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STRENGTH ANALYSIS OF A CYLINDER FROM CARBON-CARBON COMPOSITE MATERIAL AS A RESULT OF TEMPERATURE DEFORMATION AND SHRINKAGE

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Using the solution of the Lame problem, we analyzed the stress-strain state (SSS) of a cylinder made of carbon forced carbon composite material (CFC) for the case of fixed ends. It is shown that the thermal stress safety factor is close to four. In addition, an engineering analysis of the strength of CFC cylinder due to possible shrinkage of the fibers was carried out in the work. It is shown that the safety factor is equal to 0.386 and can be the cause of the destruction of the CFC during manufacture and operation.

INTRODUCTION

Carbon-carbon composite materials of a cylindrical shape, when manufactured by the method of thermogradient gas-phase compaction, are under conditions of uneven heating along the radius, when the temperature inside the cylinder is maximum. The reinforcement directions coincide with the three axes of the rectangular Cartesian coordinate system. The paper proposes to use the solution of the Godolin-Lame problem [1] to analyze the stress-strain state of such cylinders when their ends do not have axial displacements. The article [2] provides a complete solution of the Lame problem for a composite, on average, isotropic cylinder with a polymer matrix that has a nonlinear physical law for optimal filling of it with reinforcing fibers. The paper [3] presents the initial data and the results of calculations of cylinders made of CFC – elements of molds as isotropic materials, taking into account plastic deformation by the finite element method. Article [4] describes the technique and results of tensile testing of high-modulus single carbon fibers impregnated with a thermoplastic matrix solution. The tensile strength of the fiber type T700SC, UMT49, UMT400 is 4.9 GPa.

Figs. 1 and 2 shows a thick-walled cylinder in the polar coordinate system *r, θ, z*, which is in a plane stress state. Mechanical stresses σ_r, σ_θ, σ_z, acting on planes with normal *r, θ, z,* will be the principal stresses.

RESULTS AND DISCUSSION

1. THERMAL STRESS OF A CYLINDER MADE OF CFC

For the case when the temperature changes along the radius of the cylinder according to a linear law:

$$t(r) = T\frac{r_2 - r}{r_2 - r_1}, \quad (1)$$

where $T = t_1 - t_2$; t_1 and t_2 – are temperatures on the inner and outer surfaces of the cylinder, respectively, mechanical stresses can be calculated using the formulas [1]:

$$\sigma_r = \frac{E\alpha T}{3(1-\mu)(r_2 - r_1)}\left[r - \frac{r_1^3}{r^2} - \left(1 - \frac{r_1^2}{r^2}\right)\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right]; \quad (2)$$

$$\sigma_\theta = \frac{E\alpha T}{3(1-\mu)(r_2 - r_1)}\left[2r + \frac{r_1^3}{r^2} - \left(1 + \frac{r_1^2}{r^2}\right)\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right]; \quad (3)$$

$$\sigma_z = \frac{E}{1-\mu}\left[\frac{2\mu}{r_2^2 - r_1^2}\int_{r_1}^{r_2}\alpha t(r) r dr + (1-\mu)\varepsilon_z - \alpha t(r)\right], \quad (4)$$

where α is the coefficient of temperature linear expansion of the material (CTLE); E is the modulus of elasticity; μ is the Poisson's ratio of CFC, which correspond to the average temperature of the wall. Let's take into account that r_1 is a lot less than r_2, and it's not too much impact on the result. We can't accept that $r_1 = 0$ because according to [1] radial displacement is:

$$u(r) = C_1 r + C_2\frac{1}{r},$$

where C_1, C_2, are the constants of the solution equation integration of the Lame problem. When substituting $r = 0$, we get infinity, that is, the Lame problem is solved not for a solid, but for a thick-walled cylinder.

