the carrier concentration in a semiconductor plasma can take values within a very wide range.

In [4], the process of excitation of bulk longitudinal potential and transverse electromagnetic waves by a relativistic electron bunch propagating along the axis of a cylindrical waveguide completely filled with a polar semiconductor was studied. To transport relativistic electron (positron) bunches in the paraxial region, a vacuum channel is required. In the presence of a semiconductor-vacuum interface, surface plasmon-phonon electromagnetic waves are added to the spectrum of bulk waves. Our goal is to study the intensities of wake surface plasmon-phonon waves, their frequency spectrum and space-time structure of the excited surface wakefield.

All crystalline compounds have a mixed covalent-ionic bond. The presence of an ionic bond in polar semiconductor compounds will affect the polarization properties of semiconductors and, accordingly, the frequency dispersion of their permittivity. The appearance of a phonon component in the permittivity, in turn, will lead to a significant change in the spectra of longitudinal (potential) and transverse (vortex) electromagnetic oscillations in semiconductors.

1. STATEMENT OF THE PROBLEM

The waveguide system is made in the form of a tube of semiconductor material. The inner region $r < a$ is a vacuum cylindrical cavity. The region $b > r > a$ is filled with a homogeneous semiconductor. The outer surface of the dielectric tube $r = b$ is covered with a perfectly conductive metal film that completely shields the electromagnetic field in the semiconductor waveguide. A tubular relativistic electron bunch propagates in a vacuum channel with a current density

$$\vec{j}_b = -\vec{e}_z \frac{Q}{t_b}\frac{\delta(r-r_0)}{2\pi r_0} T\left(\frac{t-z/v_0}{t_b}\right), \quad \rho_b = j_{bz}/v_0, \qquad (1)$$

where r is radial coordinate, r_0 is bunch radius, z is longitudinal coordinate, v_0 is electron bunch velocity, $\vec{e}_z$ is unit vector in the direction of the longitudinal axis, Q is full bunch charge. The function $T\left((t-z/v_0)/t_b\right)$ describes the longitudinal profile of the bunch, t_b its characteristic duration. This function satisfies the normalization condition

$$\int_{-\infty}^{\infty} T(\tau)d\tau = 1.$$

Below we will consider only isotropic media. The expression for the permittivity of isotropic polar semiconductors has the form [5]

$$\varepsilon(\omega) = \varepsilon_{opt}\left(\frac{\omega_L^2-\omega^2}{\omega_T^2-\omega^2} - \frac{\omega_p^2}{\omega^2}\right), \qquad (2)$$

ε_{opt} is optical permittivity, ω_L and ω_T are frequencies of the longitudinal and transverse phonons, $\omega_p = \omega_l/\sqrt{\varepsilon_{opt}}$ is plasma frequency, $\omega_l = \sqrt{4\pi n_e e^2/m_e}$ is Langmuir frequency, n_e is free carrier concentration, m_e is effective mass of electrons, e is electron charge. The first term in the expression for the permittivity (2) is due to the contribution to the permittivity of the electric polarization of the ion subsystem of the crystal, and the second term takes into account the polarization of free carriers (plasma).

2. DISPERSION PROPERTIES OF SURFACE PLASMON-PHONON WAVES OF A SEMICONDUCTOR WAVEGUIDE

Let's stop briefly on the dispersion properties of surface waves in a semiconductor waveguide with permittivity (2). Surface waves of the considered waveguide are described by the equation

$$\varepsilon(\omega) = -\frac{1}{q_v a}\frac{I_1(q_v a)}{I_0(q_v a)} q_d a \frac{\Delta_0(q_d a, q_d b)}{\Delta_1(q_d a, q_d b)}, \qquad (3)$$

where

$$q_v = \sqrt{k^2 - k_0^2}, \quad q_d = \sqrt{k^2 - k_0^2\varepsilon(\omega)};$$

k is longitudinal wave number, $k_0 = \omega/c$.

$$\Delta_n(q_d a, q_d b) = I_0(q_d b) K_n(q_d a) - $$
$$-(-1)^n I_n(q_d a) K_0(q_d b) .$$

