https://doi.org/10.46813/2024-153-073

ON THE EFFICIENCY OF WAVE-PARTICLE INTERACTION

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The efficiency of acceleration of charged particles during cyclotron resonances and acceleration of particles in a vacuum without an external magnetic field are considered. It is shown that the statement formulated in Lawson-Woodward theorem that relativistic particles do not exchange energy with an external electromagnetic wave in a vacuum is not sufficiently justified.

PACS: 41.75.Jv; 06.30.Gv; 05.45.Xt

INTRODUCTION

Integrals of motion of nonlinear equations are the key objects in the description of resonance processes. When studying many process, these quantities are determined in the first place. Cyclotron resonances (CRs) play a particularly important role among resonances. They are the most commonly used and also the best studied. It is enough to note that these resonances determine the main mechanism of high-frequency heating of plasma in thermonuclear devices. As for the resonance conditions, they represent some relations between the characteristics of the fields and the parameters of the system under which resonant interaction is possible. In general, it is rather difficult to theoretically describe such conditions, since they can be found in an explicit form only using the description in terms of new dependent variables. There can be several such replacements of dynamic variables. As a result, it is possible to formulate new additional resonance conditions for new variables, as well as new equations for finding new variables. Therefore, it is more convenient to determine the resonance conditions taking into account the equations for the dependent variables. On this consideration, in particular, the concept of nonlinear cyclotron resonance arose. The analysis of nonlinear resonances allows a deeper understanding of the dynamics of the studied system.

As an example, we can point out the discovery of the effect of unlimited stochastic acceleration of charged particles during cyclotron resonances [1]. Previously, it was assumed that such unlimited acceleration was possible only at autoresonance. The general tendency during HF heating of the plasma is an increase in the intensity of the HF field. Taking into account this tendency led the authors [2, 3] to the need to take it into account in the conditions of cyclotron resonances. The consequence of such consideration is the appearance of new conditions of resonance, as well as a radical change in the dynamics of particles under these conditions. Notice the most important features of them. The time evolution of particles became stepwise and new regimes with chaotic dynamics appeared. In [4, 5], the same problem of particle dynamics was solved using a slightly different change of variables. It was possible not only to obtain the same results (steps and chaotic regimes), but to understand in detail the dynamics of particles at the steps.

When studying new cyclotron resonances, attention was drawn to ***two features***. ***First***: if the wave propagates strictly along a constant magnetic field (the wave vector has only one component), then all additional cyclotron resonances (except autoresonance) disappear. ***Second***: if, under the conditions of new CRs, the magnitude of the external magnetic field is reduced, then the density of resonance conditions increases significantly (Fig. 1 below). Taking these two features into account allowed the authors of [3–5] to formulate conditions for the effective acceleration of charged particles by a field of wave in a vacuum without a constant external magnetic field.

This paper presents the studies of the conditions and efficiency of acceleration of charged particles both at new CRs and without an external magnetic field. In particular, the dynamics of particles was studied under the conditions of the Lawson-Woodworth theorem [6, 7].

1. STATEMENT OF THE PROBLEM AND BASIC EQUATIONS

Consider a charged particle that moves in the external magnetic field $\vec{H}_0$ and in the field of a plane electromagnetic wave, which has the following components:

$$\begin{aligned} \boldsymbol{\varepsilon} &= \mathrm{Re}(\mathbf{E}\exp(i\omega t - i\mathbf{kr})), \\ \mathbf{H} &= \mathrm{Re}\left(\frac{c}{\omega}[\mathbf{kE}]\exp(i\omega t - i\mathbf{kr})\right), \end{aligned} \tag{1}$$

where $\mathbf{E} = E_0\boldsymbol{\alpha}$, $\boldsymbol{\alpha} = \{\alpha_x, i\alpha_y, \alpha_z\}$ is the wave polarization vector.

We choose a coordinate system in which the wave vector of the wave has only two components k_x and k_z. It is also convenient to use the following dimensionless dependent and independent variables:

$$\mathbf{p} \to \mathbf{p}/mc,\ \tau \to \omega t,\ \mathbf{r} \to \frac{\omega}{c}\mathbf{r}.$$

The equations of motion in these variables are the following:

$$\frac{d\mathbf{p}}{d\tau}=\left(1-\frac{\mathbf{kp}}{\gamma}\right)\mathrm{Re}\left(\boldsymbol{\varepsilon}e^{i\psi}\right)+\frac{\omega_H}{\gamma}[\mathbf{ph}]+ \tag{2}$$
$$+\frac{\mathbf{k}}{\gamma}\mathrm{Re}\left[(\boldsymbol{\varepsilon}\cdot\mathbf{p})e^{i\psi}\right],$$
$$\mathbf{v}=\frac{d\mathbf{r}}{d\tau}=\frac{\mathbf{p}}{\gamma},\ \dot{\psi}=\frac{d\psi}{d\tau}=1-\frac{\mathbf{kp}}{\gamma},$$

where $\mathbf{h}=\mathbf{H}/H_0$, $\omega_H=eH_0/mc\omega$, $\boldsymbol{\varepsilon}=\varepsilon_0\boldsymbol{\alpha}$, $\varepsilon_0=(eE_0/mc\omega)$ is a wave-force parameter (nonlinear parameter), $\psi=\tau-\mathbf{kr}$, $\mathbf{k}$ is the unit vector in the direction of the wave propagation, $\gamma=(1+\mathbf{p}^2)^{1/2}$ is the particle's dimensionless energy (measured in units of mc^2), $\mathbf{p}$ is the particle momentum, H_0 is the amplitude of the external magnetic field.

