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DYNAMICS OF DEFORMATION FORCES IN SINGLE-PARTICLE SPECTRA OF ODD 2s1d-SHELL NUCLEI

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With help of our newly-developed evolutionary model-independent approach we have found that with the increase in mass number of all studied odd 2s1d-shell nuclei ^{23}Na, ^{27}Al, ^{31}P, and ^{35,37}Cl the dominant deformation of the shape of these nuclei in low-laying single-particle states changes from quadrupole to hexadecapole and further to hexacontatetrapole. The single states and the continuous sets of states with abnormally weak deformation are found. The points of shape phase transitions with a change in the multipolity of deformation are found in the single-particle spectrum of ^{37}Cl nucleus.

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INTRODUCTION

It is well known that the smaller the scale on which the surface of the nucleus is substantially distorted (i.e., the more the multipolity of deformation of nuclear surface) and the greater the magnitude of this distortion, the smaller the scale on which the collective forces act, which give rise to a stable deformation of nuclei in the ground and low-laying single-particle excited states, and the greater the magnitude of these forces (see, e.g., Refs. [1, 2] or any textbook on nuclear physics).

To study the dynamics of the properties of these forces with the increase of nuclear mass number and/or the single-particle excitation energy in odd 2s1d-shell nuclei, we apply our newly-developed evolutionary model-independent approach [3], using which it is possible to extract the angular dependence (deformation) of the potential of the self-consistent field of odd nuclei, both in the ground and single-particle excited states, directly from experimental data on energies, spins and parities of the ground and low-laying single-particle excited states and the measured probabilities of electromagnetic transitions between them.

Our first preliminary results obtained with help of new method show that the shape of odd 2s1d-shell nuclei appears to be much more complicated than it might be expected for light nuclei and that the hexadecapole or even hexacontatetrapole deformations could be not just a small correction to the quadrupole deformation [4–6]. Moreover, some studied odd 2s1d-shell nuclei have states or sequences of states in which nuclei are significantly less deformed as compared to other states in their single-particle spectra, indicating the existence of shape phase transition: the change of the state of the only one nucleon – the valence proton the spin and parity of which determine the spin and parity of the whole nucleus – causes the shape phase transition from the high-symmetry phase – spherical nucleus state (states) – to the low-symmetry phase – deformed nucleus state (states).

Presently, the dynamics of nuclear shape caused by a change in the number of nucleons in the nucleus is mainly studied (see, e.g., [7–9]). However, the same nucleus in different single-particle states can have different shapes too and, in principle, the shape phase transition can be caused not only by changing the number of nucleons in the nucleus, but also by changing the states of nucleons in the nucleus. Regardless of the method of calculation, the shape of the nucleus in the single-particle state strongly influences its wave function. The wave functions of the initial and final states of the nucleus largely determine the probability of an electromagnetic transition between them. Therefore, the experimentally observed probabilities of electromagnetic transitions are a valuable source of information about the shape of the nucleus in various single-particle states.

The generalized nucleus model (in the form of Nilsson model [10, 11]) allows, in principle, to calculate the equilibrium deformation of the nucleus in any single-particle state. In fact, Nilsson model with spin-orbit coupling describes the sequence of shape phase transitions because it predicts spherically symmetric equilibrium shape of the equipotential surface of a nucleus if all states with the shell number N and the total momentum I are occupied. However, the probabilities of electromagnetic transitions can only be calculated between single-particle states with the same deformation. In a number of works [12–18], the modification of Nilsson model was proposed, in which the deformation of the nucleus is considered as a dynamic parameter, that is, the initial and final states are assumed to have different deformations. Thus, during the transition, the state of core nucleons changes alongside the state of the odd nucleon.

The modified Nilsson model enabled to calculate the probabilities of electromagnetic transitions between single-particle states, taking into account their different deformations. The assumption of the dynamic nature of the deformation of single-particle states of odd 2s1d-shell nuclei significantly reduced the discrepancy between the measured and calculated probabilities of some $E2$-transitions. However, it appeared impossible to adequately describe the entire set of experimental data, including energies, spins and parities of the ground and single-particle excited states, as well as the probabilities of both E- and M-transitions between them.

That is why we have devised a procedure that is able to extract the angular dependence (deformation) of the potential of the self-consistent field of the nucleus in the ground and single-particle excited states directly from experimental data on energies, spins, and parities of the states of nuclei, as well as the measured probabilities of electromagnetic transitions between these states [3]. The goal of this procedure is to study the dynamics of the properties of the collective forces which give rise to a stable deformation of odd 2s1d-shell nuclei in the ground and low-laying single-particle excited states with the increase of nuclear mass number and/or the single-particle excitation energy and to search for possible shape phase transitions in these nuclei.

1. DEFORMED-SHELL-MODEL SINGLE-PARTICLE HAMILTONIAN

We restrict ourselves to the case of an axially symmetric nucleus with an additional symmetry plane perpendicular to the symmetry axis. We chose a single-particle harmonic-oscillator Hamiltonian with the spin-orbit interaction (see, e.g., Refs. [10,11]) in the form:

$$H = \hbar\omega(H_0 + H_1),\ H_0 = (-\Delta + r^2)/2,$$
$$H_1 = -r^2\varphi(\theta)/2 - 2\kappa(\mathbf{l}\cdot\mathbf{s})[1-\varphi(\theta)], \tag{1}$$

where r is the reduced coordinate; $1/\sqrt{1-\varphi(\theta)}$ is the reduced radius of the equipotential surface of the nuclear potential; θ is the polar angle, $\theta \in [0;\pi/2]$; $\varphi(\theta)$ is the function that describes the shape of the equipotential surface, $\varphi(\pi-\theta) \equiv \varphi(\theta)$, $d\varphi/d\theta \equiv 0$ at the points $\theta = 0$ and $\theta = \pi/2$; $r^2\varphi(\theta)$ is the coupling of the particle with the symmetry axis; $(\mathbf{l}\cdot\mathbf{s})$ is the spin-orbit interaction; $(\mathbf{l}\cdot\mathbf{s})\varphi(\theta)$ is the coupling of the spin-orbit interaction with the symmetry axis; $\hbar\omega = 41A^{-1/3}(1+\varepsilon)$ MeV is the energy scale; $A = N + Z$ is the nucleus mass number; N and Z are the numbers of neutrons and protons in the nucleus; ε takes into account the deviation of the energy scale from its simple estimate.