First of all, we note that on the plane ω, k the region of existence of slow surface waves is limited by inequalities

$$q_v^2 > 0,\ q_d^2 > 0. \quad (4)$$

These inequalities ensure the surface nature of the waves in the vacuum channel and the medium, and on the other hand, the permittivity must be negative

$$\varepsilon(\omega) < 0 . \quad (5)$$

Inequalities (4), (5) are equivalent to the following

$$\omega < kc,\ \omega_{lp}^{(+)} > \omega > \omega_T ,\ \omega_{lp}^{(-)} > \omega > 0 , \quad (6)$$

$\omega_{lp}^{(\pm)}$ are frequencies of high-frequency and low-frequency volume longitudinal plasmon-phonon oscillations. These frequencies

$$\omega_{lp}^{(\pm)2} = \frac{\omega_L^2 + \omega_p^2}{2} \pm \sqrt{\left(\frac{\omega_L^2 + \omega_p^2}{2}\right)^2 - \omega_T^2 \omega_p^2} , \quad (7)$$

are determined from the dispersion equation

$$\varepsilon(\omega) = 0 .$$

Note that at $k \to \infty$, the dispersion equation (3) transforms into the equation

$$\varepsilon = -1 ,$$

which determines the two limiting frequencies of HF and LF surface plasmon-phonon waves

$$\omega_{ls}^{(\pm)^2} = \frac{1}{2}\left[\omega_{li}^2 + \omega_{le}^2 \pm \sqrt{\left(\omega_{li}^2 + \omega_{le}^2\right)^2 - 4\omega_{le}^2 \omega_T^2}\right]. \quad (8)$$

Here

$$\omega_{li}^2 = \frac{\varepsilon_{st} + 1}{\varepsilon_{opt} + 1}\omega_T^2 ,\ \omega_{le}^2 = \frac{\varepsilon_{opt}}{\varepsilon_{opt} + 1}\omega_p^2 ,$$

$\varepsilon_{st} = \varepsilon_{opt}\omega_L^2 / \omega_T^2$ is static permittivity ($\varepsilon_{st} > \varepsilon_{opt}$).

Dispersion curves of HF and LF surface plasmon-phonon oscillations at $k \to \infty$ always reach limiting frequencies (8). In the limiting cases of solid-state plasma of low $\omega_T^2 >> \omega_p^2$ and high plasma density $\omega_T^2 << \omega_p^2$, the expressions for the limiting frequencies (8) are simplified

$$\omega_{ls}^{(+)} = \omega_{li}\left(1 + \frac{1}{2}\frac{\Delta\varepsilon}{\varepsilon_{st} + 1}\frac{\omega_{le}^2}{\omega_{li}^2}\right) < \omega_L ,$$

$$\omega_{ls}^{(-)} = \omega_p \sqrt{\frac{\varepsilon_{opt}}{\varepsilon_{st} + 1}} < \omega_p ,\ \omega_T^2 >> \omega_p^2 ,$$

$$\omega_{ls}^{(+)} = \omega_{le}\left(1 + \frac{1}{2}\frac{\Delta\varepsilon}{\varepsilon_{opt} + 1}\frac{\omega_T^2}{\omega_{lp}^2}\right) < \omega_p ,$$

$$\omega_{ls}^{(+)} = \omega_T\left(1 - \frac{1}{2}\frac{\Delta\varepsilon}{\varepsilon_{opt}}\frac{\omega_T^2}{\omega_{le}^2}\right) < \omega_T ,\quad \omega_p^2 >> \omega_T^2 .$$

Let us now stop on the behavior of the dispersion curves of HF and LF surface plasmon-phonon oscillations near the left boundary of the region of existence of a surface wave $\omega = kc$. In this vicinity, the phase velocities of surface waves are close to the speed of light in vacuum; therefore, these waves are of the greatest interest from the point of view of their wake excitation by relativistic electron bunches. In the vicinity of this boundary, the condition

$$q_v a << 1 . \quad (9)$$

This condition makes it possible to significantly simplify the dispersion equation (3) and represent it in the form

$$\varepsilon(\omega) = -\frac{q_d a}{2}\frac{\Delta_0(q_d a, q_d b)}{\Delta_1(q_d a, q_d b)} . \quad (10)$$

At the boundary of the existence of surface waves $k = \omega / c$, the quantity q_d can be represented in form

$$q_d = \frac{\omega}{c}\sqrt{1 - \varepsilon(\omega)} .$$

Together with equation (10), we have a transcendental equation for determining the initial frequencies $\omega_{in}^{(\pm)}$, from which the dispersion curves of HF and LF surface plasmon-phonon waves begin. Each value of the boundary frequency $\omega_{in}^{(\pm)}$ corresponds to the longitudinal wave number $k_{in}^{(\pm)} = \omega_{in}^{(\pm)} / c$.

