

NEW METHODOLOGIES FOR ASSESSING CAVITATION LOAD AND RESISTANCE OF STRUCTURAL MATERIALS AND PROTECTIVE COATINGS IN NUCLEAR POWER INDUSTRY. PART II. POLYFRACTIONAL APPROACH

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A fractional approach to cavitation erosion process as proposed previously by one of the authors is explained. The polyfractional erosion process is considered as a superposition of a series of monofractional ones and described by a differential kinetic equation with a set of parameters constituting “the erosion resistance matrix”. The fundamental methodological considerations are supplemented by describing the most typical applications including prediction of erosion of structural materials under given cavitation load, derivation of cavitation erosion fatigue characteristics, retrieval of effective erosive load basing on performance of reference materials. Finally, extending the methodology onto thin and thick protective coatings is proposed.

1. POLYFRACTIONAL APPROACH TO CAVITATION EROSION MODELLING

The concept of cavitation erosion rate depending essentially on the energy absorbed by the impinged solid and material capability to accommodate resulting internal stresses (cavitation strength) is due to A.P. Thiruvengadam [1]. However, right at the beginning it was recognized that the key problems in modelling cavitation erosion progress using this approach lied in determining the cavitation energy flux efficiency, i.e. the ratio of energy absorbed to the energy delivered, and cavitation strength of the material. It has been and remains still quite clear that both these factors depend essentially on the qualitative features of the delivered energy flux on the one hand and on the widely conceived material properties (including the properties of its surface layer) on the other one.

While the qualitative and quantitative features of cavitation can be described by various characteristics, the development of piezoelectric pressure sensor and digital data processing techniques showed soon numerous advantages of using cavitation signal amplitude histograms for the purpose of cavitation load analyses. In fact it was already in mid sixties that F.G. Hammitt proposed the concept of cavitation threshold representing the cavitation signal amplitude which a single cavitation pulse has to exceed in order to make any harm to the impinged surface. In the next decade the reference to this concept was made by numerous authors including F.G. Hammitt himself [2]. At the same time it was also recognized that the cavitation signal amplitude histogram may be used to estimate the total energy flux delivered to a unit surface area of the impinged surface using the following formula:

$$ME = \sum_{i=1}^N ME_i = \frac{\tau}{2\rho c} \sum_{i=1}^N n_i p_i^2, \quad (1)$$

where n_i is the number of pulses with an average amplitude p_i from the amplitude fraction i delivered per unit

time to the data acquisition system by a pressure sensor of a particular type; ρ and c are the density of the liquid and the sound celerity; $\tau = 10^{-5}$ s is the time constant, often interpreted as the average pulse duration [3, 4].

Formula (1) has been derived from the acoustic model of pressure wave power density. In fact, due to a number of reasons explained in reference [5], the *ME* factor represents merely a parameter closely correlated with the cavitation impingement energy flux density rather than the realistic value of this physical quantity. Furthermore, while the volume loss after cavitation load exposure of given duration is always a rising function of the *ME* factor, the ratio of losses of 2 materials tested in pairs at 2 different cavitation conditions depends apparently on these conditions. This observation was no surprise for the IMP PAN (*Institute of Fluid Flow Machinery of the Polish Academy of Sciences*) team when analyzing the results of the ICET (*International Cavitation Erosion Test*) project in the end of 1990-ies [6]. However, the above analysis only confirmed that cavitation testing under single load conditions may appear insufficient for selection of the most appropriate structural material due to qualitative discrepancy between cavitation load amplitude distribution under field and lab conditions. Therefore, the first author of this text has proposed replacing the scalar sum (1) with the *ME* vector, consisting of N components ME_i corresponding to individual fractions $\{i\}$. At the same time, it has been decided to limit the maximum number of fractions to $N = 6$ with the lower limit duplicated with each fraction number. Furthermore, it has been assumed that the mean erosion depth (*MDE*) curve due to a single fraction load can be described using some analytical function $U = U(\mathbf{R}, Y)$, where $\mathbf{R}$ is the cavitation resistance vector dependent on the material properties, and Y is the surface density of the supplied energy as measured by means of the ME_i energy flux density factor. In order to sum up the contributions of individual fractions, the

principle of differential superposition is used as proposed originally in reference [7] and explained in Fig. 1.