2. PROBABILITIES OF ELECTROMAGNETIC TRANSITIONS BETWEEN SINGLE-PARTICLE STATES WITH DIFFERENT DEFORMATIONS

To determine the matrix element of the single-particle multipole operator

$$\mathsf{M} = \sum_{s=1}^{A} \hat{t}_s, \tag{2}$$

we consider two sets of occupied single-particle states calculated using the Hamiltonian (1) with two different functions $\varphi(\theta)$, which form two Slater determinants $\Psi\{u_j\}$ and $\Psi'\{v_i\}$

$$\{u_j\},\ j = 1,\ldots,N,N+1,\ldots,N+Z; \tag{3}$$
$$\{v_i\},\ i = 1,\ldots,N,N+1,\ldots,N+Z.$$

The matrix element of M, taken between $\Psi\{u_j\}$ and $\Psi'\{v_i\}$, is equal to

$$(\Psi',\mathsf{M}\Psi) = \sum_{s=1}^{A} \left|\mathbf{M}^s\right|, \tag{4}$$

where the elements of determinants $\left|\mathbf{M}^s\right|$ are as follows

$$\mathbf{M}^s_{ij} = \begin{cases} (v_i, \hat{t}_s u_j),\ i = s, \\ (v_i,\ \ u_j),\ i \neq s. \end{cases} \tag{5}$$

The reduced electric and magnetic multipole transition probabilities between the initial and final states with IK and $I'K'$, where I and K are the total momentum and its projection take the form ($\lambda < K + K'$) [12–14]:

$$B(E\lambda; IK \to I'K') = e^2\left[1 + (-1)^\lambda \frac{Z}{A^\lambda}\right] \times$$
$$\left(\frac{\hbar}{m\omega}\right)^\lambda \frac{2\lambda+1}{4\pi} \left|\langle I\lambda KK'-K \mid I'K'\rangle\right|^2 \left|\left|\mathbf{N}\right| \sum_{s=N+1}^{N+Z} \left|\mathbf{Z}^s_E\right|\right|^2; \tag{6}$$

$$B(M\lambda; IK \to I'K') = \left(\frac{e\hbar}{2mc}\right)^2 \left(\frac{\hbar}{m\omega}\right)^{\lambda-1} \frac{2\lambda+1}{16\pi} \times \tag{7}$$
$$\left|\langle I\lambda KK'-K \mid I'K'\rangle\right|^2 \left|\left|\mathbf{Z}\right| \sum_{s=1}^{N} \left|\mathbf{N}^s\right| + \left|\mathbf{N}\right| \sum_{s=N+1}^{N+Z} \left|\mathbf{Z}^s_M\right|\right|^2;$$

$$\mathbf{N}_{ij} = (v_i, u_j) = \delta_{N_i N_j} \sum_{l\Lambda} a^i_{l\Lambda} a^j_{l\Lambda};$$
$$\mathbf{N}^s_{ij} = \begin{cases} G^{ij}_{M\lambda},\ i = s, \\ \mathbf{N}_{ij},\ i \neq s, \end{cases} \tag{8}$$
$$i, j = 1,\ldots,N;$$

$$\mathbf{Z}_{ij} = (v_i, u_j) = \delta_{N_i N_j} \sum_{l\Lambda} a^i_{l\Lambda} a^j_{l\Lambda};$$
$$\mathbf{Z}^s_{E(M)ij} = \begin{cases} G^{ij}_{E(M)\lambda},\ i = s, \\ \mathbf{Z}_{ij},\ i \neq s, \end{cases} \tag{9}$$
$$i, j = N+1,\ldots,N+Z,$$

where $a^i_{l\Lambda}$ and $a^j_{l\Lambda}$ are the coefficients of decomposition of the functions v_i and u_j in the basis of the spherical harmonic oscillator [11]; N_i and N_j are the principal quantum numbers of states i and j; l and Λ are the angular momentum and its projection; $G^{ij}_{E(M)\lambda}$ correspond to the quantities $G_{E(M)\lambda}$ calculated in Ref. [11].

3. EVOLVING NUCLEAR SHAPES VIA EVOLUTIONARY ALGORITHM

We chose the function that describes the shape of the equipotential surface of the nuclear potential in i-th single-particle state of the nucleus in the following form:

$$\varphi^{(i)}(\theta) = \sum_{k=0}^{\infty} \varphi^{(i)}_{2k} \cos(2k\theta). \tag{10}$$

The values of the weight parameters $\left\{\varphi^{(i)}_{2k}\right\}$ (i=0 marks the ground state and i=1, ..., n mark the single-particle excited states) are determined independently for each state of the nucleus. Additional requirements imposed on the weight parameters $\left\{\varphi_{2k}^{(i)}\right\}$ are their minimum number for each level and their minimum value that ensures a good description of experimental data. Note that both ε and $\left\{\varphi_0^{(i)}\right\}$ affect the radius of spherical equipotential surface of the potential in the Hamiltonian (1). Thus, to avoid overestimation, we set $\left\{\varphi_0^{(0)}\right\}=0$.

