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ON CONSISTENCY OF DESCRIPTION OF THE $AdS_4 \times CP^3$ SUPERSTRING DYNAMICS BY THE TWO-DIMENSIONAL $OSp(4|6)/(SO(1,3)\times U(3))$ σ-MODEL

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It is examined one of the aspects of consistency of description of the superstring dynamics in the $AdS_4\times CP^3$ superspace by two-dimensional σ-model in the $OSp(4|6)/(SO(1, 3)\times U(3))$ supersymmetric coset space that is the subspace of this superspace. It is proved that equations, obtained by variation of the superstring action on the coordinates of the sector of broken supersymmetries, become consequences of fermionic equations corresponding to unbroken supersymmetries, when these coordinates turn to zero and superstring moves both in the AdS_4 and CP^3 spaces. These relations between superstring equations represent half of the Noether identities associated with local κ-symmetry of its action.

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INTRODUCTION

Construction of the quantum theory of gravitation remains a long-standing problem of theoretical physics. A possibility to construct such theory exists in the framework of the superstring theory in 10-dimensional space-time [1, 2]. It is based on the idea of relativistic extended objects – strings [3, 4] and branes [5–7] – and the concept of supersymmetry unifying bosons and fermions [8–12]. In the mid of 1990-s it was realized that application of the holographic principle in superstring theory can become important ingredient for construction of quantum gravity [13]. Manifestations of this principle were first observed in the black-hole physics [14, 15]. Subsequently G. 't Hooft suggested that it applies to quantum theory of gravity that in a bounded space-time domain can be described in terms of quantum field theory in the boundary space [16]. Concrete way of realization of the holographic principle in superstring theory in the form of the *AdS/CFT* correspondence was proposed by J. Maldacena [17–19]. According to the Maldacena's conjecture superstring theories in anti-de Sitter superspaces admit dual description as superconformal field theories in their boundary Minkowski superspaces. Presently the *AdS/CFT* correspondence is considered as the most elaborated way to define quantum gravity theory in anti-de Sitter space [20]. The first and the most well-examined example of the *AdS/CFT* dualities proposed by J. Maldacena connects the Type IIB superstring theory in the $AdS_5\times S^5$ superspace and the N=4 superconformal Yang-Mills theory in four-dimensional Minkowski space [17]. Since then there were proposed many other instances of duality between gravitational and gauge theories (see, e.g. review [21]). One of them was formulated by O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena and elaborates on the original Maldacena's example of the AdS_4/CFT_3 correspondence [22]. This ABJM duality connects the Type IIA superstring theory in the $AdS_4\times CP^3$ superspace and the D=3 N=6 superconformal Chern-Simons-matter theory with the $U(\mathrm{N})_k\times U(\mathrm{N})_{-k}$ gauge symmetry. Because of the sufficiently high symmetry presented by the $OSp(4|6)$ supergroup on the string side and in the isomorphic form of the D=3 N=6 superconformal global symmetry of the Chern-Simons-matter theory the ABJM duality can be subjected to many quantitative checks so its study attracted significant attention (see, e.g. [23–25]).

In the present note examined is the relation between fermionic equations of the Type IIA superstring in the $AdS_4\times CP^3$ superspace [26] and those of two-dimensional σ-model on the $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset manifold [27, 28] that is used to describe superstring dynamics on the basis of group-theoretic methods. In the next section discussed are limitations on applicability of the σ-model approach connected with geometry of the $AdS_4\times CP^3$ space. Then it is recapitulated the construction of the σ-model Lagrangian in terms of the Cartan 1-forms associated with generators of the $osp(4|6)/(so(1,3)\times u(3))$ quotient algebra. Further we prove that equations corresponding to variation of the superstring action on eight coordinates of the sector of broken supersymmetries become consequences of fermionic equations of the $OSp(4|6)/(SO(1,3)\times U(3))$ σ-model, when superstring propagates both in the AdS_4 and CP^3 spaces.

