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STUDY OF OCTUPOLE DEFORMATION OF NUCLEI IN CHAIN OF ISOTONS IN THE REGION OF ACTINIDES IN THE MEAN FIELD APPROXIMATION

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Calculations of the properties of even-even isotones $N = 134$ of nuclei with $Z = 86–96$ were carried out in the Hartree-Fock-Bogolyubov approximation, taking into account the axial symmetry of nuclei with forces Skyrme (SkM*, SLy4). Pairing of nucleons in nuclei is described by zero-range pairing forces of mixed type with different sets of pairing force constants. In the calculations we used constrained conditions on the parameters of the quadrupole and octupole deformations of the nuclei. It is shown that for the considered chain of isotones the octupole deformation of nuclei strongly depends on the choice of parameters of the nucleon pairing force and weakly depends on the type of parameterization of the Skyrme forces used in this study.

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INTRODUCTION

The study of the deformation of the shape of atomic nuclei characterized by the moments of the distribution of the density of nucleons in nuclei or the parameters of the deformation of nuclei currently remains one of the important problems of nuclear physics. For a significant part of known atomic nuclei experimental data indicate the presence of a quadrupole moment in the nucleon density distribution. It is usually assumed that the density of nuclear matter for deformed nuclei has axial symmetry and symmetry relative to the plane perpendicular to the axis of axial symmetry (reflective symmetry). The presence in the ground state of a stable reflective asymmetric form for some regions Z and A is confirmed by experimental data. In particular, the studies of the manifestations of the existence of octupole deformations (pear-shaped forms) of atomic nuclei are presented in [1, 2]. It is shown that nuclei with octupole shape deformation compared to nuclei with quadrupole deformation are represented by small regions Z and A.

To describe the properties of the large sets of nuclei, microscopic approaches based on the Hartree–Fock–Bogolyubov (HFB) method or the Hartree–Fock (HF) method [3] with effective forces are widely used. Approaches based on the relativistic Hartree–Bogolyubov (RHB) theory are also widely used [4]. The same approaches for calculating the properties of nuclei taking into account possible octupole deformation were used in [5–7]. In [7], the microscopic origin of the stable reflective asymmetric form in the ground state for some nuclei was studied in detail. The calculations of the properties of the large sets of nuclei presented in [5–9] show that, despite the large volume of research on the manifestations of octupole deformation of nuclei in the HFB and RHB approaches, the influence of the choice of pairing force parameters on both the calculated values of the octupole deformation parameter and the size of the region of existence of nuclei possessing octupole deformation has not been sufficiently studied.

The purpose of this work is to continue the studies [8, 9] of the influence of the choice of pairing force parameters on the magnitude of octupole deformation for nuclei in the region of actinides with $Z = 86–96$, forming the chain of isotones with $N = 134$. In contrast to the works [8, 9] the region of existence of octupole deformation was investigated as a function of Z, i.e. for different isotones. In addition, the dependence of the obtained results on the choice of Skyrme interaction SkM* or SLy4 was investigated.

1. CALCULATION METHOD AND MAIN RESULTS

As in [8, 9] to study the properties of the ground state of even-even isotones with $Z = 86–96$ and with $N = 134$, we used the HFBTHO v2.00d software package [10]. This software package implements the HFB method with the nucleon density-dependent effective Skyrme interaction. The HFBTHO v2.00d software package allows solving the system of stationary HFB equations under the assumption of axial symmetry of the shape of the nucleus with conservation or violation of the reflective symmetry in the nucleus. The HFBTHO v2.00d software package uses the expansion of single-nucleon wave functions over the basis of wave functions of an axially deformed harmonic oscillator. In the expansion of single-nucleon wave functions we took into account all the basis wave functions of the harmonic oscillator for which the principal oscillator quantum number does not exceed $N_0 = 18$. Calculations with an increased basis dimension show that the basis dimension $N_0 = 18$ is sufficient for the considered region of nuclear mass numbers.

The software package HFBTHO v2.00d allows solving the system of HFB equations with constrained conditions on the multipole moments of nuclei and also allows solving equations without constrained conditions on the deformation moments. In our calculations we used constrained conditions on the dipole, quadrupole and octupole deformations of nuclei. In all our calculations, the constrained conditions on the mass dipole moment of the nucleus are: $Q_{10} = 0$.