To determine the number and values of the weight parameters $\left\{\varphi_{2k}^{(i)}\right\}$, an approach based on the use of an evolutionary algorithm [19, 20] to fit the calculated observables to the measured ones was developed.

Our evolutionary approach operates on a population of N individuals. Each individual is a set of parameters $\left(\varepsilon,\kappa,\left\{\varphi_{2k}^{(i)}\right\}\right)$, i=0, ..., n, k=0, ..., m. Fitness of each individual reflects the quality of data fitting provided by the individual's parameters. Using the mutation operation, the algorithm evolves the initial population of poorly fitted individuals to the population of the well-fitted ones.

Analysis of experimental data begins with the assumption of the quadrupole deformation of the shapes of nucleus in the ground and single-particle excited states [the terms with k=0, 1 are left in Eq.(10)]. If the desired quality of data fitting is not achieved within this assumption, then the hexadecapole deformation comes into play [the term with k=2 is added in Eq.(10)], and so forth. After the number of terms in Eq.(10) is determined, the contribution of the last term found (say, k=2) is smoothly consistently reduced, preserving the desired quality of data fitting with that.

4. DYNAMICS OF SHAPES OF ODD 2s1d-SHELL NUCLEI IN LOW-LAYING SINGLE-PARTICLE SPECTRA

Fig.1 shows the reduced radius of the equipotential surface of the nuclear potential $1/\sqrt{1-\varphi(\theta)}$ (a) and its angular part $1/\sqrt{1-\varphi(\theta)}-1/\sqrt{1-\varphi_0}$ (b) as functions of angle, found for ^{23}Na nuclei with help of our approach, while Table1 presents the respective values of parameters of decomposition (10) (g.s. and 1– 4 e.s. denote ground and first excited states). Experimental data were taken from Refs. [21, 22].

Table 1

The values of parameters of decomposition (10) found for ^{23}Na nuclei (g.s. and 1– 4 e.s. denote ground and first excited states)

Parameter	$\varphi_0^{(i)}$	$\varphi_2^{(i)}$	$\varphi_4^{(i)}$
g.s.	0.00e-0	5.00e-3	3.71e-3
1 e.s.	6.46e-3	3.40e-2	1.23e-2
2 e.s.	-7.65e-3	6.37e-5	2.95e-5
3 e.s.	6.67e-2	2.56e-1	1.31e-1
4 e.s.	5.62e-3	7.10e-2	4.07e-2

We have achieved very good consistency between calculated and measured observables for the ^{23}Na nucleus. The shape of the ^{23}Na nucleus in the ground and in the first low-laying excited states has a dominant quadrupole deformation. The ground and first two single-particle excited states of ^{23}Na nucleus have small deformation as compared to the third excited state.

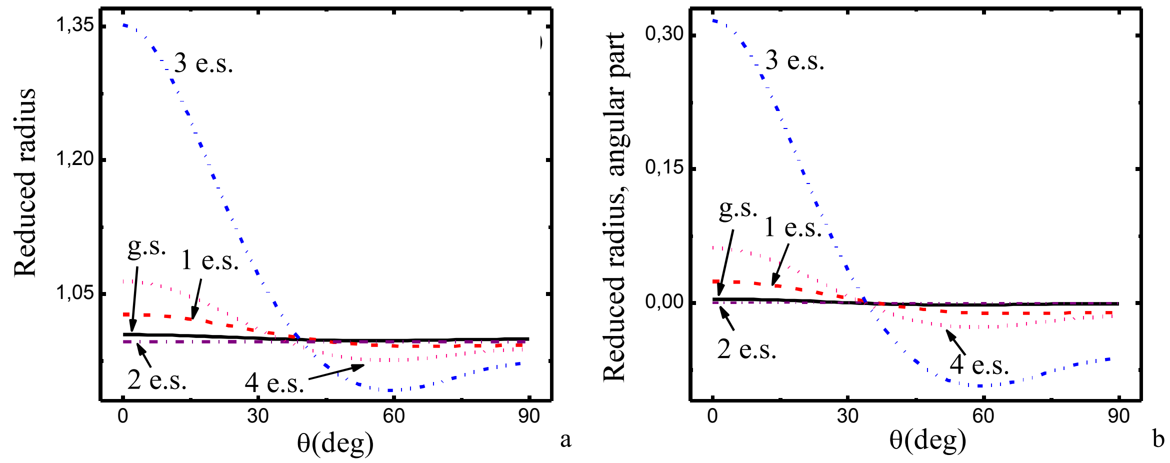

Fig. 1. Shapes for five single-particle states of ^{23}Na nucleus, calculated by our procedure. Radius of the equipotential surface of the nuclear potential (a) and its angular part as functions of angle (b). Curves marked as g.s. correspond to the ground state, curves marked as 1–4 e.s. present four low-laying excited states

This may indicate the continuous spherical phase of the ^{23}Na nucleus formed by these states and the point of phase transition into a non-spherical state (third and fourth excited states). The nucleons of the ^{23}Na nucleus do not change their characteristics, and the only particle that changes its quantum state is the valence proton, the quantum numbers of which determine the spin and parity of the ^{23}Na nucleus, both in the ground and in the excited single-particle states.

Fig. 2 shows the reduced radius of the equipotential surface of the nuclear potential (a) and its angular part (b) as functions of angle found for ^{27}Al nuclei with help of our approach, while Table 2 presents the respective values of parameters of decomposition (10) (g.s. and 1–3 e.s. denote ground and first excited states). Experimental data were taken from Refs. [21, 23].