1. RELATION BETWEEN THE $AdS_4\times CP^3$ SUPERSTRING AND THE $OSp(4|6)/(SO(1,3)\times U(3))$ σ-MODEL

The solution of the IIA supergravity equations with the $AdS_4\times CP^3$ geometry is known since the mid of 1980-s [29–32]. It was shown to preserve 24 of 32 supersymmetries [30] and its factors can be realized as the coset spaces $AdS_4=SO(2,3)/SO(1,3)$ and $CP^3=SU(4)/U(3)$ with the isometry groups $SO(2,3)$ and $SU(4)$ respectively. The interest to this background revived when the ABJM duality conjecture was put forward. Using that $SO(2,3)=Sp(4)$ and $SU(4)=SO(6)$ can be viewed as bosonic subgroups of the orthosymplectic $OSp(4|6)$ supergroup it was suggested to extend 10-dimensional $AdS_4\times CP^3$ space to the $OSp(4|6)/(SO(1,3)\times U(3))$ superspace, whose 24 Grassmann coordinates are parameters for supersymmetry generators of the $osp(4|6)$ superalgebra [27, 28]. It is the superalgebra of the $OSp(4|6)$ supergroup of isometry of the $AdS_4\times CP^3$ superspace and its $OSp(4|6)/(SO(1,3)\times U(3))$ subsuperspace.

In [27, 28] constructed was two-dimensional $OSp(4|6)/(SO(1,3)\times U(3))$ σ-model, whose dynamical equations were proved to be classically integrable along the same lines as in [33] proved was integrability of the equations of the $AdS_5\times S^5$ superstring described by two-dimensional $PSU(2,2|4)/(SO(1,4)\times SO(5))$ σ-model [34, 35]. The key difference between two σ-models is that the (10|32)-dimensional $PSU(2,2|4)/(SO(1,4)\times SO(5))$ supercoset manifold is isomorphic to the $AdS_5\times S^5$ superspace so respective σ-model applies in all dynamical sectors of the superstring, while the $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset manifold is the subspace of the $AdS_4\times CP^3$ superspace. The $AdS_4\times CP^3$ superspace includes additional eight Grassmann coordinates that correspond to broken supersymmetries. As a consequence the complete (10|32)-dimensional $AdS_4\times CP^3$ superspace cannot be realized as a supercoset manifold [26]. So the $OSp(4|6)/(SO(1,3)\times U(3))$ σ-model describes only a subsector of the $AdS_4\times CP^3$ superstring dynamics.

Natural interpretation of this σ-model is that it arises upon partial fixing the gauge for local κ-symmetry of the $AdS_4\times CP^3$ superstring action [26] by setting to zero eight coordinates for broken supersymmetries. However, as was noted already in [27], this interpretation is not always correct. Though it goes through, when the superstring moves both in the AdS_4 and CP^3 spaces, it fails, when the superstring moves, e.g. only within the anti-de Sitter space.

As is known invariance of the action under a gauge symmetry implies, in accordance with the second Noether theorem, existence of the identities satisfied on equations following from this action. Their number equals that of independent gauge parameters. The Type II superstrings are invariant under gauge κ-symmetry with 16 independent anticommuting parameters [36, 37]. So 16 of 32 fermionic equations of the Type II superstring, that result from the action variation on the superspace Grassmann coordinates, are not independent but follow from other 16 fermionic equations. In the $AdS_4\times CP^3$ superspace, whenever vanishing conditions of eight coordinates for broken supersymmetries can be considered as the partial κ-symmetry gauge, the $OSp(4|6)/(SO(1,3)\times U(3))$ σ-model action should be invariant under eight-parameter κ-symmetry. It is the remnant of 16-parameter κ-symmetry of the complete $AdS_4\times CP^3$ superstring action. This implies that out of 24 fermionic equations of the σ-model eight are consequences of other 16 equations. The latter determine dynamics of 16 gauge-invariant fermions that are physical degrees of freedom of the $AdS_4\times CP^3$ superstring.