If, together with condition (8), the condition is also satisfied

$$q_d a >> 1 , \quad (11)$$

then instead of Eq. (10) in the vicinity of the boundary $\omega = kc$ we obtain

$$\varepsilon(\omega) = -\frac{k_0 a\sqrt{1 - \varepsilon(\omega)}}{2}\tanh\left(k_0 L\sqrt{1 - \varepsilon(\omega)}\right). \quad (12)$$

In the limiting case of a thick layer $q_d L >> 1$, equation (12) for determining the initial frequency is simplified and takes the form

$$\frac{\varepsilon(\omega)}{\sqrt{1 - \varepsilon(\omega)}} = -\frac{k_0 a}{2} . \quad (13)$$

Since the right side of this equation is large, then from this equation we find the approximate values of the frequencies from which the dispersion curves of HF and LF plasmon-phonon surface waves begin

$$\omega_{in}^{(+)} = \omega_T\left(1 + \frac{\mu_T}{2}\right). \quad (14)$$

$$\omega_{in}^{(-)} = \omega_p\left(\frac{\varepsilon_{opt}}{P_e}\right)^{1/4} , \quad (15)$$

where $\mu_T = \dfrac{4\Delta\varepsilon}{k_T^2 a^2} << 1$, $P_e = \omega_p^2 a^2 / 4c^2$. The applicability condition of formula (14) has the form

$$\kappa_L a >> 2\frac{\omega_p}{\omega_T\sqrt{\varepsilon_{opt}}} ,\quad \kappa_L a >> 2\sqrt{\Delta\varepsilon} ,$$

and formula (15) is valid when

$$P_e >> \varepsilon_{opt} .$$

Thus, in the case of a thick layer, the dispersion curve of the HF surface plasmon-phonon wave on the boundary straight line $\omega = ck$ begins with a frequency slightly higher than the frequency of transverse phonons, and the boundary LF surface plasmon-phonon wave is significantly lower than the plasma frequency.

Consider now the limiting case of a thin layer

$$q_d L \ll 1 . \quad (16)$$

In this case, the equation for determining the boundary frequency (12) is simplified and takes the form

$$\frac{\varepsilon(\omega)}{1-\varepsilon(\omega)} = -\frac{k_0^2 aL}{2} . \quad (17)$$

The right side of this equation is the product of the large $k_0 a \gg 1$ and small $k_0 L \ll 1$ parameters. Therefore, it can take, generally speaking, an arbitrary value. However, the requirement $\varepsilon(\omega) < 0$ imposes a restriction on the value of the right side of equation (17)

$$k_0^2 aL/2 < 1 .$$

To simplify the analysis, we will assume that the layer is so thin that the more stringent condition $k_0^2 aL/2 \ll 1$ is satisfied. In this case, instead of (17) we have

$$\varepsilon(\omega) = -\frac{k_0^2 aL}{2} . \quad (18)$$

It follows from this equation that the boundary frequencies of surface plasmon-phonon oscillations are close to the eigen frequencies of bulk longitudinal plasmon-phonon oscillations (7)

$$\omega_{in}^{(+)^2} = \omega_{lp}^{(+)^2}\left(1 - \frac{1}{2\varepsilon_{opt}} L_{lp}^{(+)} \frac{\omega_{lp}^{(+)^2} aL}{c^2}\right), \quad (19)$$

$$\omega_{in}^{(-)^2} = \omega_{lp}^{(-)^2}\left(1 + \frac{1}{2\varepsilon_{opt}} L_{lp}^{(-)} \frac{\omega_{lp}^{(-)^2} aL}{c^2}\right), \quad (20)$$

$$L_{lp}^{(\pm)} = \frac{\omega_{lp}^{(\pm)2} - \omega_T^2}{\omega_{lp}^{(+)2} - \omega_{lp}^{(-)2}} .$$

In the case under consideration, the initial frequencies of the dispersion curves are close to the upper boundaries of the regions of existence of HF and LF surface plasmon-phonon waves (6).

