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KEEPING AN ELECTRON BEAM IN POTENTIAL WELL MAGNETIC SOLENOIDAL FIELD

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An intense cylindrical magnetic field is considered in which potential well is created with the help of additional magnets. It is shown that the motion of electrons can be associated with model of three-dimensional oscillations in potential well. The results of numerical simulation of electron trajectories in gradient magnetic field with point secondary emission cathode located in the middle of the system are presented. The formation of the beam with energy of 55 keV in the radial and longitudinal directions during its transportation in solenoidal field with large gradient is considered. The operating modes of the gun are obtained, in which the particle experiences stable three-dimensional vibrations. The influence of the initial conditions during emission on the occurrence of oscillations is studied. It is shown that for the given electron energy and the fixed magnetic field, the parameter that determines the stability of particle motion is the polar angle of entry relative to the axis of the cylindrical magnetic field.

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INTRODUCTION

At KIPT, an electron accelerator based on a magnetron gun was created, which uses an axial electron beam to irradiate metal targets [1-4]. As an object for research, the magnetron gun has the advantage that, on the one hand, it can be modified to conduct experiments of various types, on the other hand, to obtain quantitative dependences, it is possible to use computational methods and compare comparison of numerical results with experimental data. A possible direction is a combination of two matched gun magnetic systems to create a potential well of the magnetic field [4, 6]. With the selected shape and depth of the well, the motion of a particle placed in it will be stable [5].

In this paper, the parameters of an electron beam as it moves in a solenoidal field with a longitudinal trap formed by a magnetic field and the construction of a model for the motion of an electron beam are studied. The aim of the study was to create a model for the motion of an electron beam with an energy of tens of keV in a gradient magnetic field, to study the formation of electron trajectories, the dependence of particle dynamics on initial conditions, and the distribution of a magnetic field. The results of modeling on the motion of the electron beam are presented. The simulation results show the possibility of establishing the phenomenon of oscillatory motion under experimental conditions.

RESEARCH METHODOLOGY

The use of intense magnetic fields with an increased gradient of their intensity makes it possible to consider new phenomena in the spatial picture of the evolution of electron beams. On the basis of the Hamiltonian formalism of the motion of electrons in a magnetic field, a software tool has been constructed that allows numerical simulation of the dynamics of electron flows in gradient magnetic field.

Earlier, in the course of numerical experiments [4, 6], the effect of reverse motion of electrons in the gradient magnetic field of a magnetron gun was discovered.

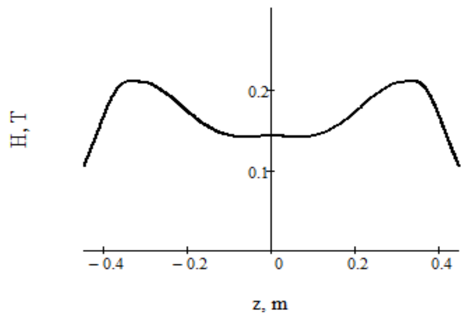


Fig. 1. Longitudinal tension distribution magnetic field H(z)

The distribution of the longitudinal magnetic field H(z) along the axis of the system and the transportation channel, which was used in the numerical simulation, is shown in Fig. 1. The source of electrons is placed in the center (z=0), while the particles are given energy E=55 keV.

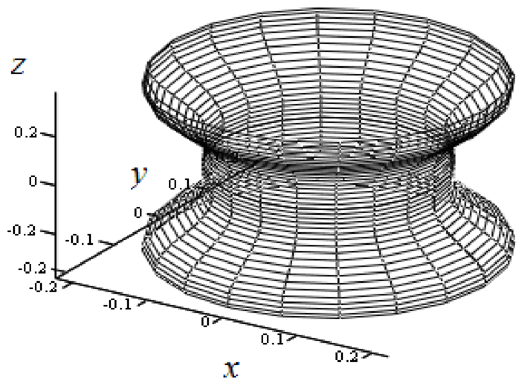


Fig. 2. Magnetic field profile H(z)

The three-dimensional profile of the magnetic field strength distribution H(z) is shown in Fig. 2. This type of H(z) distribution was chosen due to the large gradient at the boundaries of the field well and its smoothness in the central region.