The parameters of the quadrupole and octupole deformations of the nucleon distribution density are determined similarly to the work [6]:

$$\beta_2 = Q_{20} / \left(\sqrt{\frac{16\pi}{5}} \frac{3}{4\pi} A R_0^2 \right), \quad (1)$$

$$\beta_3 = Q_{30} / \left(\sqrt{\frac{16\pi}{7}} \frac{3}{4\pi} A R_0^3 \right), \quad (2)$$

where Q_{20} and Q_{30} are the mass quadrupole and octupole moments of nuclei as in [6], A is the number of nucleons in the nucleus, and $R_0 = 1.2A^{1/3}$ fm.

The solutions of the HFB equations corresponding to the most bound state of the nucleus were determined from finding the deepest minimum of the calculated surfaces of the total energies of nuclei E with constrained conditions on the mass parameters of the quadrupole β_2 and octupole β_3 deformations. By the binding energy of the nucleus B we will mean $B = -E$. The total energy surfaces of the nuclei were calculated in the range of quadrupole deformation parameters $-0.4 \le \beta_2 \le 1.0$ and octupole deformation $0 \le \beta_3 \le 0.3$.

In our calculations we used the Skyrme force parameter sets SkM* [11] and SLy4 [12]. Nucleon pairing is described by zero-range pairing forces that depend on the nucleon density. We used the following form for the pairing forces [9]:

$$\upsilon(\vec{r}_1,\vec{r}_2) = V_\tau \left\{ 1 - \alpha \left[\frac{\rho(\vec{R})}{\rho_0} \right] \right\} \delta(\vec{r}_1 - \vec{r}_2), \quad (3)$$

where V_τ is the pairing force parameter for neutrons ($\tau = n$) and protons ($\tau = p$), α is the parameter that determines the type of pairing (α = 0, 0.5, 1 – volume, mixed and surface types of pairing, respectively), $\rho_0 = 0.16$ fm^{-3}, $\vec{r} = \vec{r}_1 - \vec{r}_2$ is the relative radius-vector and $\vec{R} = (\vec{r}_1 + \vec{r}_2)/2$ radius-vector of the center of mass of interacting nucleons. In the present work, we used only mixed-type pairing forces. Pair correlations are usually described using the pairing energy gap Δ_τ for neutrons and protons. In this paper we use the method for calculating Δ_τ described in [10] and implemented in the HFBTHO v2.00d software code.

When discussing the influence of pairing forces on the calculated properties of the isotones we have considered, it is useful to use the average energy gaps of nuclei Δ_τ for neutrons Δ_n and protons Δ_p, comparing them with the energy gaps calculated from the even-odd differences in the masses of neighboring nuclei. This allows us to compare the calculated average energy gaps of nuclei Δ_τ with empirical values obtained on the basis of data extracted from the experiment. In the present work, for each of the isotones with N=134 considered by us, the parameters of the pairing forces V_τ were adjusted for the best agreement between the Δ_τ calculated in the HFB calculations with Skyrme forces and the values of the empirical energy gaps calculated from the even-odd differences in the masses of neighboring nuclei. This significantly distinguishes these calculations from the calculations given in works [8, 9], in which octupole deformation and the properties of isotope chains were studied at a fixed value V_τ for all isotopes of the chain under consideration. To calculate the neutron and proton energy gaps of even-even nuclei from even-odd differences in the masses of nuclei, we used the differences in three and four binding energies of neighboring nuclei, expressed in terms of single-particle of neutron separation energy S_n or proton separation energy S_p in neighboring nuclei [13, 14]:

$$\Delta_n^{(3)}(N,Z) = -\frac{1}{2}[S_n(N+1,Z) - S_n(N,Z)], \quad (4)$$

$$\Delta_p^{(3)}(N,Z) = -\frac{1}{2}[S_p(N,Z+1) - S_p(N,Z)], \quad (5)$$

$$\Delta_n^{(4)}(N,Z) = -\frac{1}{4}[S_n(N+1,Z) + S_n(N-1,Z) - 2S_n(N,Z)], \quad (6)$$

$$\Delta_p^{(4)}(N,Z) = -\frac{1}{4}[S_p(N,Z+1) + S_p(N,Z-1) - 2S_p(N,Z)]. \quad (7)$$

Single-particle energies of separation of a neutron S_n or a proton S_p in nuclei are taken from the tables given in [14]. The results of fitting the parameters of the pairing forces are given in the Table. For convenience, we will designate the obtained optimal values V_τ for Skyrme forces SkM* as P1, and for Skyrme forces SLy4 as P4. To study the influence of changes in the parameters of pairing forces on the deformation of nuclei, we calculated the properties of nuclei with changed parameters P1 and P4, which we designate as: P2 = 0.95·P1, P3 = 1.05·P1, P5 = 0.95·P4, P6 = 1.05·P4.