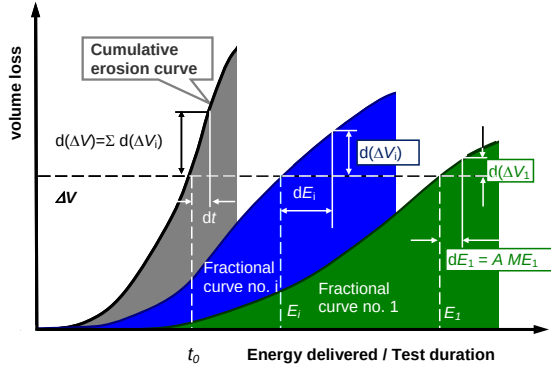


Fig. 1. The course of differential superposition of monofractional curves. A – eroded surface area

As shown in references [5, 6] the superposition principle can be also written down in form of the general equation of polyfractional erosion process kinetics.

$$\frac{dMDE}{dt} = \sum_{i=1}^N ME_i \frac{\partial U}{\partial Y_i} (R_i, Y_i) \Big|_{Y_i = \Theta(R_i, MDE)}, \quad (2)$$

with Θ as the inverse function of U with respect to the density of energy delivered Y .

The authors of this paper use the analytical function

$$U = MDE_0 [\ln(Y/Y_0 + 1)]^\beta, \quad (3)$$

with MDE_0 , Y_0 , and β denoting the subsequent components of the R vector. The above function is based on a proposal put forward in 1985 by L. Sitnik [8] in order to model cumulative erosion curves in a single fraction approach. Its significant advantage is easy analytical reversibility which allows to write down Θ function as

$$\Theta(R; MDE) = Y_0 \left[\exp \left(\frac{MDE}{MDE_0} \right)^{1/\beta} - 1 \right]. \quad (4)$$

Substituting expressions (3) and (4) into equation (2) yields the special equation of polyfractional erosion process kinetics in the following form (5):

$$\frac{d(MDE)}{dt} = \sum_{i=1}^N ME_i \frac{\beta_i MDE_{0i}}{Y_{0i}} \left(\frac{MDE}{MDE_{0i}} \right)^{1-1/\beta_i} \exp \left[- \left(\frac{MDE}{MDE_{0i}} \right)^{1/\beta_i} \right]. \quad (5)$$

Equations (2) and (5) can be easily solved using the fourth order Runge-Kutta procedure. The obvious boundary condition is $MDE(0) = 0$. In most cases ($\beta_i > 1$ for all fractions) one should expect also the zero first derivative condition. As Runge-Kutta algorithm uses the starting point derivative only when establishing the approximate MDE value in the first time step mid, one can avoid the possible numerical problems by calculating this value by simple arithmetic adding the contributions coming from individual fractions according to equation (3).

Repeated solving of any of equations (2) or (5) is a key component of the fractional analysis algorithm allowing to determine the cavitation resistance matrix $R = [R_1, R_2, \dots, R_N]^T$. The basis of the analysis are erosion tests carried out at several different loads described by the ME vectors. The elements of the R matrix are determined by fitting the erosion curves following from equations (2) or (4) to a set of volume loss data. The

relevant numerical procedure consists in minimizing the goal function (6) with respect to the R matrix.

$$S(R) = \sum_{i=1}^K \frac{\gamma_i}{M_i} \sum_{j=1}^{M_i} [MDE_{ij} - CEF(R, ME_i; t_{ij})]^2,$$

where CEF denotes the equation (2) solution for material characterised by erosion resistance matrix R at erosive load state i as defined by the ME_i erosive load vector; K is the number of test series conducted under different test conditions; M_i is the number of experimental points in test series i ; i, j are subscripts referring to the j -th experimental point established in the i -th erosive load state; γ_i weigh coefficient of the load state i .

The details of the procedure are discussed in references [5] and [9] while typical results in form of experimental volume loss and the monofractional MDE vs delivered energy density E curves are shown in Fig. 2.