MATHEMATICAL MODEL

To construct the mathematical model of the solution, we use the axial symmetry of the problem. In the polar coordinate system (r, z, ϑ), we use the Hamiltonian of an electron in a magnetic field in a cylindrical coordinate system [5]

$$H = \frac{p_r^2 + p_z^2}{2m} + \frac{1}{2m}\left(\frac{p_\vartheta}{r} - e_0 A\right)^2, \qquad (1)$$

where e_0, m are the electron charge and rest mass; p_r, p_z, p_ϑ are the canonical momenta; A is the magnetic potential. Taking into account azimuthal symmetry, it is convenient to write it in the form $A(r,z) = Brf(z)$, where $f(z)$ is the function of the longitudinal coordinate, which we will use below, and B is the magnetic field strength at the point under consideration.

Hamiltonian (1) corresponds to 6 equations of motion for coordinates and momenta, from which it follows that the formation of the particle trajectory is influenced by both the distribution of the magnetic field H along the z axis and its derivative dH/dz. In the system of these 6 equations, let's move, using the speed of light c, from the current time t to the variable $S=ct$ (the length of the path that the particle has traversed). It is necessary to add initial conditions to equations (3) for r_0, z_0, ϑ_0, as well as for r_0', z_0', θ_0'. For canonical pulses, we make the replacement $p_r = e_0 B q_r$, $p_z = e_0 B q_z$, $p_\vartheta = e_0 B q_\vartheta$ and arrive at the system of equations

$$\begin{cases} r' = \mu q_r, \\ z' = \mu q_z, \\ \vartheta' = \mu\left(q_\vartheta / r^2 - f(z)\right), \\ q_r' = \mu r\left(q_\vartheta / r^2 - f(z)\right)\left(q_\vartheta / r^2 + f(z)\right), \\ q_z' = \mu r^2\left(q_\vartheta / r^2 - f(z)\right) df(z)/dz, \\ q_\vartheta' = 0. \end{cases} \qquad (2)$$

In equations (2) $\mu = e_0 B / mc$, $Bf(z)$ is a function that describes the magnetic field strength along the axis z. We choose the amplitude B so that the function $Bf(z)$ can be used over the entire range of possible values z. It is necessary to add initial conditions to equations (3) for r_0, z_0, ϑ_0, as well as for q_{r0}, q_{z0}, $q_{\vartheta 0}$.

The problem can be formulated as the problem of finding, on a given interval, a solution to the system of ordinary differential equations with fixed initial conditions. This made it possible to obtain solutions to a system of equations in the form of 6-dimensional arrays describing trajectories. The algorithm for numerically finding solutions is described in detail in [4, 6].

NUMERICAL SIMULATION

On Fig. 3–5 show the results of numerical simulation of the trajectories of an electron with energy E =55 keV in the gradient solenoidal magnetic field, performed under initial conditions of $z_0 = 0$ mm, $r_0 = 21$ mm, the azimuth ϑ_0 is uniformly distributed in the interval (0, 2π). Further (E_0=511 keV is the rest energy)

$$q_z'\big|_0 = \frac{1}{\mu}\sqrt{\frac{2E}{E_0}}, \qquad q_\theta'\big|_0 = r_0^2 f(z_0). \qquad (3)$$

As for the radial angle of entry into the transportation system, its value proved effective for holding only for values in the region of about $r_0' = 0.015$ rad and in the region of about $r_0' = $-0.015 rad, accordingly.

The results of numerical simulation are shown in the figures in the range of electron trajectory lengths from S=0 m to S=8.3 m.

On Fig. 3 shows a typical dependence of the vertical coordinate z on the current time of flight (trajectory length). It can be seen that the particle oscillates in volume and is retained.