Finally, consider the limiting case of the vacuum channel of a small radius

$$q_d a \ll 1,\ q_d b \ge 1 .$$

In this limiting case, dispersion equation (10) takes the form

$$\varepsilon(\omega) = -\frac{q_d^2 a^2}{2} \ln\left(\frac{1}{\nu q_d a}\right),$$

ν is Euler's constant. Solving this equation by the method of successive approximations, we find the values of the initial frequencies of the plasmon-phonon oscillation branches

$$\omega_{in}^{(+)^2} = \omega_{lp}^{(+)^2}\left(1 - L_{lp}^{(+)} \frac{\kappa_{lp}^{(+)^2} a^2}{2\varepsilon_{opt}} \ln\frac{1}{\nu\kappa_{lp}^{(+)} a}\right),$$

$$\omega_{in}^{(-)^2} = \omega_{lp}^{(-)^2}\left(1 + L_{lp}^{(-)} \frac{\kappa_{lp}^{(-)^2} a^2}{2\varepsilon_{opt}} \ln\frac{1}{\nu\kappa_{lp}^{(-)} a}\right),$$

where $k_{lp}^{(\pm)} = \omega_{lp}^{(\pm)}/v_0$. The initial frequency at which the dispersion curve begins is slightly below the frequency of bulk plasmons. In this case the initial longitudinal wavenumbers are $k_{in}^{(\pm)} = \omega_{in}^{(\pm)}/c$.

3. DETERMINATION OF THE WAKE SURFACE ELECTROMAGNETIC FIELD IN A POLAR SEMICONDUCTOR WAVEGUIDE

Let us determine the surface wake field of an electron bunch, which has the form of an infinitely thin tubular bunch (1), which propagates in the vacuum channel of a semiconductor waveguide. We will solve the problem by the Fourier transform method. We expand the quantities in the Maxwell equations into the Fourier integrals over frequencies

$$\left(\vec{E},\vec{H}\right) = \int_{-\infty}^{\infty} \left(\vec{E}_\omega, \vec{H}_\omega\right) e^{-i\omega\bar{t}} d\omega,$$

$$\left(\vec{j}_b, \rho_b\right) = -\frac{Q}{t_b}\frac{\delta(r-r_0)}{2\pi r_0}\left(\vec{e}_z, \frac{1}{v_0}\right)\int_{-\infty}^{\infty} T_\omega e^{-i\omega\bar{t}} d\omega,$$

where $\bar{t} = t - z/v_0$,

$$\left(\vec{E}_\omega, \vec{H}_\omega\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\vec{E}(t), \vec{H}(t)\right) e^{i\omega t} dt,$$

$$T_\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} T(t) e^{i\omega t} dt.$$

The longitudinal component of the Fourier amplitude of the field $E_{z\omega}$ in the vacuum channel $r < a$ is described by the equation

$$\frac{1}{r}\frac{d}{dr} r \frac{dE_{z\omega}}{dr} - \kappa_v^2 E_{z\omega} = -\frac{2i\omega}{v_0^2\gamma_0^2}\frac{Q}{t_b} T_\omega \frac{\delta(r-r_0)}{r_0} ,$$

γ_0 is relativistic factor of the bunch. There is no electron bunch in a semiconductor. Therefore the equation for the Fourier amplitude is homogeneous

$$\frac{1}{r}\frac{d}{dr} r \frac{dE_{z\omega}^{(d)}}{dr} - \kappa_d^2 E_{z\omega}^{(d)} = 0 ,$$

where $\kappa_v^2 = k_l^2 - k_0^2 \equiv k_l^2/\gamma_0^2$, $\kappa_d^2 = k_l^2 - k_0^2\varepsilon,\ k_l = \omega/v_0$.