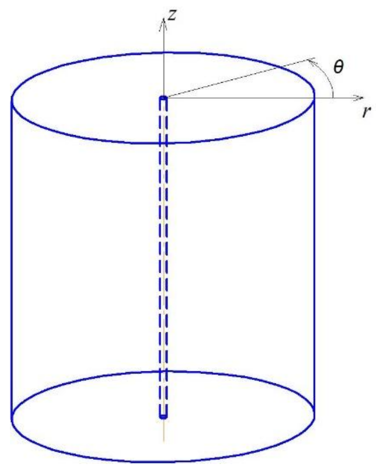

Fig. 1. General view of the CFC cylinder

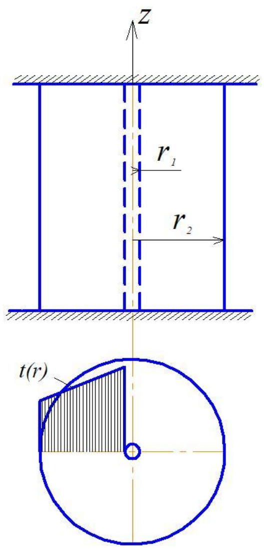

Fig. 2. Calculation scheme of the cylinder

To calculate the axial stresses σ_z for the temperature distribution (1), we find in (4) a definite integral:

$$\int_{r_1}^{r_2} \alpha t(r) r dr = \alpha \frac{T}{r_2 - r_1} \left(\frac{r_2^3 - 3r_1^2 r_2 + 2r_1^3}{6} \right)$$

and substitute into (4) the expression for ε_z through Hooke's law:

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \mu\sigma_r - \mu\sigma_\theta) + \alpha t(r) = const .$$

Then we obtain a linear equation with respect to, from which we have:

$$\sigma_z = \sigma_r + \sigma_\theta - \frac{E\alpha T}{3} \frac{r_2^3 - 3r_1^2 r_2 + 2r_1^3}{(r_2 - r_1)^2 (r_2 + r_1)} . \qquad (5)$$

Expression (5) is expression for calculating the axial stress σ_z for the case when the ends of the cylinder cannot move in the axial direction.

Consider practical numerical examples. The values of the elastic modulus E=18 GPa and Poisson's ratio μ=0.19 of CFC are taken as in [3]; also in the calculations were taken: t_1 = 1500 °C, t_2 = 1000 °C, r_1 = 2 mm, r_2 = 130 mm, $\alpha = 3 \cdot 10^{-6}$ 1/deg. Tensile strength $[\sigma_+]_B$ = 110 MPa, compressive strength $[\sigma_-]_B$ = 100 MPa [3]. Fig. 3 shows the distribution of the stress tensor components along the radius of the cylinder. In the distribution of the considered temperature, the most loaded is the inner surface of the cylinder. Maximum circumferential stress takes place at $r = r_1$ σ_θ = –22.05 MPa axial stress σ_z = –31.15 MPa.

Stress intensity is given by:

$$\sigma_i = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]} = 27.7\text{MPa},$$

where $\sigma_1 = 0$, σ_2 = –22.05 MPa, σ_3 = –31.15 MPa.

According to the Coulomb-Mohr strength criterion [3] for brittle materials, the stress intensity should be less than the tensile strength limit:

$$\sigma_i = 27.7 \text{ MPa} < [\sigma_+]_B = 110 \text{ MPa}. \qquad (6)$$

Table 1 shows the results of calculations of the principal stresses of the cylinder on its inner surface at different values of r_2. The obtained data show that the stressed state of the cylinder does not change significantly with the increase of r_2, which coincides with the data known for the case of loading the cylinder with internal pressure [1].