The similar dependence, but for the radial component r, is shown in Fig. 4. It can be seen that the particle, being held in the volume, oscillates, flying around the center in the region of r =0.05 m.

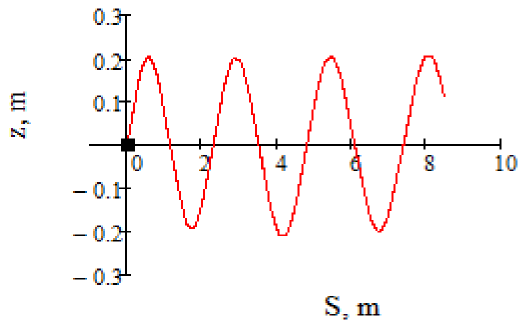


Fig. 3. Dependence of the longitudinal coordinate z on the length of the trajectory S of the particle, the start of the trajectory is indicated by the square

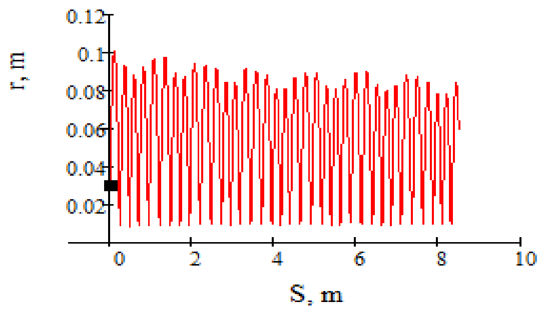


Fig. 4. Dependence of the radial coordinate r on the length of the trajectory S of the particle, the start of the trajectory is indicated by the square

The phase map of oscillations in the (x,y)-plane is shown in Fig. 5 for two particles. The first of them (*red*) started with the condition $r_0' = 0.015$ rad, the second one (*black*) started with the condition $r_0' =$-0.015 rad. It can be seen that the region of stable oscillations consists of two sub-regions, each of which corresponds to the initial condition r_0'.

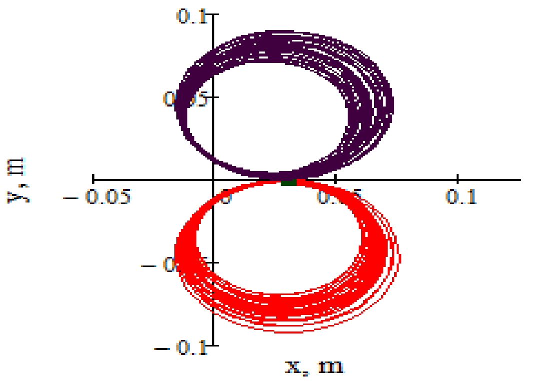


Fig. 5. Phase map of oscillations of two particles, the start of the trajectories is indicated by the square

On Fig. 6 shows the volumetric trajectory of one of the particles in the interval *S*=8 m with the initial condition $r_0' = 0.015$ rad (see Fig. 5). The electron fills the surface of the cylinder.

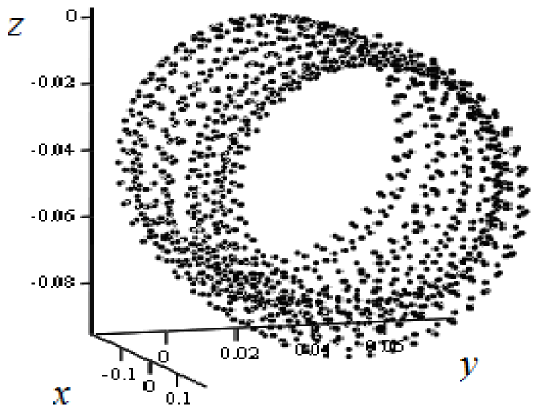


Fig. 6. Volumetric particle trajectories at interval S=8 m, $r_0' = 0.015$ rad

On Fig. 7 shows the phase (*z*,*r*)-map of one particle (see Fig. 5) on the interval *S*=8 *m* with the initial condition $r_0' = 0.015$ rad.