Let us divide the semiconductor waveguide into three regions: two regions in the vacuum channel

$$E_{z\omega}^{(v1)}(r) = E_{z\omega}^{(v)}(r < r_0 < a),\ E_{z\omega}^{(v2)}(r) = E_{z\omega}^{(v)}(a > r > r_0)$$

and the third region $b > r > a$ is a semiconductor tube

$$E_{z\omega}^{(d)}(r) = E_{z\omega}^{(d)}(b > r > a) .$$

The electromagnetic field must satisfy the following boundary conditions. On the surface of an ideally conducting lateral metal film, the longitudinal component of the electric field vanishes

$$E_{z\omega}^{(d)}(r = b) = 0 .$$

At the semiconductor-vacuum boundary, the tangential components of the electric and magnetic fields are continuous

$$E_{z\omega}^{(d)}(r = a) = E_{z\omega}^{(v2)}(r = a) ,$$

$$H_{\varphi\omega}^{(d)}(r = a) = H_{\varphi\omega}^{(v2)}(r = a) .$$

In the considered case of an infinitely thin bunch, the Fourier amplitude of the magnetic field $H_{\varphi\omega}^{(v)}(r)$ undergoes a jump on the surface $r = r_0$ along which the ring electron bunch propagates

$$H_{\varphi\omega}^{(v2)}(r = r_0 + 0) - H_{\varphi\omega}^{(v1)}(r = r_0 - 0) = -\frac{2Q}{c r_0 t_b} T_\omega .$$

The longitudinal component of the electric field on this surface is continuous

$$E_{z\omega}^{(v2)}(r=r_0)=E_{z\omega}^{(v1)}(r=r_0).$$

Having obtained the solution of the boundary problem formulated above, we find expressions for the Fourier components of the wake electromagnetic field in each of the regions of the semiconductor waveguide. As a result, for the components of the electromagnetic field in the paraxial region of the vacuum channel, we obtain the following expressions

$$E_z^{(v1)}(r,\bar t)=i\frac{2Q}{a^2t_b}\int_{-\infty}^{\infty}e^{-i\omega\bar t}T(\omega)\frac{d\omega}{\omega}\frac{I_0(\kappa_v r)}{I_0(\kappa_v a)}\left[\frac{I(\kappa_v r_0)}{I_0(\kappa_v a)}\frac{1}{D(\omega)}+\right.$$
$$\left.+\kappa_v^2a^2\Delta_0(\kappa_v r_0,\kappa_v a)\right], \qquad (21)$$

$$H_\varphi^{(v1)}(r,\bar t)=\frac{2Q}{a^2t_b}\int_{-\infty}^{\infty}e^{-i\omega\bar t}T(\omega)\frac{d\omega}{\kappa_v c}\frac{I_1(\kappa_v r)}{I_0(\kappa_v a)}\left[\frac{I_0(\kappa_v r_0)}{I_0(\kappa_v a)}\frac{1}{D(\omega)}+\right.$$
$$\left.+\kappa_v^2a^2\Delta_0(\kappa_v r_0,\kappa_v a)\right], \qquad (22)$$

where

$$\Delta_n(\kappa_d r,\kappa_d b)=I_0(\kappa_d b)K_n(\kappa_d r)-$$
$$-(-1)^nI_n(\kappa_d r)K_0(\kappa_d b),$$
$$\kappa_d=\sqrt{k_l^2-k_0^2\varepsilon(\omega)}\equiv k_l\sqrt{\gamma_0^{-2}+\beta_0^2\left[1-\varepsilon(\omega)\right]},$$
$$\kappa_v=\sqrt{k_l^2-k_0^2}\equiv k_l/\gamma_0,$$
$$D(\omega)=\frac{1}{\kappa_v a}\frac{I_1(\kappa_v a)}{I_0(\kappa_v a)}+\frac{\varepsilon}{\kappa_d a}\frac{\Delta_1(\kappa_d a,\kappa_d b)}{\Delta_0(\kappa_d a,\kappa_d b)}. \qquad (23)$$

Fourier integrals (21), (22) can be conveniently represented as a convolution. So, for example, for the longitudinal component of the electric field in the axial region, we have the following integral representation

$$E_z^{(v1)}(r,\bar t)=\frac{Q}{t_b}\int_{-\infty}^{\infty}T\left(\frac{t_0}{t_b}\right)E_{gv}(r,\bar t-t_0)dt_0, \qquad (24)$$

$$E_{gs}(r,\bar t-t_0)=\frac{i}{a^2\pi}\int_{-\infty}^{\infty}e^{-i\omega(\bar t-t_0)}\frac{d\omega}{\omega}\frac{I_0(\kappa_v r)}{I_0(\kappa_v a)}\left[\frac{I(\kappa_v r_0)}{I_0(\kappa_v a)}\frac{1}{D(\omega)}+\right.$$
$$\left.+\kappa_v^2a^2\Delta_0(\kappa_v r_0,\kappa_v a)\right]. \qquad (25)$$