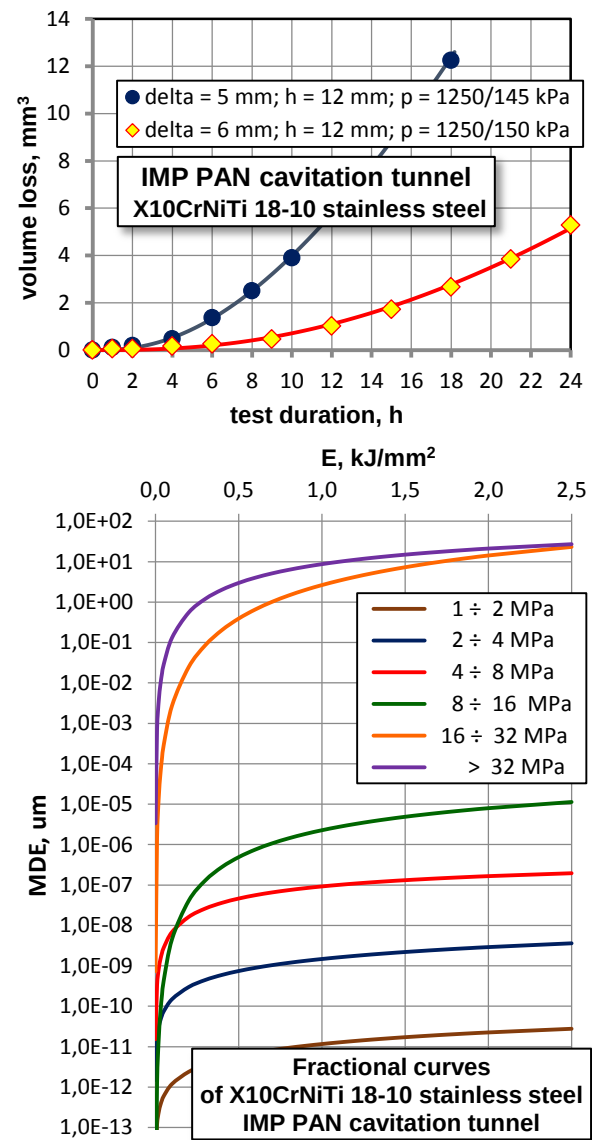


Fig. 2. The X10CrNiTi 18-10 stainless steel volume loss curves following from the IMP PAN double-barricade cavitation tunnel tests under different test parameters and the monofractional $MDE = MDE(E)$ curves resulting from the relevant fractional analysis.

Energy flux density established basing on the PCB 113B pressure transducer signal

2. STRUCTURAL MATERIAL APPLICATIONS

EROSION CURVE PREDICTION

The most obvious application of the above discussed model consists in using the kinetic equation (2) or (5) for predicting the volume loss curve under given cavitation load. It is also the simplest method to verify the whole procedure. The key weakness of the procedure lies in limited reliability with which the erosive load can be determined – both at the lab rig for which cavitation resistance matrix can be established by a direct method and at the streamlined surface for which erosion progress is to be predicted.

The traditional direct methodology is using piezoelectric pressure transducers with high resonance frequency. The well-known disadvantages of this methodology consist in limited resonance frequency of the sensor, as well as indiscernibility between signals resulting from highly localized pressure impacts (single or collective) and those due to a pressure wave acting on the whole sensor membrane. While high resonance frequencies can be reached by applying the PVD technology, the inability to determine the area affected by the pressure pulse remains quite a severe drawback of the methodology.

One can try to avoid the aforementioned ambiguities by replacing the pressure signal histogram with that of cavitation pulse energy as determined from the size distribution of cavitation pits in a soft reference material (e.g. copper) after a short exposure [5]. The specific pulse energy coefficient (pulse energy per pit volume unit) as available in literature [10] can be used for this purpose. Determining pulse histograms by means of numerical methods is slowly becoming an ever more attractive alternative as it can be often employed both at the laboratory erosion test rig and at the flow system in the field.

In case cavitation load cannot be determined in the lab and in the field using the same direct technique, using the reference material test method can be recommended for determining cavitation load in the field.