For the sample of 500 particles, the evolution of the density distribution on the *z*=0 plane was simulated. The initial condition along the vertical is uniform: $|z_0| \le 0.07$ m. Half of the particles from the sample of them started with the condition $r_0' = 0.015$ rad, the second half were given the condition $r_0' =$ -0.015 rad.

From Fig. 8 one can see the dynamics of the phase Poincaré (*x*,*y*)-map on the plane *z*=0. Each of the particles experiences in a given cylindrical magnetic field with a potential well a complex helical oscillation, the form of which is determined by the initial condition.

From Fig. 3–8 it can be seen that with an increase in the time of flight (the length of the trajectory), the particle oscillates in volume and is retained. Three-dimensional stable oscillations of an electron are carried out in the region of the minimum of the magnetic field H(*z*). In this region, the distribution of the field along the axis can be approximately represented by a parabolic dependence in *z*.

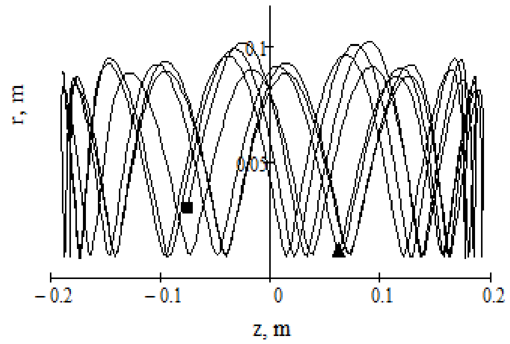


Fig. 7. Phase map of particle oscillations on the interval S=8 m; $r_0' = 0.015$ rad, start of the trajectory is indicated by the square, triangle – finish

Longitudinal oscillations of the electron are subsequently repeated many times, which was verified by performing special calculations.

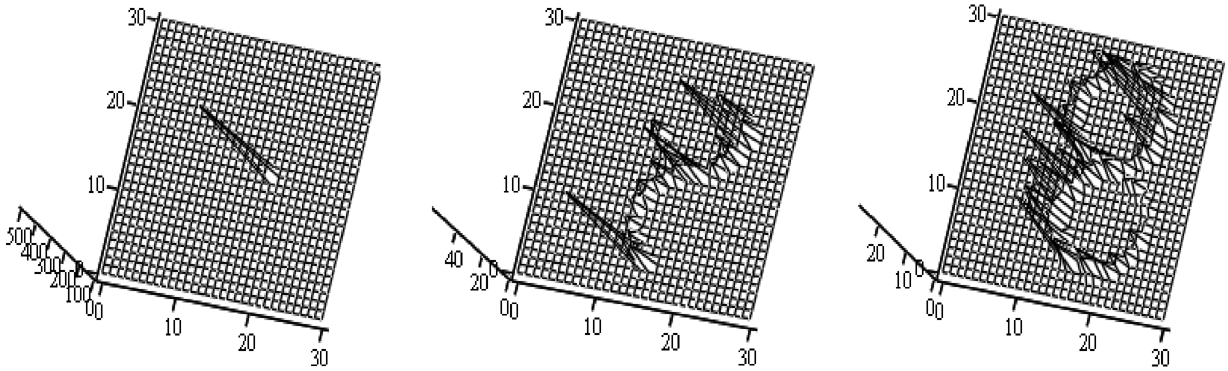
In the future, this longitudinal oscillation of the electron can be repeated many times (at the selected angle r_0'). The given electron trajectories can be compared to a model of the passage of light rays in a medium with a gradient refractive index depending on the radius.

At other chosen angles r_0', which are close enough to $r_0' =$ -0.014 *rad*, their form of oscillations changes, but the type of oscillations and their stability remain unchanged. The given trajectories of electrons can be compared with the model of the passage of light rays in a medium with a gradient refractive index depending on the radius [4, 6].