Parameters of pairing forces P1 for neutrons V_n and protons V_p for nuclei with N = 134 in HFB calculations with Skyrme forces SkM*. In brackets are the parameters of pairing forces P4 for Skyrme forces SLy4

Nucleus	$-V_n$, MeV·fm^3	$-V_p$, MeV·fm^3
Rn	271.88 (317.75)	348.56 (369.00)
Ra	271.88 (310.00)	348.56 (360.00)
Th	278.51 (317.75)	340.06 (351.00)
U	278.51 (317.75)	340.06 (351.00)
Pu	285.14 (325.50)	340.06 (351.00)
Cm	285.14 (325.50)	340.06 (351.00)

Figs. 1, 2 show the comparison of the energy gaps Δ_n and Δ_p for neutrons and protons of the N = 134 isotones, which were calculated with the Skyrme interaction SkM* and SLy4 with the sets of pairing force parameters P1–P6, with the values of the energy gaps calculated from the even–odd differences in the masses of neighboring nuclei (4)–(7). As can be seen from Fig. 1, for the SkM* forces, the calculated Δ_n in the HFB, with the pairing force parameters P2 and P3, give, respectively, underestimated and overestimated values compared to $\Delta^{(3)}$ and $\Delta^{(4)}$, which are obtained, respectively, from (4)–(7). At the same time, Δ_n calculated in the HFB with the forces P1 obtained after their fitting are in good agreement with $\Delta^{(3)}$ and $\Delta^{(4)}$. For Δ_p, the pairing forces

P2 and P3 also give, respectively, underestimated and overestimated values of the energy gap compared to $\Delta^{(3)}$ and $\Delta^{(4)}$. The calculations of Δ_p (for Skyrme forces SkM*) with the pairing forces P1 agree somewhat worse with the values of $\Delta^{(3)}$ and $\Delta^{(4)}$ than for the calculations of Δ_n, despite the fitting of the parameters. The pairing forces P1 give the best agreement of Δ_p with $\Delta^{(3)}$.

For the Skyrme forces SLy4 and the parameters P5 and P6, as well as for the Skyrme forces SkM* with the pairing forces P2 and P3, we obtain, respectively, underestimated or overestimated values of Δ_n and Δ_p compared to $\Delta^{(3)}$ and $\Delta^{(4)}$ for neutrons and protons. The P4 pairing forces for the SLy4 Skyrme forces provide the best description of the energy gaps.

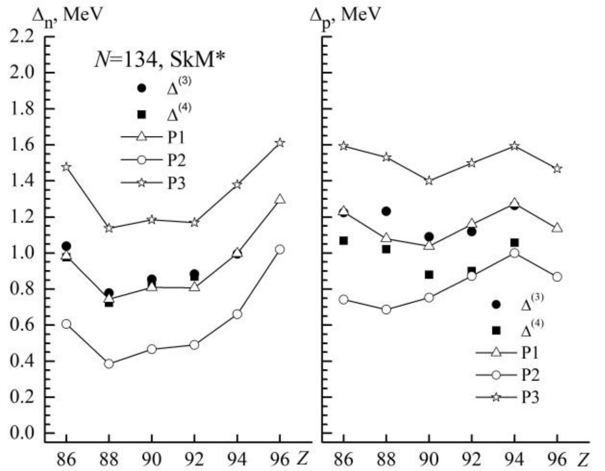

Fig. 1. Comparison of neutron and proton energy gaps of N = 134 isotones, calculated with the Skyrme interaction SkM for different sets of pairing force parameters, with the values of energy gaps calculated from even-odd differences in the masses of neighboring nuclei*

It is difficult to use the surfaces of total energy $E(\beta_2, \beta_3)$ to analyze the influence of the value of the pairing force parameter on the values of the quadrupole deformation parameter β_2 and octupole deformation β_3, since for the same isotone for different pairing force options these surfaces can differ significantly in the value of total energy. For such analysis, it is more convenient to use $\Delta E_3 = E(\beta_2, \beta_3) - E(\beta_2^{'}, \beta_3 = 0)$.