EROSIVE LOAD RETRIEVAL BASED ON THE REFERENCE MATERIAL TESTS

The main concept of the method is determining the **ME** cavitation load vector using erosion curves of a set of reference materials with cavitation resistance matrices resulting from erosion tests at a reference test rig under strictly defined cavitation loads. In order to determine the erosive cavitation load under any other conditions (e.g. at another test rig) the model curves of reference materials are to be fitted to erosion test data (Fig. 3). For this purpose one has to minimise the goal function (6)

$$\Xi(\mathbf{ME}) = \sum_m \gamma_m \sum_{j=1}^{N_m} [MDE_{jm} - \text{CEF}(\mathbf{R}_m, \mathbf{ME}; t_{jm})]^2, \quad (6)$$

where j – a subscript identifying the (t_{jm}, V_{jm}) experimental point at the erosion curve of material m ; $\mathbf{R}_m$ – erosion resistance matrix of material m ; N_m – number of experimental points at material m curve; γ_m – weigh coefficient of erosion curve m .

Further details of the procedure – including discussion of the weigh coefficients – are to be found in references [5] and [9].

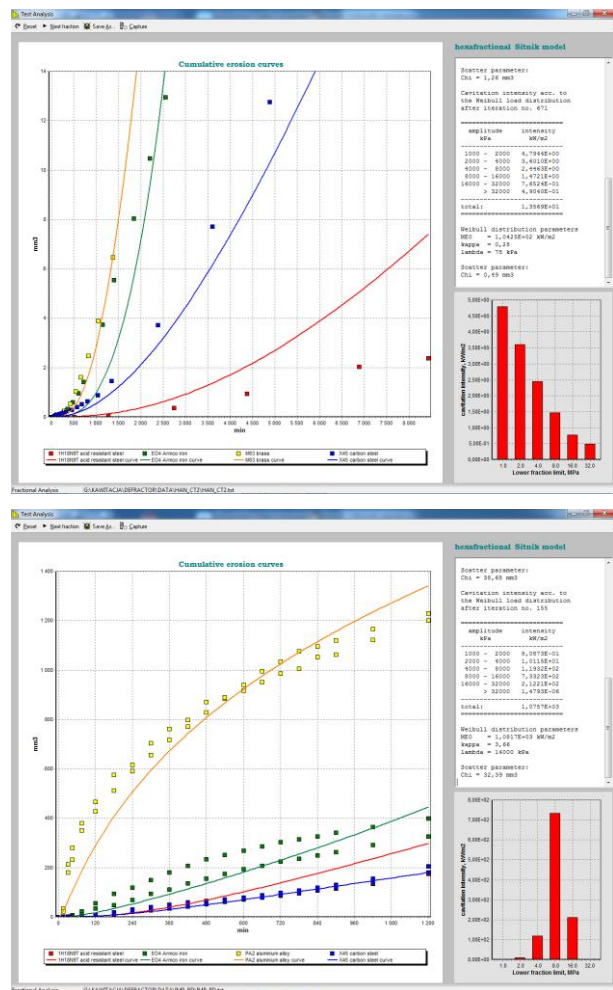


Fig. 3. Screenshots from the DEFRACOR IMP PAN proprietary software, showing fitting of the model curves to the volume loss data and established cavitation loads at the double-barricade tunnel (University of Hannover, upper diagram) and the rotating disk rig (IMP PAN, lower diagram)

CAVITATION FATIGUE CHARACTERISTICS

The fractional erosion curves can be easily used to characterize the fatigue properties of a structural material under cavitation load. For this purpose, the classic fatigue fracture criterion is to be replaced by some other one. The natural candidate is the erosion incubation period. With monofractional erosion curve available, duration of this period at any location can be easily expressed in terms of the number of load cycles per unit of the impinging surface area. As the use of nominal incubation period definition according to the ASTM G32 standard can be burdened with excessive scatter it has been decided to use the $MDE = 10 \mu\text{m}$ condition instead. Fig. 4 shows a set of cavitation fatigue strength characteristics with $MDE = 10 \mu\text{m}$ “material failure” criterion as established for various materials tested at the IMP PAN cavitation tunnel. The analogy with the classic Wöhler curves is quite apparent. In distinction to the fractional curves, which can be used almost exclusively for rather sophisticated erosion prediction pur-

poses, the erosive fatigue strength curves can be used for swift comparative assessment of various structural material performance under load of different force or pressure pulse characteristics.