Table 2

The values of parameters of decomposition (10) found for ^{27}Al nuclei (g.s. and 1–3 e.s. denote ground and first excited states)

Parameter	$\varphi_0^{(i)}$	$\varphi_2^{(i)}$	$\varphi_4^{(i)}$
g.s.	0.00e-0	-3.11e-2	-7.40e-2
1 e.s.	1.16e-2	-3.04e-3	-1.50e-2
2 e.s.	5.76e-5	1.58e-2	-1.14e-2
3 e.s.	0.00e-0	3.50e-2	2.58e-3

We have achieved very good consistency between the calculated and measured observables for the ^{27}Al nucleus. The hexadecapole deformation dominates in the ground state of the ^{27}Al nucleus. Then, with the increase of the excitation energy, its contribution decreases and almost vanishes for the third excited state.

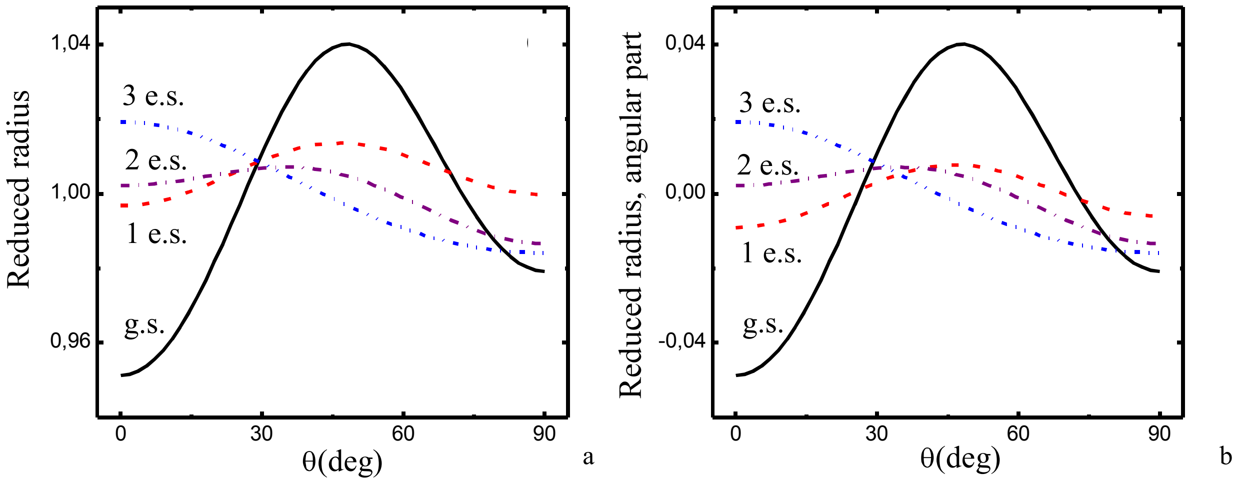

Fig. 2. Shapes for four single-particle states of ^{27}Al nucleus, calculated by our procedure. Radius of the equipotential surface of the nuclear potential (a) and its angular part as functions of angle (b). Curves marked as g.s. correspond to the ground state, curves marked as 1–3 e.s. present four low-laying excited states

The contribution of the quadrupole deformation is small for the ground and first excited states, but it increases with the increase of the excitation energy and becomes dominant for the third excited state of the ^{27}Al nucleus. All studied states of the ^{27}Al nucleus are abnormally weakly deformed, which may point on the existence of the continuous spherical phase of the ^{27}Al nucleus formed by these states. Starting from the first excited state, the core nucleons of the ^{27}Al nucleus do not change their characteristics and the only particle that changes its quantum state is the valence proton, the quantum numbers of which determine the spin and parity of the ^{27}Al nucleus, both in the ground and in the excited single-particle states.

Fig. 3 shows the reduced radius of the equipotential surface of the nuclear potential (a) and its angular part (b) as functions of angle found for ^{31}P nuclei with help of our approach, while Table 3 presents the respective values of parameters of decomposition (10) (g.s. and 1–3 e.s. denote ground and first excited states). Experimental data were taken from Refs. [21, 24].

We have achieved very good consistency between the calculated and measured observables for the ^{31}P nucleus.

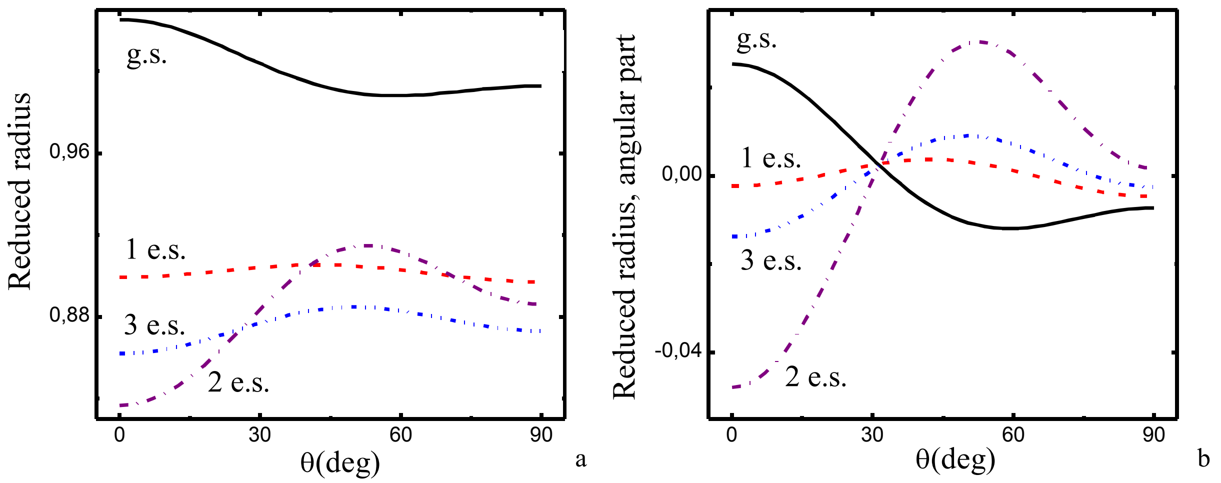

Fig. 3. Shapes for four single-particle states of ^{31}P nucleus, calculated by our procedure. Radius of the equipotential surface of the nuclear potential (a) and its angular part as functions of angle. Curves marked as g.s. correspond to the ground state (b), curves marked as 1–3 e.s. present four low-laying excited states