Table 1

Stress on the inner surface of the cylinder at different values of the outer diameter

r_2, mm	Stress, MPa		
	σ_r	σ_θ	σ_z
130	0	-22.05	-31.15
220	0	-22.12	-31.20
350	0	-22.16	-31.21

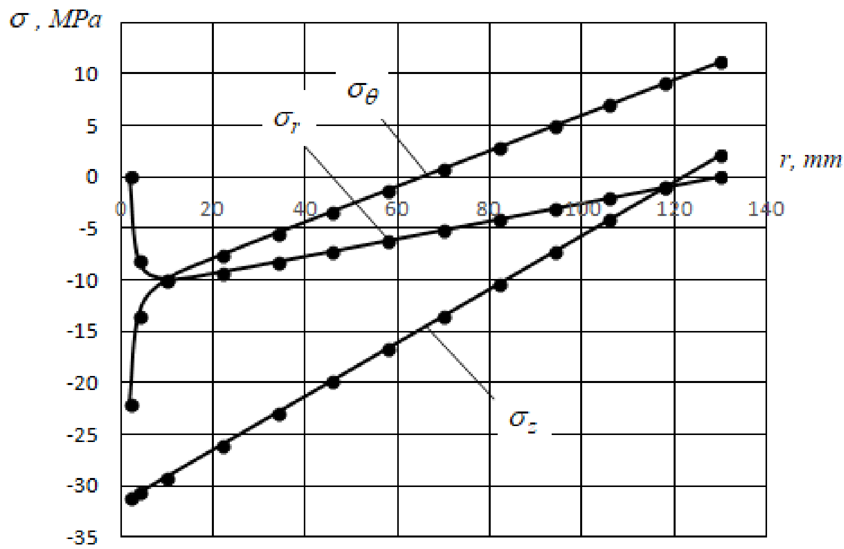

Fig. 3. Distribution of stresses σ_r , σ_θ , σ_z along the radius of the cylinder

To determine the displacement, we will use the formula [1]:

$$u=\frac{1}{r}\frac{1+\mu}{1-\mu}\int_{r_1}^{r}\alpha t(r)rdr+C_1 r+\frac{C_2}{r}, \quad (7)$$

where $C_1=\frac{(1+\mu)(1-2\mu)}{1-\mu}\frac{1}{r_2^2-r_1^2}\int_{r_1}^{r_2}\alpha t(r)rdr-\mu\varepsilon_z$;

$$C_2=\frac{1+\mu}{1-\mu}\frac{r_1^2}{r_2^2-r_1^2}\int_{r_1}^{r_2}\alpha t(r)rdr .$$

After calculating the integrals in (7), we obtain:

$$u=\frac{(1+\mu)}{r(1-\mu)}\frac{\alpha T\left(3r_2r^2-2r^3-3r_1^2r_2+2r_1^3\right)}{6(r_2-r_1)}+C_1 r+\frac{C_2}{r}, \quad (8)$$

where $C_1=\frac{(1+\mu)(1-2\mu)}{6(1-\mu)}\frac{\alpha T\left(r_2^3-3r_2r_1^2+2r_1^3\right)}{(r_2-r_1)^2(r_2+r_1)}-\mu\varepsilon_z$;

$$C_2=\frac{(1+\mu)r_1^2}{(1-\mu)}\frac{\alpha T\left(r_2^3-3r_2r_1^2+2r_1^3\right)}{(r_2-r_1)^2(r_2+r_1)}.$$

The calculation confirms that ε_z does not depend on the radius; it was also established that the value $\mu\varepsilon_z = 4.59\cdot10^{-7}$ is three orders of magnitude smaller than $C_1 = const = 2.31\cdot10^{-4}$.

Fig. 4 shows the distribution of radial displacements along the radius. Analysis of the graph shows that all movements are positive, the minimum and maximum values are:

$$u\big|_{r=r_1}=1.207\cdot10^{-6}\text{ m}, u\big|_{r=r_2}=7.846\cdot10^{-5}\text{ m}.$$