The function $E_g(r,\bar t-t_0)$ has a simple physical meaning. It describes the longitudinal component of the wake electric field excited by an infinitely thin in the radial and longitudinal directions of an electron bunch that has entered into a semiconductor waveguide at a time $t=t_0$ and has a unit charge (Green's function).

The integrands in (25) have only simple poles, which are the roots of the equation

$$D(\omega)=0. \qquad (26)$$

The roots of equation (26) determine the frequencies of the eigen waves of the dielectric waveguide, synchronous with the relativistic electron bunch. Within the framework of the considered model, a dielectric waveguide has a formally infinite number of branches of bulk slow electromagnetic waves (bulk polaritons) and two surface electromagnetic waves (surface LF and HF plasmon-phonon oscillations). Cherenkov excitation by a relativistic electron bunch of bulk polaritons was considered in paper [4] for the model of a semiconductor waveguide with full filling. The presence of a vacuum channel does not introduce qualitative features into the picture of excitation of bulk polaritons. Below we will focus on investigation the physical picture of excitation of only surface plasmon-phonon oscillations. We note that there are no poles $\varepsilon(\omega)=0$ in the integrands (21), (22). Therefore, a relativistic electron bunch propagating in a vacuum channel does not excite longitudinal bulk plasmon-phonon ocsillations. A similar situation occurs with wake excitation of surface waves in a plasma waveguide [6].

Calculating the residues in the poles $\omega=\pm\omega_{ls}^{(\pm)}-i0$ corresponding to the surface HF and LF plasmon-phonon waves, we find the expression for the Green's function (25)

$$E_{gv}(r,\bar t)=\frac{2}{a^2}\vartheta(\bar t)\left[\frac{I_0\left(\kappa_{vs}^{(+)}r_0\right)I_0\left(\kappa_{vs}^{(+)}r\right)}{\Lambda_s^{(+)}I_0^2\left(\kappa_{vs}^{(+)}a\right)}\cos\omega_{ls}^{(+)}\bar t+\right.$$
$$\left.+\frac{I_0\left(\kappa_{vs}^{(-)}r_0\right)I_0\left(\kappa_{vs}^{(-)}r\right)}{\Lambda_s^{(-)}I_0^2\left(\kappa_{vs}^{(-)}a\right)}\cos\omega_{ls}^{(-)}\bar t\right], \qquad (27)$$

where $\kappa_{vs}^{(\pm)}\equiv\kappa_v(\omega_{ls}^{(\pm)})$, $\Lambda_s^{(\pm)}\equiv\Lambda(\omega_{ls}^{(\pm)})$,

$$\Lambda(\omega)=\omega^2\frac{\partial D(\omega^2)}{\partial\omega^2}. \qquad (28)$$

The field of a surface wave excited by an electron bunch of finite dimensions is found by summing the fields of elementary ring charges and currents according to formula (24). As a result, we obtain the following expressions for the longitudinal component of the surface electric field in the vacuum channel

$$E_{zs}^{(v)}=\frac{2Q}{a^2}\left[\frac{1}{\Lambda_s^{(+)}}\frac{I_0\left(\kappa_{vs}^{(+)}r\right)I_0\left(\kappa_{vs}^{(+)}r\right)}{I_0^2\left(\kappa_{vs}^{(+)}a\right)}Z(\omega_{ls}^{(+)}\bar t)+\right.$$
$$\left.+\frac{1}{\Lambda_s^{(-)}}\frac{I_0\left(\kappa_{vs}^{(-)}r\right)I_0\left(\kappa_{vs}^{(-)}r_0\right)}{I_0^2\left(\kappa_{vs}^{(-)}a\right)}Z(\omega_{ls}^{(-)}\bar t)\right]. \qquad (29)$$

Here

$$Z(\omega t)=\frac{1}{t_b}\int_{-\infty}^{\tau}T\left(t_0/t_b\right)\cos\omega(t-t_0)dt_0.$$

The function $Z(\omega\tau)$ describes the distribution of the wakefield at a frequency ω in the longitudinal direction at each moment of time.