Table 3

The values of parameters of decomposition (10) found for ^{31}P nuclei (g.s. and 1–3 e.s. denote ground and first excited states)

Parameter	$\varphi_0^{(i)}$	$\varphi_2^{(i)}$	$\varphi_4^{(i)}$
g.s.	0.00e-0	7.10e-2	4.03e-2
1 e.s.	-2.31e-1	6.18e-3	-1.80e-2
2 e.s.	-2.87e-1	-9.50e-2	-9.00e-2
3 e.s.	-3.04e-1	-2.40e-2	-3.50e-2

Except for the ground state that has a quadrupole deformation, first excited states of the ^{31}P nucleus are characterized by the dominating hexadecapole deformation. Thus, with the increase in the nuclear mass number, the nuclei in the middle of the 2s1d-shell change the deformation of their first low-laying excited single-particle states from the dominant quadrupole to the dominant hexadecapole.

All studied states of the ^{31}P nucleus are abnormally weakly deformed, which may point on the existence of the continuous spherical phase of the ^{31}P nucleus formed by these states. Thus, the ^{27}Al and ^{31}P nuclei, which occupy the 2s1d-shell middle, have forms very close to spherical both in the ground and in the low-laying single-particle excited states. This conclusion contradicts to the common belief that the nuclei in the shell middle are deformed the most. Some of the core nucleons of the ^{31}P nucleus change their characteristics with the increase of the excitation energy. Spin and parity of the ^{31}P nucleus both in the ground and in the excited single-particle states is determined by the quantum state of the valence proton.

Fig. 4 shows the reduced radius of the equipotential surface of the nuclear potential (a) and its angular part (b) as functions of angle found for ^{35}Cl nuclei with help of our approach, while Table 4 presents the respective values of parameters of decomposition (10) (g.s. and 1–3 e.s. denote ground and first excited states).

Note that in order to replicate well the experimental data on ^{35}Cl nucleus it was not necessary to account for the contribution of the basis function $\cos(4\theta)$ in the decomposition (10). Thus, the values of the parameters $\varphi_4^{(i)}$ were set to zero. Experimental data were taken from Refs. [21, 25].

We have achieved very good consistency between the calculated and measured observables for the ^{35}Cl nucleus. The contribution of hexacontatetrapole deformation [$\cos(6\theta)$] is crucial, while the contribution of hexadecapole deformation [$\cos(4\theta)$] is absent. This conclusion is very important because the existing nuclear models do not predict and do not account for the deformations, whose multipolity is significantly more complicated than quadrupole [$\cos(2\theta)$] for lightweight odd 2s1d-shell nuclei.

The deformation of the ^{35}Cl nucleus in its ground state is relatively small, which may point on its spherical form. The first and especially second excited states are strongly deformed with dominant hexacontatetrapole deformation. The contributions of quadrupole and hexacontatetrapole deformations to the form of the third excited state are approximately the same. This observation indicates that the change of the state of the only one nucleon – the valence proton the spin and parity of which determine the spin and parity of the ^{35}Cl nucleus – causes the shape phase transition from the high-symmetry phase – spherical ground state – to the low-symmetry phase – deformed excited states.

Table 4

The values of parameters of decomposition (10) found for ^{35}Cl nuclei (g.s. and 1–3 e.s. denote ground and first excited states)

Parameter	$\varphi_0^{(i)}$	$\varphi_2^{(i)}$	$\varphi_6^{(i)}$
g.s.	0.00e-0	-6.96e-2	-3.11e-2
1 e.s.	-5.75e-2	-1.43e-1	-3.22e-1
2 e.s.	-2.03e-2	3.23e-2	-5.20e-1
3 e.s.	-8.93e-2	-1.39e-1	-1.47e-1

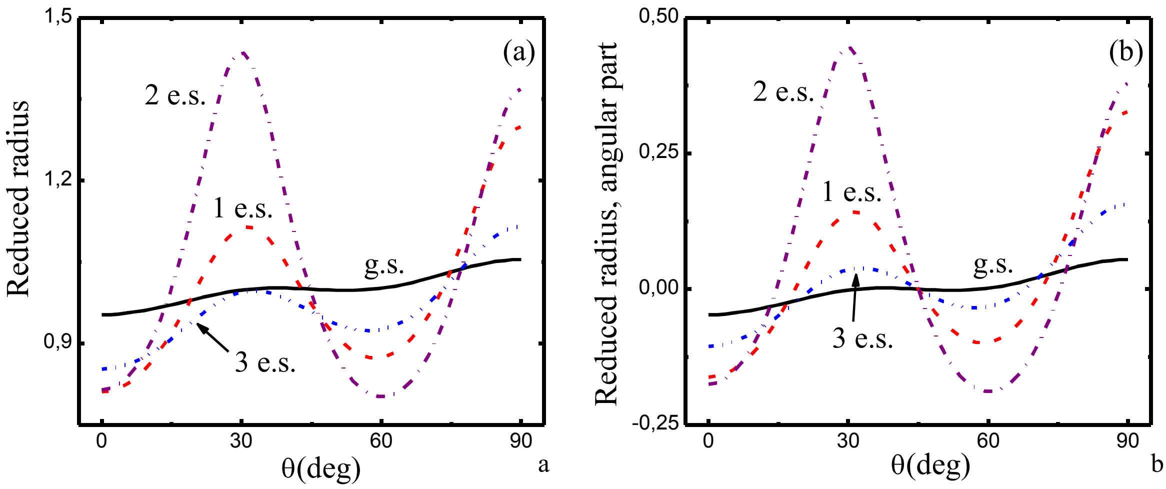

Fig. 4. Shapes for four single-particle states of ^{35}Cl nucleus, calculated by our procedure. Radius of the equipotential surface of the nuclear potential (a) and its angular part as functions of angle (b). Curves marked as g.s. correspond to the ground state, curves marked as 1–3 e.s. present four low-laying excited states