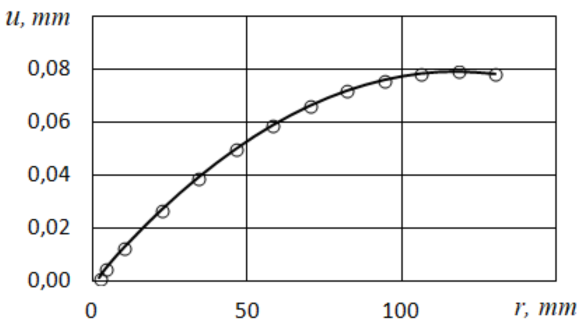

Fig. 4. Distribution of displacements u along the radius

To take into account the thermal expansion of the cylinder at the direction of axis *z*, we will use [5]:

$$N_z^T=\frac{\alpha\Delta Tl}{\frac{l}{EF}+\frac{1}{C}}, \quad (9)$$

where $\Delta T=\frac{t_1+t_2}{2}=1250\ ^o\text{C}$, $l=1.09$ m, cross-sectional area is $F=0.053\text{ m}^2$, current supply spring stiffness is $C=2\cdot10^{\ 4}\text{H/m}$. With such data, we get that the axial force has a very insignificant value $N_z^T=8.175$ H, which can be ignored in the future.

Safety factor:

$$\eta=\frac{|\sigma_+|_B}{\sigma_i}=\frac{110}{27.7}=3.97>1.$$

2. STRESS-STRAIN STATE OF A CYLINDER MADE OF CFC DUE TO POSSIBLE SHRINKAGE OF THE FIBERS

When creating high-temperature composites, there is a rule: "the temperature of the final processing of the fiber should not be less than the working temperature of the composite". This is due to possible changes in the structure of the fibers under the influence of high temperatures, leading to shrinkage. Also, a similar effect can be observed during prolonged exposure of the fibers at temperatures lower than but close to the final processing temperature, or may be due to radiation exposure, etc [6]. For products with a diameter of about 260 mm, the shrinkage is $\delta = 0.5...1\% = 0.65...1.3$ mm.

Let us find out the stress-strain state of the cylinder, which is caused by external pressure p_2, which causes displacement on its outer surface $\delta = 0.5\% = 0.65$ mm. To do this, we write an expression for displacements on the outer surface of the cylinder under external pressure [1] and equate it to δ:

$$u=\frac{(-p_2)r_2}{E\left(r_2^2-r_1^2\right)}\left[(1-\mu)r_2^2+(1+\mu)r_1^2\right]=\delta. \quad (10)$$

From here we find that

$$p_2=\frac{\delta}{A}=\frac{-0.65\cdot10^{-3}}{-5.85\cdot10^{-12}}=111.05\text{ MPa},$$

where

$$A=\frac{-r_2}{E\left(r_2^2-r_1^2\right)}\left[(1-\mu)r_2^2+(1+\mu)r_1^2\right]=-5.85\cdot10^{-12}\,\frac{\text{m}^3}{\text{H}}.$$

For a cylinder loaded with external pressure $p_2 = 111.05$ MPa, the components of SSS are determined by formulas [1]:

$$\sigma_r^{sh}=\frac{p_2r_2^2}{r_2^2-r_1^2}\left[\frac{r_1^2}{r^2}-1\right]; \quad (11)$$

$$\sigma_\theta^{sh}=-\frac{p_2r_2^2}{r_2^2-r_1^2}\left[\frac{r_1^2}{r^2}+1\right]. \quad (12)$$

The results of calculations according to formulas (11), (12) are shown in Fig. 5. The greatest stresses that arise act on the inner surface:

$$\sigma_\theta=-222.15\text{ MPa}.$$

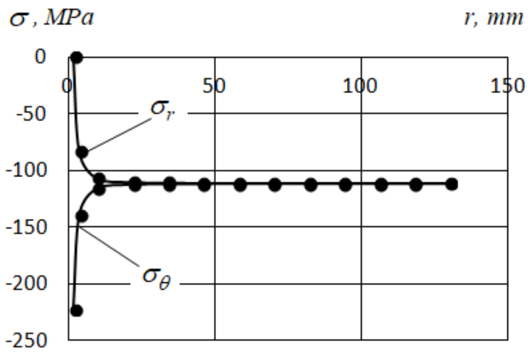

Fig. 5. Stress distribution due to shrinkage along the radius of the cylinder

Let us find out the ultimate stress in an CAC element with dimensions of 1.5x2.5 mm, if the tensile strength of the fiber $[\sigma_+]_B$ = 2 GPa, the fiber thread contains