When superstring moves entirely in the AdS_4 space, the σ-model action becomes invariant under 12-parameter κ-symmetry transformations [27]. This implies that the σ-model describes dynamics only of 12 physical fermionic degrees of freedom of the $AdS_4\times CP^3$ superstring, while other four were among eight superspace coordinates for broken supersymmetries that were set to zero. Therefore in this case conditions of vanishing of these eight coordinates include four gauge-fixing conditions and four gauge-invariant initial data constraints. The latter are known in field theory [38], gravity [39] and string theory [40–43] and select the subspace in the space of field configurations of a theory that is preserved in the process of evolution governed by theory's dynamical equations. So this dynamical sector of the $AdS_4\times CP^3$ superstring should be described by equations that follow from the complete action [26].

Above consideration indicates that the question of consistency of description of the $AdS_4\times CP^3$ superstring dynamics by the $OSp(4|6)/(SO(1,3)\times U(3))$ σ-model is quite non-trivial. In [27] it was checked that the number of physical gauge-invariant degrees of freedom described by the σ-model matches that of the $AdS_4\times CP^3$ superstring in the sector, where it moves both in the AdS_4 and CP^3 spaces. Also there was verified that in the Penrose limit of the $AdS_4\times CP^3$ space-time the quadratic part of the σ-model Lagrangian coincides with the quadratic part of the superstring Lagrangian. Note that the form of superstring Lagrangian up to quadratic order in fermions is known for arbitrary curved background satisfying superfield constraints of Type II supergravities [44].

Here we study the relation between fermionic equations of the σ-model and superstring, when it propagates both in the AdS_4 and CP^3 spaces. In this case eight equations obtained by variation of the superstring action on coordinates of the sector of broken supersymmetries are not satisfied identically, when these coordinates are set to zero, and cannot be obtained from the σ-model action, whereas other 24 fermionic equations of the superstring reduce to fermionic equations of the σ-model. It was proved in [45] that to leading order in the Grassmann coordinates parameterizing the $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset space these eight equations become consequences of fermionic equations of the σ-model, when superstring moves both in AdS_4 and CP^3 spaces. In this note it is proved that this result holds in general irrespective to parameterization of the $OSp(4|6)/(SO(1,3)\times U(3))$ superspace, i.e. to all orders in the fermionic coordinates. This is achieved by using the representation for superstring equations in terms of the Cartan forms that encode local geometry of the $OSp(4|6)/(SO(1,3)\times U(3))$ manifold.

2. TWO-DIMENSIONAL $OSp(4|6)/(SO(1,3)\times U(3))$ σ-MODEL

As is known geometric objects of a symmetric space G/H with the isometry group G are constructed out of differential Cartan forms taking value in the isometry algebra g. Namely, vielbein 1-form is identified with the Cartan forms for the quotient algebra g/h and connection 1-form – with Cartan forms related to the stability algebra h. This identification is generalized to the case of supercoset spaces $\underline{G}/\underline{H}$. The $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset manifold belongs to the family of supercoset manifolds with isometry superalgebras $\underline{g}$, whose (anti)commutation relations are invariant under four-element automorphism Υ. The stability subalgebra generators are invariant under this automorphism. So on such supercoset manifolds classically integrable two-dimensional σ-models can be defined [33], [46–48]. Invariance of isometry superalgebras under this Z_4 automorphism Υ induces division of their generators into four eigenspaces with eigenvalues ± 1 and $\pm i$

$$\Upsilon(g_{(k)}) = i^{(k)} g_{(k)}, \quad k = 0, 1, 2, 3. \tag{1}$$

So (anti)commutation relations of $\underline{g}$ can be presented in the Z_4-graded form