Note that in the most interesting case

$$\kappa_{vs}^{(\pm)}a\equiv\frac{\omega_{ls}^{(\pm)}a}{v_0\gamma_0}<<1 \qquad (30)$$

of a ultra-relativistic electron bunch, the longitudinal component of the electric field of the surface field in the vacuum channel is almost uniform over the channel cross section. The expression for the longitudinal component of the electric field (29) in the vacuum channel takes the form

$$E_{zs}^{(v)}=\frac{2Q}{a^2}\left[\frac{1}{\Lambda_s^{(+)}}Z(\omega_{ls}^{(+)}\bar t)+\frac{1}{\Lambda_s^{(-)}}Z(\omega_{ls}^{(-)}\bar t)\right]. \qquad (31)$$

For an electron bunch with a Gaussian longitudinal profile

$$T(t_0/t_b)=\frac{1}{\sqrt{\pi}}e^{-t_0^2/t_b^2},$$

the wake surface field in the "wave" zone $\omega_{ls}^{(\pm)}\bar t>>1$ is a superposition of HF and LF plasmon-phonon monochromatic waves.

$$E_{zs}^{(v)} = E_w^{(+)} T_s^{(+)} \cos \omega_{ls}^{(+)} \bar{t} + E_w^{(-)} T_s^{(-)} \cos \omega_{ls}^{(-)} \bar{t}, \quad (32)$$

where

$$E_w^{(\pm)} = \frac{2Q}{a^2} \frac{1}{\Lambda_s^{(\pm)}}, \ T_s^{(\pm)} = \exp\left(-\frac{\omega_{ls}^{(\pm)^2} t_b^2}{4} \right).$$

Let us study expressions for the field components of a surface wake wave in a number of limiting cases. First of all, consider the limiting case of a vacuum channel of small radius, when, along with inequality (30), the condition $\kappa_d a << 1$ is fulfilled. In this limiting case, the frequencies of HF and LF surface waves are close to the frequencies of bulk longitudinal plasmon-phonon oscillations $\omega_{lp}^{(\pm)}$. Accordingly, the structure of wake HF and LF surface waves continuously transforms into the structure of the field of volumetric longitudinal oscillations

$$E_{zs}^{(v)}(r, \bar{t}) = \frac{2Q}{\varepsilon_{opt}} \Big[k_{lp}^{(+)2} L_{lp}^{(+)} T_s^{(+)} \ln \frac{2}{k_{lp}^{(+)} a} \cos \omega_{lp}^{(+)} \bar{t} + $$

$$+ k_{lp}^{(-)2} L_{lp}^{(-)} T_s^{(-)} \ln \frac{2}{k_{lp}^{(-)} a} \cos \omega_{lp}^{(-)} \bar{t} \Bigg].$$

The above consideration is valid when the radius of the vacuum channel and, accordingly, the transverse size of the electron bunch is significantly less than the wavelength of bulk plasmons $a << \lambda_L / 2\pi$, $\lambda_L = 2\pi c / \omega_L$. More realistic is the case when the dimensions of the vacuum channel and the electron bunch are of the same order of magnitude or exceed the length of the excited surface wave. We assume that the condition is satisfied

$$k_0 a >> 1. \quad (33)$$

Then, the more the inequality $\kappa_d a >> 1$ is fulfilled. The bunch is strongly relativistic $\gamma_0^2 >> 1$, so that requirement (30) is still satisfied. Under these conditions, the spectrum equation (26), (23) for determining the frequency of the surface wake wave takes the form (12). In the case of a thick layer

$$\kappa_d L >> 1, \quad (34)$$

equation (12) is simplified and takes the form (13). Accordingly, the frequencies of wake HF and LF surface waves for relativistic electron bunches practically coincide with the initial frequencies (14), (15) of the HF and LF branches of surface plasmon-phonon oscillations. The amplitudes of HF and LF surface waves take the values

$$E_w^{(+)} = \frac{4Q}{a^2} \mu_T, \ E_w^{(-)} = \frac{4Q}{a^2}.$$

Since the parameter μ_T is small $(\mu_T << 1)$, in the considered case of a thick semiconductor layer, LF surface plasmon-phonon wave is predominantly excited and, in fact, a single-wave mode of excitation of the wake surface field is realized.