$N = 2\times 6\cdot 10^3$ microfibers, and the diameter of one microfiber is $d_м = 6\cdot 10^{-6}\ m$. Then the area of one microfiber:

$$S_m = \frac{\pi d_m^2}{4} = \frac{\pi\cdot\left(6\cdot 10^{-6}\right)^2}{4} = 28.27\cdot 10^{-12}\ \mathrm{m}^2\cdot$$

Area of carbon fiber thread is:

$$S = NS_m = 12\cdot 10^3 \times 28.27\cdot 10^{-12} = 3.39\cdot 10^{-7}\ \mathrm{m}^2\cdot$$

The breaking force of the thread is:

$$P = \left[\sigma_+\right]_B S = 2\cdot 10^9 \times 3.39\cdot 10^{-7} = 678.6\ N \cong 67.8\ \text{кг}\cdot$$

In fact, microfibers perceive the load differently, so the thread will be able to withstand less force.

Let's calculate the ultimate stress in the CFC element without taking into account the bearing capacity of the pyrocarbon matrix:

Area of the element measuring 1.5 x 2.5 mm is:

$$S_c = 1.5\cdot 10^{-3}\times 2.5\cdot 10^{-3} = 3.75\cdot 10^{-6}\ \mathrm{m}^2\cdot$$

Limit stress in the element:

$$\sigma_c = \frac{P}{S_c} = \frac{678.6}{3.75\cdot 10^{-6}} = 180.96\ \mathrm{MPa}\cdot$$

Thus, within the accepted assumptions, the coefficient of safety factor for shrinkage:

$$\eta_{sh} = \frac{\sigma_c}{\sigma_{eqvIII}} = \frac{180.96}{222.15} = 0.8145 < 1.$$

When $\delta = 1.0\ \% = 1.3\ \mathrm{mm}$ we get:

$p_2 = 222.1\ \mathrm{MPa}$, $\sigma_\theta = -444.3\ \mathrm{MPa}$, $\eta = 0.407 < 1$.

In physical terms, a safety factor of less than 1 means the possible destruction of the material. It is this destruction that was repeatedly observed during the pyrocompaction of preforms made from fibers with a low (1100…1500 °C) final processing temperature.

3. COMBINED EFFECT OF TEMPERATURE GRADIENT AND SHRINKAGE

The results obtained indicate that the maximum stresses on the inner surface of the cylinder arise due to the possible shrinkage of the fiber $\sigma_\theta^{sh} = -444.30\ \mathrm{MPa}$ and ten times exaggerate the temperature stresses in absolute value $\sigma_\theta^T = -22.05\ \mathrm{MPa}$. Possible shrinkage of the fibers and uneven heating occur simultaneously, according to the principle of superposition:

$$\sigma_\theta^{sh} + \sigma_\theta^T = -444.3 - 22.05 = -466.35\ \mathrm{MPa}\cdot \qquad (13)$$

Surround stresses on the inner surface after shrinkage increase with additional temperature stresses by 5%. Remaining safety factor:

$$\eta = \frac{\sigma_к}{\left|\sigma_\theta^{sh} + \sigma_\theta^t\right|} = \frac{180.96}{466.35} = 0.386 < 1.$$

Let's get the value of the safety factor for different amounts of shrinkage in the range from 0.1 to 1%. To do this, we calculate the external pressure p_2, stress σ_θ^{sh}, $\sigma_\theta^{sh} + \sigma_\theta^t$ and safety factor η using formula (13). The calculation results are shown in Table 2 and Fig. 6.