$$[g_{(j)}, g_{(k)}\} = g_{(j+k)\,\mathrm{mod}\,4}. \tag{2}$$

In case of the $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset space the $so(1,3)\oplus u(3)$ algebra of its stability group plays the role of $g_{(0)}$ subalgebra of the $osp(4|6)$ superalgebra. If generators of the $sp(4)=so(2,3)$ and $so(6)=su(4)$ subalgebras of $osp(4|6)$ superalgebra are $M_{\underline{mn}}=\{M_{0'm'}, M_{m'n'}\}$ ($\underline{m}$, $\underline{n}$=0', 0, 1, 2, 3, m', n'=0, 1, 2, 3) and $V_A{}^B=\{V_a{}^b, V_a{}^4, V_4{}^b\}$ (A, B=1, 2, 3, 4, a, b=1, 2, 3), then $g_{(0)}=\{M_{m'n'}, V_a{}^b\}$ and $g_{(2)}=\{M_{0'm'}, V_4{}^a, V_a{}^4\}$. Under the Z_4 automorphism generators from $g_{(0)}$ and $g_{(2)}$ have eigenvalues +1 and -1 respectively. Linear combinations of the above introduced generators define generators of the conformal algebra of three-dimensional Minkowski space $conf(1, 2)=so(2, 3)$

$$D = -2M_{0'3},\ P_m = M_{0'm} - M_{3m},$$
$$K_m = M_{0'm} + M_{3m},\ m = 0,1,2. \tag{3}$$

The $osp(4|6)$ superalgebra belongs to the family of superalgebras for which supersymmetry generators transform in the spinor representations of bosonic subalgebras [49]. So transferring to isomorphic realization of the $so(2,3)$ algebra as conformal algebra results in isomorphic realization of the $osp(4|6)$ superalgebra as D=3 N=6 superconformal algebra $sconf(3|6)$. Its 24 supersymmetry generators are given by 12 Poincare supersymmetry generators Q^a_μ, $\overline{Q}_{\mu\, a}$ and 12 special conformal supersymmetry generators S^a_μ, $\overline{S}_{\mu\, a}$ that transform as two-component $sl(2,R)\subset so(2,3)$ spinors (μ=1,2) and in the (anti)fundamental representation of $su(3)\subset su(4)$ (a=1, 2, 3). Then generators from $g_{(1)}$ and $g_{(3)}$ eigenspaces are defined by

$$Q_{(1)\mu}{}^a = Q^a_\mu + iS^a_\mu,\ \overline{Q}_{(1)\mu\, a} = \overline{Q}_{\mu\, a} - i\overline{S}_{\mu\, a};$$
$$Q_{(3)\mu}{}^a = Q^a_\mu - iS^a_\mu,\ \overline{Q}_{(1)\mu\, a} = \overline{Q}_{\mu\, a} + i\overline{S}_{\mu\, a}. \tag{4}$$

The whole set of (anti)commutation relations of generators of the D=3 N=6 superconformal algebra can be found in our work [50].

Left Cartan forms associated with the generators of the D=3 N=6 superconformal algebra are defined by the relation [51]

$$\begin{aligned} C(d) = G^{-1}dG = \omega^m(d)P_m + c^m(d)K_m \\ + \Delta(d)D + G^{mn}(d)M_{mn} \\ + \Omega_a(d)T^a + \Omega^a(d)T_a + \Omega_a{}^b(d)V_b{}^a \\ + \omega^\mu_a(d)Q^a_\mu + \overline{\omega}^{\mu\, a}(d)\overline{Q}_{\mu\, a} \\ + \chi_{\mu\, a}(d)S^{\mu\, a} + \overline{\chi}^a_\mu(d)\overline{S}^\mu_a \in sconf(3|6). \end{aligned} \tag{5}$$