Let us now consider the limiting case of a thin semiconductor layer $\kappa_d L << 1$. In this limiting case, the spectrum equation has the form (18), and the frequencies of HF and LF wake surface plasmon-phonon waves are determined by expressions (19), (20) and are close to the frequencies of longitudinal bulk plasmon-phonon oscillations. For the amplitudes of wake HF and LF surface plasmon-phonon oscillations, from formula (32) we obtain

$$E_w^{(+)} = 2Q L_{lp}^{(+)} \frac{k_{ls}^{(+)^2}}{\varepsilon_{opt}} \frac{L}{a}, \quad E_w^{(-)} = -2Q L_{lp}^{(-)} \frac{k_{ls}^{(-)^2}}{\varepsilon_{opt}}.$$

Here $k_{ls}^{(\pm)} = \omega_{ls}^{(\pm)} / v_0$. If the frequency of the HF surface wave significantly exceeds the frequency of the LF surface wave, then the amplitude of the HF wave will also significantly exceed the amplitude of the LF surface wave.

CONCLUSIONS

In the presence of a vacuum channel in a semiconductor waveguide, surface HF and LF plasmon-phonon waves appear in the eigenwave spectrum. Surface plasmon-phonon waves of a semiconductor waveguide exist in the frequency range where the permittivity of a polar semiconductor is negative. Note that there are no bulk transverse polaritons in this frequency range.

The dispersion properties of surface waves in a polar tubular semiconductor waveguide are studied. The process of excitation of wake surface HF and LF plasmon-phonon oscillations by a relativistic electron bunch propagating in a vacuum channel has also been studied. The amplitudes of excited wake HF and LF surface waves and their dependence on the parameters of polar semiconductors are determined. The ultrarelativistic electron bunch will always excite wake waves with frequencies very close to the initial frequencies of the corresponding eigen wave branches of a polar semiconductor waveguide.

REFERENCES

1. K.L. Bane, P.B. Wilson. *Wakefields and Wakefield Acceleration.* 1984. SLAC-PUB-3528.
2. P. Chen, J.M. Dawson, T. Katsouleas. Acceleration of Electrons by the Interaction of a Bunched Electron Beam with a Plasma // *Phys. Rev. Lett.* 1985, v. 54, № 7, p. 693.
3. T. Katsouleas. Physical mechanisms in the plasma wake-field accelerator // *Phys. Rev. A.* 1986, v. 337, № 3, p. 2056.
4. V.F. Balakirev, I.N. Onishchenko. Excitation of a wake fields by a relativistic bunch in a polar semiconductor waveguide // *Problems of Atomic Science and Technology. Series "Nuclear Physics Investigations".* 2022, № 3, p. 76-81.
5. N.L. Dmitruk, V.L. Litovchenko, V.G. Strizhevsky. *Surface polaritons in semiconductors and dielectrics.* Kiev: "Naukova Dumka", 1989, 376 p.
6. V.A. Balakirev, V.I. Karas, A.P. Tolstoluzskii. Excitation of wake field in a plasma with radial density variation // *Plasma Physics Reports*. 1997, v. 23, № 4, p. 290-298.

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ЗБУДЖЕННЯ ПОВЕРХНЕВИХ ПЛАЗМОН-ФОНОННИХ КОЛИВАНЬ РЕЛЯТИВІСТСЬКИМ ЕЛЕКТРОННИМ ЗГУСТКОМ У ПОЛЯРНОМУ НАПІВПРОВІДНИКОВОМУ ХВИЛЕВОДІ

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Досліджено процес збудження поверхневих кільватерних полів релятивістським згустком електронів у полярному напівпровідниковому хвилеводі з вакуумним каналом для транспортування згустка електронів. Досліджено спектри та просторово-часову структуру збудженого поверхневого кільватерного поля в такій системі. Показано, що збуджене повне поверхневе кільватерне поле в терагерцовому та інфрачервоному діапазонах частот складається з полів НЧ- і ВЧ-гібридних плазмон-фононних коливань. Визначено амплітуди збуджених НЧ- та ВЧ-поверхневих плазмон-фононних хвиль.