Table 2

Total safety factor under the influence of temperature gradient and shrinkage

δ, %	δ, mm	p_2, MPa	σ_θ^{sh}, MPa	$\sigma_\theta^{sh} + \sigma_\theta^t$, MPa	η
0.1	0.13	22.22	44.28	66.33	2.728
0,2	0.26	44.44	88.55	110.60	1.636
0.3	0.39	66.67	132.83	154.88	1.168
0.4	0.52	88.89	177.10	199.15	0.909
0,5	0,65	111.11	221.38	243.43	0.743
0.6	0.78	133.33	265.65	287.70	0.629
0.7	0.91	155.56	309.93	331.98	0.545
0.8	1.04	177.78	354.20	376.25	0.481
0.9	1.17	200.00	398.48	420.53	0.430
1.0	1.30	222.22	442.75	464.80	0,389

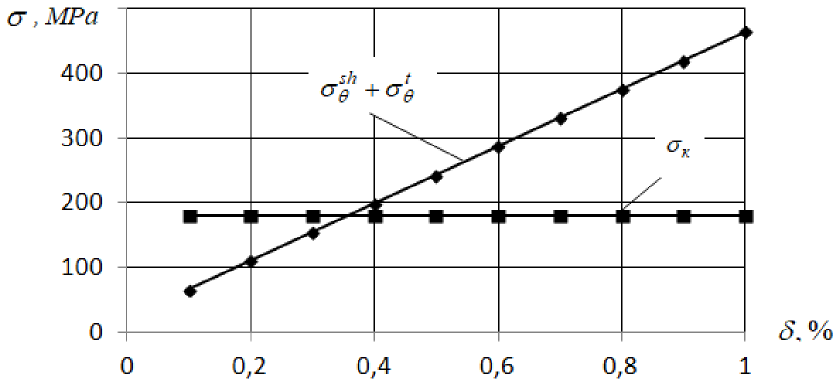

Fig. 6. Stress under the combined action of the temperature gradient and shrinkage

CONCLUSIONS

In this paper, the stress-strain state of a cylinder made of a carbon-carbon composite material was studied for the case of a temperature gradient from the center to the outer surface according to a linear law from 1500 to 1000 °C. The mathematical model of deformation is constructed using the solution of the Lame problem, which is common for the case of a cylinder fixed at the ends. It has been established that the safety factor for these physical characteristics is close to 4.

The problem of modeling the process of occurrence of residual stresses in the process of thermogradient gas-phase compaction of the CFC is also very relevant.

The safety factor of the cylinder with possible shrinkage of the fibers and the temperature gradient is found, which is less than unity and is equal to 0.386.

Surrounding stresses due to shrinkage at the most stressed area – on the inner surface of the cylinder are increased by temperature stresses by 5%.

These calculations explain the destruction of individual preforms during pyrocompaction, which we observed earlier. Also, this effect must be taken into account when creating carbon composite materials that work under irradiation.

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АНАЛІЗ МІЦНОСТІ ЦИЛІНДРА З ВУГЛЕЦЬ-ВУГЛЕЦЕВОГО КОМПОЗИТНОГО МАТЕРІАЛУ ВНАСЛІДОК ТЕМПЕРАТУРНИХ ДЕФОРМАЦІЙ І УСАДКИ

М.В. Мельтюхов, І.В. Гурін, Я.В. Кравцов

За допомогою рішення задачі Ляме у роботі проведено аналіз напружено-деформованого стану (НДС) циліндра з вуглець-вуглецевого композитного матеріалу (ВВКМ) для випадку закріплених торців. Показано, що коефіцієнт запасу по термонапруженням близький до чотирьох. Крім того, у роботі проведено інженерний аналіз міцності циліндра з ВВКМ внаслідок можливої усадки волокон. Показано, що коефіцієнт запасу міцності дорівнює 0,386 і може бути причиною руйнування ВВКМ під час виготовлення та експлуатації.