Some of the core nucleons of the ^{35}Cl nucleus change their characteristics with the increase of the excitation energy. Spin and parity of the ^{35}Cl nucleus both in the ground and in the excited single-particle states is determined by the quantum state of the valence proton.

Table 5

Set 1 of the values of parameters of decomposition (10) found for ^{37}Cl nuclei (g.s. and 1–3 e.s. denote ground and first excited states)

Parameter	$\varphi_0^{(i)}$	$\varphi_2^{(i)}$	$\varphi_4^{(i)}$	$\varphi_6^{(i)}$
g.s.	0.00e-0	-2.38e-1	-2.15e-1	1.66e-1
1 e.s.	-3.25e-2	-1.37e-1	7.09e-2	1.12e-1
2 e.s.	-7.54e-2	-3.27e-2	-8.12e-2	-3.64e-1
3 e.s.	-1.08e-2	2.15e-1	2.06e-1	0.00e-0

Table 6

Set 2 of the values of parameters of decomposition (10) found for ^{37}Cl nuclei (g.s. and 1–3 e.s. denote ground and first excited states)

Parameter	$\varphi_0^{(i)}$	$\varphi_2^{(i)}$	$\varphi_4^{(i)}$	$\varphi_6^{(i)}$
g.s.	0.00e-0	-2.80e-1	-2.46e-1	2.23e-1
1 e.s.	3.83e-2	-3.38e-2	2.45e-1	4.37e-1
2 e.s.	2.94e-2	1.59e-1	1.41e-1	0.00e-0
3 e.s.	-9.13e-2	1.91e-2	-3.67e-2	-5.23e-1

Table 7

Set 3 of the values of parameters of decomposition (10) found for ^{37}Cl nuclei (g.s. and 1–3e.s. denote ground and first excited states)

Parameter	$\varphi_0^{(i)}$	$\varphi_2^{(i)}$	$\varphi_4^{(i)}$	$\varphi_6^{(i)}$
g.s.	0.00e-0	-2.50e-2	-2.35e-1	9.10e-2
1e.s.	-4.51e-2	-1.76e-1	1.08e-2	0.00e-0
2e.s.	-6.44e-2	-2.78e-2	-7.23e-2	-3.35e-1
3e.s.	2.43e-2	2.70e-1	2.75e-1	1.46e-1