They can be presented as the sum

$$C(d) = C_{(0)}(d) + C_{(2)}(d) + C_{(1)}(d) + C_{(3)}(d), \tag{6}$$

where each summand takes value in respective eigenspace under the Z_4 automorphism

$$\begin{aligned} C_{(0)}(d) = 2G^{3m}(d)M_{3m} + G^{mn}(d)M_{mn} \\ + \Omega_a{}^b(d)V_b{}^a \in g_{(0)}, \\ C_{(2)}(d) = 2G^{0'm}(d)M_{0'm} + \Delta(d)D \\ + \Omega_a(d)T^a + \Omega_a(d)T^a \in g_{(2)}, \\ C_{(1)}(d) = \omega_{(1)a}^{\ \mu}(d)Q_{(1)\mu}{}^a + \overline{\omega}_{(1)}{}^{\mu\, a}(d)\overline{Q}_{(1)\mu\, a} \in g_{(1)}, \\ C_{(3)}(d) = \omega_{(3)a}^{\ \mu}(d)Q_{(3)\mu}{}^a + \overline{\omega}_{(3)}{}^{\mu\, a}(d)\overline{Q}_{(3)\mu\, a} \in g_{(3)}. \end{aligned} \tag{7}$$

Cartan forms $G^{0'm}(d)=1/2(\omega^m(d)+c^m(d))$, $\Delta(d)$ and $\Omega_a(d)$, $\Omega^a(d)$ associated with the $g_{(2)}$ generators are identified with bosonic components of the $OSp(4|6)/(SO(1,3)\times U(3))$ supervielbein and fermionic Cartan forms

$$\begin{aligned} \omega_{(1)a}^{\ \mu}(d) = \frac{1}{2}\left(\omega^\mu_a(d) + i\chi^\mu_a(d)\right), \\ \omega_{(3)a}^{\ \mu}(d) = \frac{1}{2}\left(\omega^\mu_a(d) - i\chi^\mu_a(d)\right) \end{aligned} \tag{8}$$

and c.c. are identified with its fermionic components. Cartan forms for the $g_{(0)}$ generators $G^{3m}(d) = -1/2(\omega^m(d)-c^m(d))$, $G^{mn}(d)$ and $\Omega_a{}^b(d)$ are identified with the connection 1-form on the $OSp(4|6)/(SO(1,3)\times U(3))$ supermanifold.

The $OSp(4|6)/(SO(1,3)\times U(3))$ σ-model action has known form [27, 28]

$$S_{\sigma-\mathrm{model}} = \int d^2\xi\left(L_{kin} + L_{WZ}\right), \tag{9}$$

where ξ^i are local world-sheet coordinates (i,j=(τ, σ)). In [51] kinetic and Wess-Zumino terms of the Lagrangian were expressed in terms of mappings of the Cartan forms in conformal basis to the world sheet

$$\begin{aligned} L_{kin} = -\frac{1}{2}\gamma^{ij}\left(G_i^{0'm}G^{0'}_{j\,m} + \Delta_i\Delta_j + \Omega_{ia}\Omega^a_j\right) \\ = -\frac{1}{2}\gamma^{ij}\left(\frac{1}{4}(\omega_{im} + c_{im})(\omega^m_j + c^m_j) + \Delta_i\Delta_j + \Omega_{ia}\Omega^a_j\right) \end{aligned} \tag{10}$$

and

$$\begin{aligned} L_{\mathrm{WZ}} = -\varepsilon^{ij}\left(\omega_{(1)ia}^{\ \ \mu}\varepsilon_{\mu\nu}\overline{\omega}_{(3)j}{}^{\nu a} + (1\leftrightarrow 3)\right) \\ = -\frac{1}{2}\varepsilon^{ij}\left(\omega^{\ \mu}_{ia}\varepsilon_{\mu\nu}\overline{\omega}^{\nu a}_j + \chi_{i\mu a}\varepsilon^{\mu\nu}\overline{\chi}_{j\nu}{}^a\right). \end{aligned} \tag{11}$$

3. RELATION BETWEEN FERMIONIC EQUATIONS OF THE *AdS*$_4\times$*CP*^{3} SUPERSTRING AND THE *OSp*(4|6)/(*SO*(1,3)×*U*(3)) σ-MODEL