It is worthwhile to mention that the relationship between cavitation resistance and fatigue strength was studied over 20 years ago by W. Będkowski et al. [11]. The authors correlated the time to reach maximum ero-

sion rate at a Lichtarowicz cell run under variable inlet pressure conditions with the number of effective stress cycles as required for specimen failure at a classic fatigue test rig run under a random pulse generator control. In distinction to the abovementioned investigation, the fractional approach allows to study correlation between the cavitation erosion test results and those of a standard fatigue strength test.

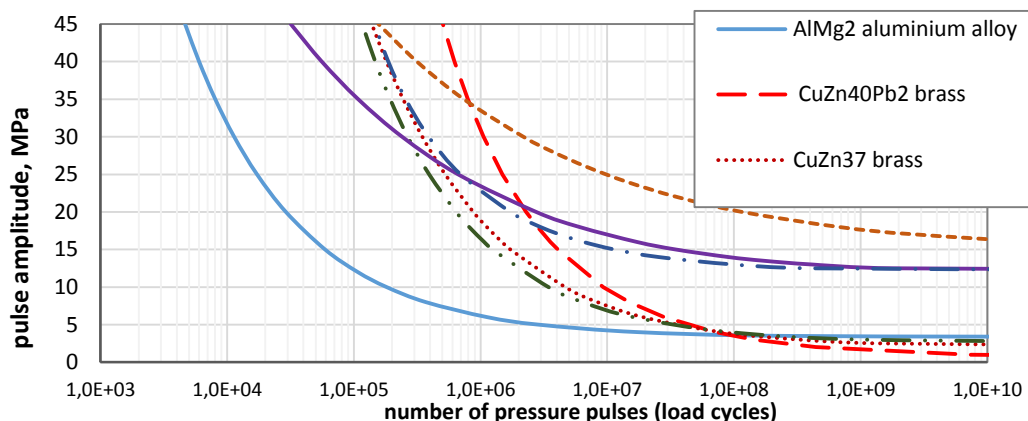


Fig. 4. Cavitation fatigue characteristics of selected structural materials derived from the fractional analysis results of the cavitation tunnel tests at IMP PAN lab

3. PROTECTIVE COATINGS

INTRODUCTORY CONSIDERATIONS

In distinction from the structural materials in which case the volume loss and erosion penetration depth are generally considered the main parameters of material degradation degree, the degradation of a protective coating can be quantified either by the percentage of coating surface removed or the percentage of coating thickness penetrated. This division clearly reflects the macroscopic division between two main coating destruction mechanisms:

- detachment due to overcoming coating adhesion to the substrate material (mainly by shear forces);
- penetration due to mechanisms typical for structural materials.

For the purpose of this paper, the above two degradation mechanisms will be the basis for coating classification as “thin” or “thick”. Of course, in reality, both these mechanisms may coexist. Furthermore, the prevalence of one or the other may strongly depend on the kind and intensity of load applied.

In case of coatings – in the same way as in the case of structural materials – it is extremely advantageous to consider erosion a stochastic process. The probability of total destruction (destruction degree) due to a single fraction process can be represented analytically by one of the well-known probability functions, e.g. the Weibull distribution function,

$$WDF(Y) = 1 - \exp[-(Y/Y_0)^\alpha], \quad (7)$$

with α and Y_0 representing the Weibull function parameters. Weibull functions have been used extensively by numerous researchers, e.g. [12–14], due to their flexibility enabling reproduction of numerous well-known statistical distributions. In particular, with $\alpha = 1$ and 2, the exponential and Rayleigh distributions can be

reproduced whereas Weibull functions with high α values resemble the Gauss curves.

For simplicity, further considerations will address thin coatings, which implies that the process of coating detachment will be analysed. However, the reasoning will remain valid also in case of coating penetration process provided the fraction of surface removed is replaced by the mean depth of erosion MDE related to the coating thickness, δ .

THIN COATING EROSION MODEL

Let the A_0 and $A(t)$ symbols denote the substrate material surface area under consideration and the area still covered with the protective coating after cavitation load exposure of duration t , respectively. In such a case the $F(t) = 1 - A(t)/A_0$ expression can be interpreted as the removal probability of this coating after time t , and the expression $G(t) = A(t)/A_0$ as the probability of its survival. The probability that the coating is still there at the time point $t + dt$, provided that it was preserved at the time point t , can be written down as the quotient $G(t + dt)/G(t)$. The same remains valid if only one load fraction acts on the surface. However, as mentioned, in this case it is possible to replace the G function by some analytical function $W = W(\mathbf{R}, Y)$ with $\mathbf{R}$ denoting the set of function parameters (erosion resistance vector) and Y – density of energy delivered to the impinging surface. The probability of coating survival after the single-fraction energy with density $Y + dY$ has been delivered, provided that the coating was still there after supply of energy with density Y , will be $W(\mathbf{R}, Y + dY)/W(\mathbf{R}, Y)$.