Note that, to more accurate determination the minimum value of the total energy of the nucleus after calculations with constrained conditions, we carried out calculations without constrained conditions on β_2 and β_3 in the vicinity of the minimum of the $E(\beta_2, \beta_3)$ dependence. Such calculations were carried out for all isotones $N = 134$ considered in the work with all variants of pairing forces P1–P6. The results of such refined calculations without constrained conditions on β_2 and β_3 are given below in Figs. 3, 4.

In Figs. 3, 4, for all isotones $N = 134$ considered in the work, the results of calculations with Skyrme forces SkM* and SLy4 of the deformation parameters of nuclei β_2 and β_3, as well as the value $\Delta E_3 = E(\beta_2, \beta_3) - E(\beta_2^{'}, \beta_3 = 0)$ are shown.

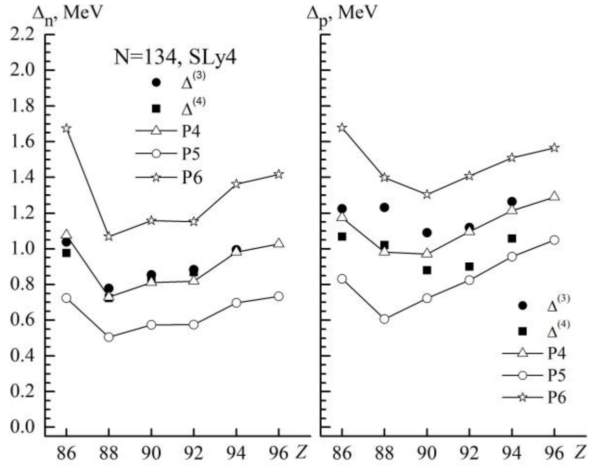

Fig. 2. Comparison of neutron and proton energy gaps of N = 134 isotones calculated with the Skyrme SLy4 interaction for different sets of pairing force parameters with the values of energy gaps calculated from even-odd mass differences of neighboring nuclei

The energy ΔE_3 that represents the contribution to the total energy of the nucleus from taking into account the octupole deformation of the nuclei will be called the octupole energy of the nucleus. The calculations shown in Figs. 3, 4 were performed with different sets of parameters of the pairing forces, which were discussed above. As can be seen from Figs. 3, 4, the choice of the magnitude of the pairing forces significantly affects the values of the deformation parameters β_2 and β_3, as well as the magnitude of the octupole energy ΔE_3. The results shown in Figs. 3,4 show that an increase in the pairing force leads not only to a decrease or complete disappearance of the quadrupole and octupole deformation of nuclei, but also, accordingly, to a decrease or complete disappearance of the octupole energy. This result for the chain of isotones we have considered coincides with the results we have previously obtained for some chains of isotopes in this region of mass numbers [8, 9]. From Figs. 3, 4 it is also evident that the extent of the region of existence of the octupole deformation β_3 in the space (N, Z) depends on the choice of the type of Skyrme interaction.

The results of calculations of β_3 for Skyrme forces SkM* and pairing forces P3 show that the region of existence of β_3 extends from $Z = 88$ to $Z = 94$. In the case of using pairing forces P1 obtained as a result of the above-described fitting, this region expands towards smaller Z.

For SLy4 forces, the extent of the region of existence of octupole deformations β_3 approximately coincides with the results for Skyrme forces SkM*, but extends into the region of large Z.

As can be seen from the results presented in this work and the results of [8, 9], in the calculations it is necessary to control the magnitude of the pairing forces by comparing the calculated pairing energy gaps with their empirical values. In the calculations of the properties of $N = 134$ isotones presented in this work, such a comparison allows us to give preference to the pairing forces P1 and P4 for the Skyrme forces SkM* and SLy4, respectively.