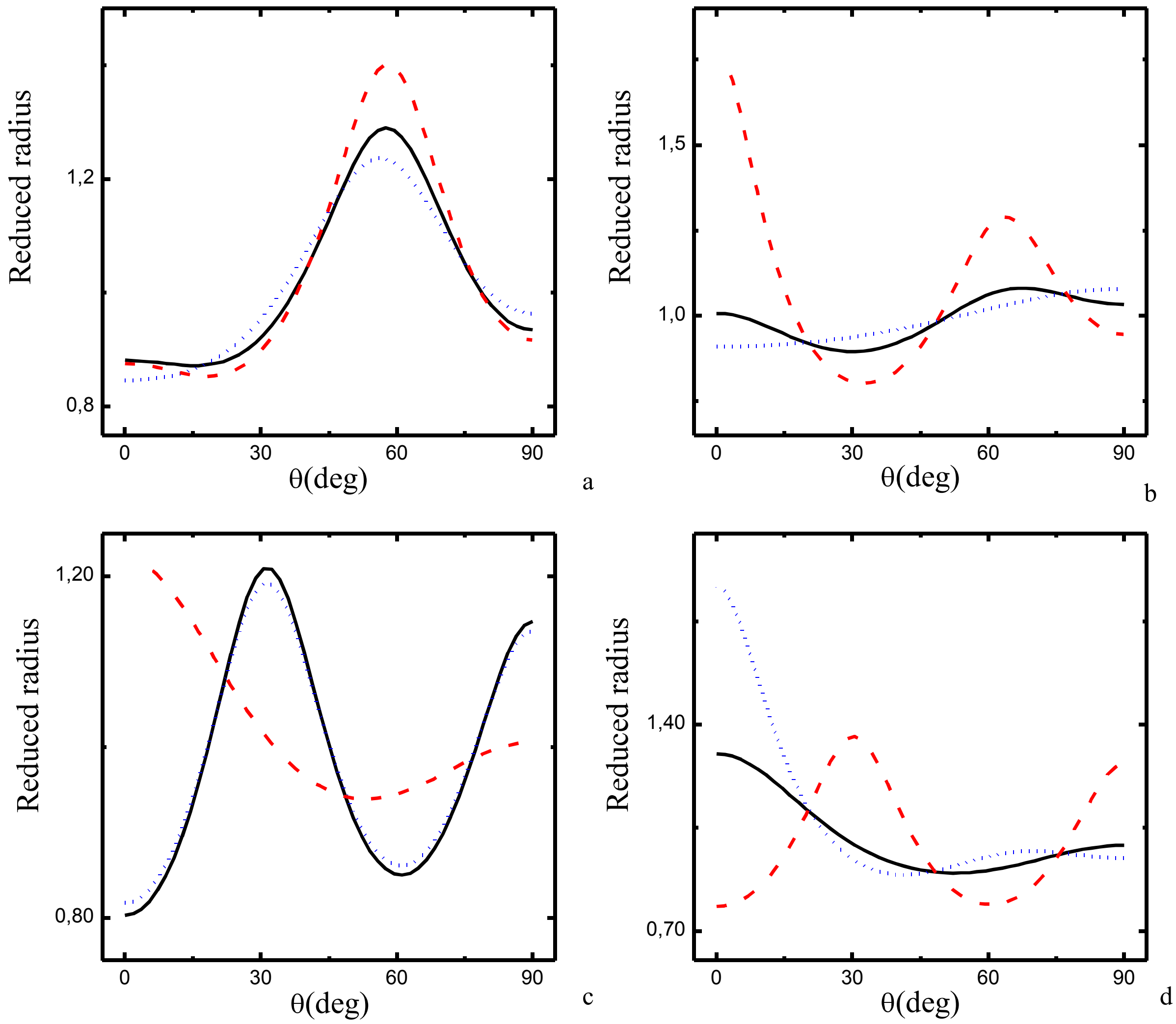

Fig. 5. Radii of the equipotential surface of the nuclear potential as functions of angle for four single-particle states of ^{37}Cl nucleus calculated by our procedure: a – ground state, b–d – 1–3 excited states. Black solid curves are calculated with parameters $\varphi_0^{(i)},\varphi_2^{(i)},\varphi_4^{(i)},\varphi_6^{(i)}$ taken from Table 5. Red dashed curves – calculations with these parameters taken from Table 6. Blue dotted curves – calculations with these parameters taken from Table 7

Very good replication of the experimental data for the ^{37}Cl nucleus [21, 26] has required taking into account the contributions of the quadrupole, hexadecapole and hexacontatetrapole deformations. At the same time, it has appeared that one of the parameters $\varphi_6^{(i)}$ responsible for the contribution of the hexacontatetrapole deformation to the form of the ^{37}Cl nucleus excited states can be set to zero without loss in the quality of data fitting. Tables 5–7 contain three sets of parameters $\varphi_0^{(i)},\varphi_2^{(i)},\varphi_4^{(i)},\varphi_6^{(i)}$ for which we have achieved very good consistency between the calculated and measured observables for the ^{35}Cl nucleus. Figs. 5, 6 show the reduced radii of the equipotential surface of the nuclear potential and their angular parts as functions of angle calculated using parameters $\varphi_0^{(i)},\varphi_2^{(i)},\varphi_4^{(i)},\varphi_6^{(i)}$ given in Tables 5–7, Figs. 5, 6,a show results of calculations for the ground state. Figs. 5, 6,b–d show the same for the three excited states. Black solid curves are the results of calculations with parameters $\varphi_0^{(i)},\varphi_2^{(i)},\varphi_4^{(i)},\varphi_6^{(i)}$ taken from Table 5. Red dashed curves are calculated with these parameters taken from Table 6. Blue dotted curves are calculated with these parameters taken from Table 7.

The appearance or disappearance of the hexacontatetrapole deformation of the form of the ^{37}Cl nucleus in its excited states is caused by the change in the quantum state of the valence proton, the spin and parity of which determine the spin and parity of the ^{35}Cl nucleus both in the ground and in the low-laying single-particle excited states.

Thus, the shape phase transitions with a change in the multipolity of deformation are found in the single-particle spectrum of ^{37}Cl nucleus. Until now, there is no information in the scientific literature on the identification of phase transitions with a change in the multipolity of deformation of the shape of any nuclei.