Equations obtained by taking variation of the σ-model action (9) on fermionic Cartan forms (8), in which differentials of coordinates of the $OSp(4|6)/(SO(1,3)\times U(3))$ superspace are replaced by their variations, can be brought to the following form

$$V^{ij}_+ M_{(1)i\hat{a}\nu}{}^{\mu\hat{b}}\begin{pmatrix} \omega_{(1)jb}{}^{\nu} \\ \overline{\omega}_{(1)j}{}^{\nu b}\end{pmatrix} = 0 \tag{12}$$

and

$$V^{ij}_- M_{(3)i\hat{a}\nu}{}^{\mu\hat{b}}\begin{pmatrix} \omega_{(3)jb}{}^{\nu} \\ \overline{\omega}_{(3)j}{}^{\nu b}\end{pmatrix} = 0\,. \tag{13}$$

Here $V^{ij}_\pm = 1/2\left(\gamma^{ij} \pm \varepsilon^{ij}\right)$ are projectors that single out (anti)self-dual parts of a world-sheet vector and matrices $M_{(1,3)i\hat{a}\nu}{}^{\mu\hat{b}}$ are defined as

$$M_{(1)\,j\hat{a}\nu}^{\;\;\mu\hat{b}} = \begin{pmatrix} \delta_a^b m_i{}^\mu{}_\nu & -i\delta_\nu^\mu \varepsilon_{acb}\Omega_i{}^c \\ -i\delta_\nu^\mu \varepsilon^{acb}\Omega_{ic} & -\delta_b^a \overline{m}_i{}^\mu{}_\nu \end{pmatrix} \quad (14)$$

and

$$M_{(3)\,j\hat{a}\nu}^{\;\;\mu\hat{b}} = \begin{pmatrix} -\delta_a^b \overline{m}_i{}^\mu{}_\nu & i\delta_\nu^\mu \varepsilon_{acb}\Omega_i{}^c \\ i\delta_\nu^\mu \varepsilon^{acb}\Omega_{ic} & \delta_b^a m_i{}^\mu{}_\nu \end{pmatrix}, \quad (15)$$

where

$$\begin{aligned} m_i{}^\mu{}_\nu &= -iG_i^{\;0'm}\sigma_m{}^\mu{}_\nu + \Delta_i\delta_\nu^\mu, \\ \overline{m}_i{}^\mu{}_\nu &= iG_i^{\;0'm}\sigma_m{}^\mu{}_\nu + \Delta_i\delta_\nu^\mu. \end{aligned} \quad (16)$$

As was shown in [50] respective fermionic equations of the superstring reduce to equations (12) and (13), when coordinates of the sector of broken supersymmetries of the $AdS_4\times CP^3$ superspace are set to zero.

In [50] there were also obtained superstring equations that correspond to variation of the action on coordinates of the sector of broken supersymmetries

$$V_+^{ij}\Omega_i{}^b\omega_{(1)\,jb}^{\;\;\nu} = V_+^{ij}\Omega_{ib}\overline{\omega}_{(1)\,j}^{\;\;\nu b} = 0\,, \quad (17)$$

$$V_-^{ij}\Omega_i{}^b\omega_{(3)\,jb}^{\;\;\nu} = V_-^{ij}\Omega_{ib}\overline{\omega}_{(3)\,j}^{\;\;\nu b} = 0\,. \quad (18)$$

Let us prove that eight equations (17) and (18) follow from equations (12) and (13) of the $OSp(4|6)/(SO(1,3)\times U(3))$ σ-model. Recall that action of the projectors $V_\pm^{ij}$ on a two-component vector factorizes

$$\begin{aligned} &V_\pm^{ij}F_j = V_\pm^i F_\mp, \\ &V_\pm^i = \frac{1}{2}\begin{pmatrix} 1 \\ \dfrac{\gamma^{\tau\sigma}\mp 1}{\gamma^{\tau\tau}} \end{pmatrix}, \quad F_\mp = \gamma^{\tau\tau}F_\tau + (\gamma^{\tau\sigma}\pm 1)F_\sigma. \end{aligned} \quad (19)$$