CONCLUSIONS

The selected type of gradient magnetic field determines the transformation of the distribution density in the phase space of the electron beam as they move in the transport channel. The interaction of electrons with an intense magnetic field leads to a "double bottleneck" effect of particle reflection. From the simulation results it follows that for the selected initial conditions of particles, adjustment of the initial conditions, in particular the angle of electron arrival, for the given configuration of a gradient magnetic field with a potential well affects the general properties of the beam motion.

So, when moving in the transport channel, the electron experiences stable three-dimensional oscillations. Thus, the possibility of confining a particle in a hole formed by a magnetic solenoidal field with a longitudinal pit-trap is shown.



S=0 *m* *S*=4 *m* *S*=8 *m*

Fig. 8. Density distributions of particles in the (x,y)-plane for different trajectory lengths S

REFERENCES

1. M.I. Ayzatsky, A.N. Dovbnya, V.V. Zakutin, N.G. Reshetnyak. Electron accelerators based on magnetron guns with secondary emission cathodes for flat and cylindrical irradiation of matirial surfaces // *Physics of Particles and Nuclear Letters*. 2012, v. 9, № 4-5, p. 720-728.

2. A.N. Dovbnya, S.D. Lavrinenko, V.V. Zakutin, et al. Surface modification of zirconium and Zr1%Nb alloy by the electron beam of the magnetron gun-based accelerator // *Problems of Atomic Science and Technology. Series "Physics of Radiation Effects and Radiation Materials Science"*. 2011, № 2, p. 39-45.

3. M.I. Ayzatsky, A.N. Dovbnya, A.S. Mazmanishvili, N.G. Reshetnyak, V.P. Romasko, I.A. Chertishchev. Studies on formation of the radially-directed electron beam generated by the magnetron gun with a secondary emission cathode // *Problems of Atomic Science and Technology. Series "Nuclear Physics Investigations" (66).* 2016, № 3(103), p. 11-16.

4. A.S. Mazmanishvili, N.G. Reshetnyak. Electron beam transversion management on exit of magnetic gun by gradient magnetic field // *Problems of Atomic Science and Technology. Series "Nuclear Physics Investigations" (72).* 2019, № 6(124), p. 106-113.

5. L.A. Artsimovich. *Controlled thermonuclear reactions*. M.: "Fizmatgiz", 1963, p. 496.

6. A.S. Mazmanishvili, N.G. Reshetnyak. Transformation of data array of cylindrical magnetic field of the magnetron gun and problem of the radial motion of electrons // *Applied Problems of Mathematical Modeling*. 2020, v. 3, № 1, p. 108-116.

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УТРИМАННЯ ЕЛЕКТРОННОГО ПУЧКА У ПОТЕНЦІЙНІЙ ЯМІ МАГНІТНОГО СОЛЕНОЇДАЛЬНОГО ПОЛЯ

О.С. Мазманішвілі, М.Г. Решетняк

Розглянуто інтенсивне циліндричне магнітне поле, в якому за допомогою додаткових магнітів створена потенційна яма. Вивчено динаміку електронного пучка з енергією в десятки килоелектронвольт у подовжньому і радіальному напрямках при його транспортуванні в синтезованому полі. Показано, що рух електронів можна порівняти з моделю тривимірних осциляцій потенційної ями. Представлені результати чисельного моделювання траєкторій електронів у градієнтному магнітному полі з точковим вторинноемісійним катодом, розташованим у середині системи. Розглянуто формування пучка з енергією 55 кеВ у радіальному та поздовжньому напрямках при його транспортуванні в соленоїдальному полі з великим градієнтом. Отримано режими роботи гармати, у яких частка відчуває стійкі тривимірні коливання. Вивчено вплив початкових умов при емісії виникнення осциляцій. Показано, що при заданій енергії електрона і фіксованому магнітному полі параметром, що визначає стійкість руху частинки, є полярний кут вльоту щодо осі циліндричного магнітного поля.