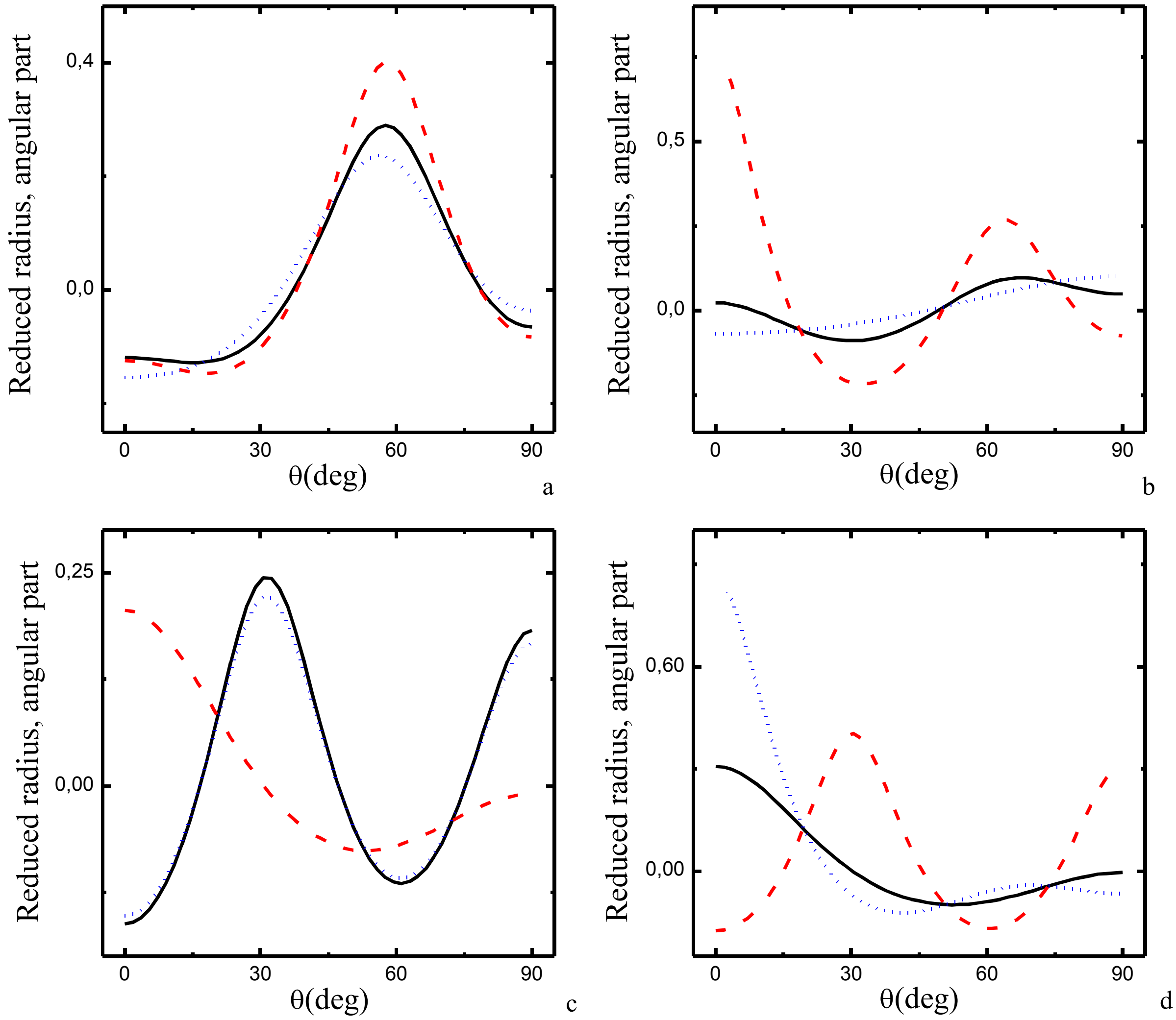

Fig. 6. Radii (angular parts) of the equipotential surface of the nuclear potential as functions of angle for four single-particle states of ^{37}Cl nucleus calculated by our procedure: a – ground state, b–d – 1–3 excited states. Black solid curves are calculated with parameters $\varphi_0^{(i)},\varphi_2^{(i)},\varphi_4^{(i)},\varphi_6^{(i)}$ taken from Table 5. Red dashed curves – calculations with these parameters taken from Table 6. Blue dotted curves – calculations with these parameters taken from Table 7

CONCLUSIONS

Preliminary results of the analysis performed using our evolutionary model-independent approach demonstrate very good consistency between the calculated and measured observables for the odd 2s1d-shell nuclei.

With the increase of mass number of all studied odd 2s1d-shell nuclei ^{23}Na, ^{27}Al, ^{31}P, and ^{35,37}Cl, the dominant deformation of the shape of nuclei in low-laying single-particle states changes from quadrupole to hexadecapole and further to hexacontatetrapole. This conclusion is very important because the existing nuclear models do not involve and do not take into account deformations, whose multipolity is significantly more difficult than quadrupole for light nuclei.

^{27}Al and ^{31}P nuclei, which occupy the 2s1d-shell middle, have forms very close to spherical both in the ground and in the low-laying single-particle excited states. This conclusion contradicts to the common belief that the nuclei in the shell middle are deformed the most.

All the studied odd 2s1d-shell nuclei have states or sequences of states in which nuclei are significantly less deformed in relation to other states in their single-particle spectra.

The change of the state of the only one nucleon – the valence proton the spin and parity of which determine the spin and parity of the whole nucleus – causes the shape phase transition from the high-symmetry phase – spherical nucleus state (states) – to the low-symmetry phase – deformed nucleus state (states).

We find that the spin and parity of all studied odd 2s1d-shell nuclei are determined by the spin and parity of the last odd (valence) proton. At the same time, some of the nucleons of the nucleus core change their characteristics, too. This means that the electromagnetic transitions between the single-particle states of these nuclei are significantly the multi-particle processes.

These conclusions indicate the manifestation of collective forces that are not related to the formation of nuclear shells, but give rise to stable deformation of nuclei in both the ground and low-laying single-particle excited states.

With the increase in the mass number of all studied odd 2s1d-shell nuclei, the radius of action of these forces decreases (multipolity increases), and their magnitude is determined by the state of the odd (valence) nucleon due to the mechanism of phase transition with the change in the shape of the nucleus.

The shape phase transitions with a change in the multipolity of deformation are found in the single-particle spectrum of ^{37}Cl nucleus.

The appearance or disappearance of the hexacontatetrapole deformation of the form of the ^{37}Cl nucleus in its excited states is caused by the change in the quantum state of the valence proton, the spin and parity of which determine the spin and parity of the ^{35}Cl nucleus both in the ground and in the low-laying single-particle excited states.

Further analysis will determine which particular excited state of the ^{35}Cl nucleus is the point of the phase transition with the appearance or disappearance of hexacontatetrapole deformation.

The question of whether the phase transition is possible with the appearance or disappearance of combinations of different multipoles of deformation of the ^{37}Cl nucleus in the single-particle spectrum requires further careful study.

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ДИНАМІКА СИЛ ДЕФОРМАЦІЇ В ОДНОЧАСТИНКОВИХ СПЕКТРАХ НЕПАРНИХ ЯДЕР 2s1d-ОБОЛОНКИ

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За допомогою нещодавно розробленого нами еволюційного безмодельного підходу ми знайшли, що зі зростанням масового числа всіх досліджених непарних ядер 2s1d-оболонки ^{23}Na, ^{27}Al, ^{31}P та ^{35,37}Cl домінуюча деформація форми цих ядер у низьколежачих одночастинкових станах змінюється з квадрупольної до гексадекапольної і далі до гексаконтатетрапольної. Знайдено окремі стани та послідовності станів з аномально малою деформацією. Точки фазових переходів зі зміною мультипольності деформації форми ядра ^{37}Cl знайдені в його одночастинковому спектрі.