So one can write equations (12) and (13) in the form

$$M_{(1)+\hat{a}\nu}^{\;\;\mu\hat{b}} \begin{pmatrix} \omega_{(1)-b}^{\;\;\nu} \\ \overline{\omega}_{(1)-}^{\;\;\nu b} \end{pmatrix} = 0 \quad (20)$$

and

$$M_{(3)-\hat{a}\nu}^{\;\;\mu\hat{b}} \begin{pmatrix} \omega_{(3)+b}^{\;\;\nu} \\ \overline{\omega}_{(3)+}^{\;\;\nu b} \end{pmatrix} = 0\,. \quad (21)$$

These equations include 12×12 matrices

$$M_{(1)+\hat{a}\nu}^{\;\;\mu\hat{b}} = \begin{pmatrix} \delta_a^b m_+{}^\mu{}_\nu & -i\delta_\nu^\mu \varepsilon_{acb}\Omega_+{}^c \\ -i\delta_\nu^\mu \varepsilon^{acb}\Omega_{+c} & -\delta_b^a \overline{m}_+{}^\mu{}_\nu \end{pmatrix} \quad (22)$$

and

$$M_{(3)-\hat{a}\nu}^{\;\;\mu\hat{b}} = \begin{pmatrix} -\delta_a^b \overline{m}_-{}^\mu{}_\nu & i\delta_\nu^\mu \varepsilon_{acb}\Omega_-{}^c \\ i\delta_\nu^\mu \varepsilon^{acb}\Omega_{-c} & \delta_b^a m_-{}^\mu{}_\nu \end{pmatrix}, \quad (23)$$

whose rank equals eight. It can be shown that matrices $m_\pm{}^\mu{}_\nu$ and $\overline{m}_\pm{}^\mu{}_\nu$ satisfy the relations

$$\begin{aligned} m_\pm{}^\mu{}_\nu \overline{m}_\pm{}^\nu{}_\lambda &= \overline{m}_\pm{}^\mu{}_\nu m_\pm{}^\nu{}_\lambda = \\ &= \delta_\lambda^\mu\left(G_\pm\cdot G_\pm + \Delta_\pm\Delta_\pm\right). \end{aligned} \quad (24)$$

The right-hand side is non-zero, when superstring moves both in AdS_4 and CP^3. To see this, apply projectors $V_\pm^{ij}$ to equations for auxiliary world-sheet metric γ_{ij} that follow from (9)

$$\begin{aligned} &G_i^{\;0'}{}_m G_j^{\;0'm} + \Delta_i\Delta_j + \frac{1}{2}\left(\Omega_{ia}{}^4\Omega_{j4}{}^a + \Omega_{ja}{}^4\Omega_{i4}{}^a\right) \\ &-\frac{1}{2}\gamma_{ij}\gamma^{kl}\left(G_k^{\;0'}{}_m G_l^{\;0'm} + \Delta_k\Delta_l + \Omega_{ka}{}^4\Omega_{l4}{}^a\right) = 0. \end{aligned} \quad (25)$$

As a result one obtains two equations

$$G_\pm^{\;0'}{}_m G_\pm^{\;0'm} + \Delta_\pm^2 + \Omega_{\pm a}\Omega_\pm{}^a = 0\,. \quad (26)$$

Taking into account that for superstring moving both in AdS_4 and CP^3 $\Omega_{\pm a}\neq 0$ and $\Omega_\pm{}^a \neq 0$ yields the above conclusion.