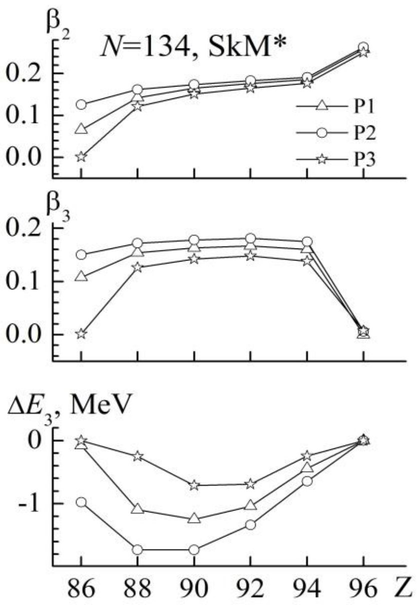

Fig. 3. Deformation parameters β_2 and β_3, as well as octupole energies, calculated with the Skyrm interaction SkM with different sets of pairing force parameters for isotones N = 134 with Z = 86–96*

We note that for isotones $N = 134$ in this range of mass numbers the character of the change in β_2, β_3 and ΔE_3 for the pairing force parameters considered by us is similar to the results obtained in [8, 9] for isotope chains.

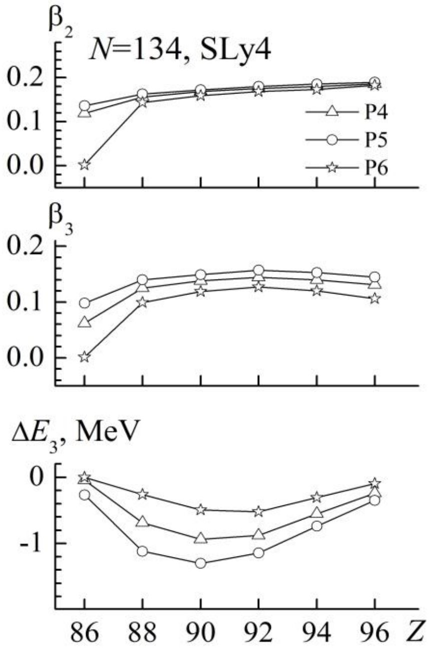

Fig. 4. Deformation parameters β_2 and β_3, as well as octupole energies, calculated with the Skyrm interaction Sly4 with different sets of pairing force parameters for isotones N = 134 with Z = 86–96

CONCLUSIONS

In this work, calculations of the properties of even-even isotones $N = 134$ in the region of actinides in the range of Z from 86 to 96 are carried out in the HFB approximation under the assumption of axial symmetry of nuclei with Skyrme forces SkM* and SLy4. The pairing of nucleons in nuclei is described by density-dependent zero-range pairing forces with different sets of constants determining the pairing force. The constrained conditions on the parameters of quadrupole and octupole deformations of nuclei were used in the calculations.

It is shown that for the considered isotones $N = 134$, the octupole deformation of nuclei strongly depends on the choice of parameters of the nucleon pairing strength. Overestimated values of the pairing force constants lead to a decrease or complete disappearance of the quadrupole and octupole deformation in the nuclei considered. It is shown that the extent of the region of existence of octupole deformation in the (N, Z) space for the region of nuclei under consideration depends both on the choice of the parameters of the pairing forces and on the choice of the type of Skyrme forces.

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ДОСЛІДЖЕННЯ ОКТУПОЛЬНОЇ ДЕФОРМАЦІЇ ЯДЕР У ЛАНЦЮЖКУ ІЗОТОНІВ В ОБЛАСТІ АКТИНОЇДІВ У НАБЛИЖЕННІ СЕРЕДНЬОГО ПОЛЯ

В.М. Тарасов, В.І. Купріков, Д.В. Тарасов

Проведено розрахунки властивостей парно-парних ізотонів N=134 ядер з Z=86–96 у наближенні Хартрі-Фока-Боголюбова з урахуванням аксіальної симетрії ядер із силами Скірма (SkM*, SLy4). Спарювання нуклонів у ядрах описується силами спарювання нульового радіуса дії змішаного типу з різними наборами констант сил спарювання. У розрахунках використовувалися накладені умови на параметри квадрупольної та октупольної деформацій ядер. Показано, що для розглянутого ланцюжка ізотонів октупольна деформація ядер сильно залежить від вибору параметрів сили спарювання нуклонів і слабо залежить від використовуваних у даному дослідженні типу параметризації сил Скірма.