If $Y = \theta(\mathbf{R}_i, G)$ is the density of fraction i energy which delivery still does not result in coating removal then – under usual assumptions of the polyfractional model [5] – the conditional probability $G(t + dt)/G(t)$ can be written down as:

$$\frac{G(t+dt)}{G(t)} = \prod_i \frac{W(\mathbf{R}_i, Y+dY)}{W(\mathbf{R}_i, Y)} \Big|_{Y=\Theta(\mathbf{R}_i, G)}, \quad (8)$$

where Θ denotes the function inverse to $W = W(\mathbf{R}, Y)$ with respect to the argument Y .

After taking logarithms of both sides, equation (8) can be rewritten as

$$\ln G(t+dt) - \ln G(t) = \sum_i [\ln W(\mathbf{R}_i, Y+dY) - \ln W(\mathbf{R}_i, Y)] \Big|_{Y=\Theta(\mathbf{R}_i, G)} \quad (9a)$$

or

$$dH = \sum_i dU(\mathbf{R}_i, Y) \Big|_{Y=\Theta(\mathbf{R}_i, H)}, \quad (9b)$$

where $H = -\ln G$ and $U = -\ln W$ while Θ is the function inverse to $U = U(\mathbf{R}, Y)$ with respect to argument Y (Fig. 5)

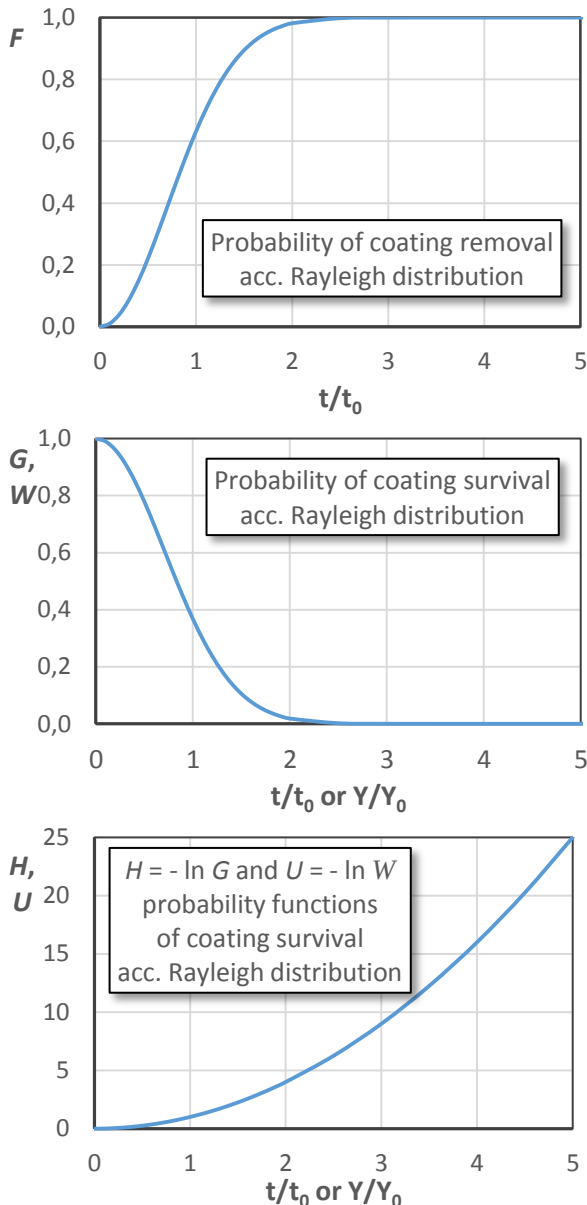


Fig. 5. The F , G (W), and H (U) probability functions in a single fractional model with model function F represented by Rayleigh distribution