Consider in detail the system of equations (20). Let us apply to it projection matrix

$$\Pi_{(1)} = \begin{pmatrix} \delta_a^b\delta_\nu^\mu & -i\dfrac{m_+{}^\mu{}_\nu}{G_+\cdot G_+ + \Delta_+^2}\varepsilon_{acb}\Omega_+{}^c \\ i\dfrac{\overline{m}_+{}^\mu{}_\nu}{G_+\cdot G_+ + \Delta_+^2}\varepsilon^{acb}\Omega_{+c} & \delta_b^a\delta_\nu^\mu \end{pmatrix} \quad (27)$$

that has rank equal eight and satisfies the condition $\Pi_{(1)}^3 - 3\Pi_{(1)}^2 + 2\Pi_{(1)} = 0$. Since $G_+\cdot G_+ + \Delta_+^2 \neq 0$ the resulting system of equations acquires the form

$$\begin{pmatrix} m_+{}^\mu{}_\nu\Omega_{+a}\Omega_+{}^b & 0 \\ 0 & -\overline{m}_+{}^\mu{}_\nu\Omega_+{}^a\Omega_{+b} \end{pmatrix}\begin{pmatrix} \omega_{(1)-b}^{\;\;\nu} \\ \overline{\omega}_{(1)-}^{\;\;\nu b} \end{pmatrix} = 0\,. \quad (28)$$

Further using (24) yields that the system (28) is equivalent to four equations

$$\Omega_+{}^b\omega_{(1)-b}^{\;\;\nu} = \Omega_{+b}\overline{\omega}_{(1)-}^{\;\;\nu b} = 0 \quad (29)$$

coinciding with (17). Analogously it can be shown that the equations (18) follow from (21).

CONCLUSIONS

The main result of this note is the proof that eight equations corresponding to variation of the $AdS_4\times CP^3$ superstring action on coordinates of the sector of broken supersymmetries are not independent but follow from other fermionic equations, when these coordinates turn to zero. This proof generalizes known result obtained to linear order in 24 Grassmann coordinates of the $AdS_4\times CP^3$ superspace that correspond to unbroken supersymmetries. It implies that in the case, when coordinates of the sector of broken supersymmetries are set to zero, all non-trivial fermionic equations of the $AdS_4\times CP^3$ superstring can be obtained from the $OSp(4|6)/(SO(1,3)\times U(3))$ σ-model action. This result represents one of the consistency checks of description of the $AdS_4\times CP^3$ superstring dynamics by two-dimensional $OSp(4|6)/(SO(1,3)\times U(3))$ σ-model. The proof relies on the assumption that superstring moves both in the AdS_4 and CP^3 spaces. This is one of the sectors of superstring dynamics, where the σ-model description is valid because there the condition of vanishing of coordinates for broken supersymmetries fixes eight of 16 parameters of the κ-symmetry of the action. It is also discussed the relation between the superstring and the σ-model in another dynamical sector, in which it moves only within the AdS_4 space. In this sector dynamics of the superstring cannot be described by the σ-model. The reason is that in such case the condition of vanishing of coordinates for broken supersymmetries contains not only those partially fixing the κ-symmetry gauge but also four initial data constraints. The latter exclude four world-sheet fermionic fields that describe physical excitations of the superstring in this dynamical sector.

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ПРО УЗГОДЖЕНІСТЬ ОПИСУ ДИНАМІКИ $AdS_4\times CP^3$-СУПЕРСТРУНИ ДВОВИМІРНОЮ $OSp(4|6)/(SO(1,3)\times U(3))$ σ-МОДЕЛЛЮ

Д.В. Уваров

Досліджується один з аспектів узгодженості опису динаміки суперструни в $AdS_4\times CP^3$-суперпросторі двовимірною σ-моделлю в $OSp(4|6)/(SO(1,3)\times U(3))$-суперсиметричному фактор-просторі, який є підпростором цього суперпростору. Доводиться, що рівняння, здобуті варіюванням дії суперструни за координатами сектора порушених суперсиметрій, стають висновками з ферміонних рівнянь, які відповідають непорушеним суперсиметріям, коли ці координати обертаються на нуль і суперструна рухається як у AdS_4, так і у CP^3-просторі. Ці співвідношення між рівняннями суперструни представляють половину тотожностей Ньотер, пов'язаних з локальною κ-симетрією її дії.