Considering that for each fraction i the relationship $Y_i = ME_i \cdot t$ remains valid, equation (9b) can be written in the form of the general equation of polyfractional coating erosion kinetics:

$$\frac{dH}{dt} = \sum_i ME_i \frac{\partial U(\mathbf{R}_i, Y)}{\partial Y} \Big|_{Y=\Theta(\mathbf{R}_i, H)}. \quad (10)$$

This equation is identical to equation (2), except for the definition of H and U functions. A similar solution is also expected. The probability of coating removal after application of erosive load energy with density Y is proposed to be defined using the WDF Weibull distribution function where α and Y_0 are the components of the coating cavitation resistance vector $\mathbf{R}_i$. Relevant substitution to equation (10) results in the special equation of polyfractional coating erosion kinetics:

$$\frac{dH}{dt} = \sum_i \frac{\alpha_i}{Y_{0i}} ME_i H^{1-1/\alpha_i}, \quad (11)$$

which can be solved by assuming the initial conditions $H(0) = 0$ and $H'(0) = 0$. When using the Runge-Kutta method with step Δt , it is recommended to replace the value $H'(0) = 0$ with the expression

$$\frac{2}{\Delta t} \sum_i \left(\frac{ME_i \Delta t}{2Y_{0i}} \right)^{\alpha_i}.$$

The cavitation resistance matrix $\mathbf{R} = [\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_N]^T$ is supposed to be determined by simultaneous fitting of model curves $H = H(\mathbf{R}, t)$ to experimental points $(t_{jk}, \ln(A_0/A_{jk}))$, where j is the identifier of experimental points resulting from tests performed in series k with load vector parameter ME_k .

We assume that, as in the case of structural materials, the knowledge of cavitation resistance matrix will make it possible not only to predict the course of coating destruction at a given cavitation load, but also to derive the fatigue characteristics. It is proposed to consider the removal of 95% of its surface area as the criterion of complete destruction.

4. FINAL REMARKS

The main reason for starting development of the methodology for assessing cavitation resistance and determining fatigue properties was the intention to provide the research tools necessary to optimize the selection and design of new structural materials and protective coatings depending on the expected erosion load. It is anticipated that this methodology will be the subject of planned research and further development.

The starting point of the technique will be to determine the erosion load of the tested samples. Direct methods involve determining the load vector from the amplitude distribution of cavitation pulses using piezoelectric pressure sensors or from the distribution of pits in a soft material (for example, pure copper) during the initial period of exposure. This latter method appears to have a number of fundamental advantages. In addition to the advantages mentioned in [5], access to modern equipment (for example, a confocal microscope) will allow detailed determination of the load distribution on the exposed surface, as well as detailed monitoring of the erosion process as a function of load on a single sample. The use of the previously described fractional technique for determining the erosion curves and fatigue characteristics of cavitation-protective coatings seems completely impossible without the use of a microscopic method for analyzing the eroded surface.

Fatigue curves as derived from the fractional erosion curves allow a quick comparison of erosion resilience of different materials and protective coatings. Developing advanced techniques to determine their fractional erosion curves will be an important step towards predicting their performance under field conditions.

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НОВІ МЕТОДИКИ ОЦІНКИ КАВІТАЦІЙНОГО НАВАНТАЖЕННЯ ТА ОПІРУ КОНСТРУКЦІЙНИХ МАТЕРІАЛІВ ТА ЗАХИСНИХ ПОКРИТТІВ У АТОМНІЙ ЕНЕРГЕТИЦІ. ЧАСТИНА II. ПОЛІФРАКЦІЙНИЙ ПІДХІД

Януш Стеллер, Володимир Сафонов

Пояснено фракційний підхід до процесу кавітаційної ерозії, запропонований раніше одним із авторів. Процес поліфракційної ерозії розглядається як суперпозиція низки монофракційних та описується диференціальним кінетичним рівнянням з набором параметрів, що становлять «матрицю ерозійної стійкості». Основні методичні міркування доповнені описом найбільш типових додатків, включаючи прогнозування ерозії конструкційних матеріалів при заданому навантаженні кавітації, отримання характеристик кавітаційної ерозійної втоми, визначення ефективного ерозійного навантаження на основі характеристик еталонних матеріалів. Нарешті, пропонується поширити цю методологію на тонкі та товсті захисні покриття.