Multiplying the first equation (2) by $\mathbf{p}$, we obtain a useful equation that describes the change in particle energy:

$$\frac{d\gamma}{d\tau}=\mathrm{Re}\left(\mathbf{v}\boldsymbol{\mathcal{E}}e^{i\psi}\right). \tag{3}$$

Equations (2) and (3) have the integrals [4]:

$$\mathbf{p}+\mathrm{Re}\left(i\boldsymbol{\varepsilon}e^{i\psi}\right)-\omega_H[\mathbf{rh}]-\mathbf{k}\gamma= \tag{4}$$
$$=\mathbf{p_0}-\mathbf{k}\gamma_0+\mathrm{Re}\left(i\boldsymbol{\varepsilon}e^{i\psi_0}\right)-\omega_H[\mathbf{r_0h}]=\mathrm{const}.$$

Here the index "0" denotes the values of the initial variables.

2. NEW VARIABLES

It is known that the change of variables can be a powerful tool that in many cases allows us to fully solve a problem and see important features of the problem that are difficult or even impossible to see in other variables. All these features of variable changes will be visible below. All analytical results are obtained using the following new dimensionless variables:

$$p_x=p_\perp\cos\theta,\ p_y=p_\perp\sin\theta,\ p_z=p_\parallel, \tag{5}$$
$$x=\xi-\frac{p_\perp}{\omega_H}\sin\theta,\ y=\eta+\frac{p_\perp}{\omega_H}\cos\theta;$$

$$p_x=p_\perp\cos\theta,\ p_y=p_\perp\sin\theta,\ p_z=p_\parallel, \tag{6}$$
$$x=\xi-\frac{p_\perp}{\omega_H\gamma}\sin\theta,\ y=\eta+\frac{p_\perp}{\omega_H\gamma}\cos\theta;$$

$$p_x=p_\perp\cos\theta,\ p_y=p_\perp\sin\theta,\ p_z=p_\parallel, \tag{7}$$
$$x=\xi-\frac{p_\perp}{\gamma\dot{\psi}}\sin\theta,\ y=\eta+\frac{p_\perp}{\gamma\dot{\psi}}\cos\theta.$$

New CR: Using the first change of variables (5) one can found the resonance conditions [2]:

$$\dot{\theta}_n=\left(1-k_zv_z-n\frac{\omega_H}{\gamma}\right)-2\varepsilon\frac{k_x}{\omega_H}J_n(\mu)\sin\theta_n, \tag{8}$$

where $\mu=k_xp_\perp/\omega_H$.

The first term represents the known condition for cyclotron resonances. The second takes into account the influence of the wave field on the resonance condition.

Expression (8) is the well-known Adler equation. This equation is widely used in synchronization theory (see, for example, [6, 7]). Note that the appearance of the second term leads to an increase in the range of parameters where resonances can exist.

Let us give an example of the condition of new unusual resonance taking the following parameters of the system under consideration: $n=0$, $k_z=0,\ k_x=1$, $\omega_H<<1$, $\varepsilon_x=\varepsilon_z=0$. In this case, condition (8) takes the form

$$\varepsilon=\frac{1}{\omega_H}J_0(p_\perp/\omega_H)\sin\theta_0,\ p_\perp=2,\ \theta_0=\pi/4. \tag{9}$$

Fig. 1 shows the conditions for new resonances. The usual cyclotron resonances in this figure are located in the form of separate points on the axis $\varepsilon=0$. New resonances are located on the lines defined by formula (9).

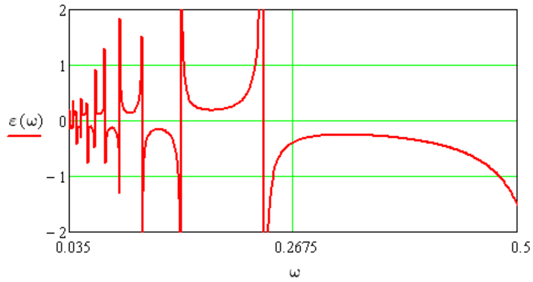

Fig. 1. Parameters ($\varepsilon,\omega=\omega_H$) for which resonances can exist. Other parameters: n=0, Kz=0.9, Pperp=3, only Ey, $\theta_0=\pi/4$, Vz=0.9

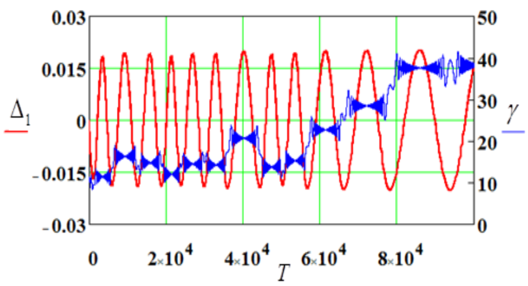

Fig. 2. Time-dependences of the energy and function Δ_1

The use of transformation (6) made it possible to discover new features of resonances. In this case, these resonance conditions take the form [3]

$$\frac{d\theta_n}{dt}=\Delta_0-\Delta_1-\Delta_2, \tag{10}$$

where $\Delta_0=\left(1-k_zv_z-n\dfrac{\omega_H}{\gamma}\right)$,

$$\Delta_1=k_xv_\perp\cos\theta\left(1-\frac{1}{\gamma}\right),\qquad \Delta_2\approx\varepsilon.$$

This condition includes three components. The first term describes the well-known conditions for CR. The last one is proportional to the intensity of the electromagnetic wave. Particular attention should be paid to the second term. This new function determines all main features of the particle dynamics (see Figs. 2). An essential element of this dynamics is its stepwise nature. The time width of the steps coincides with the period of the function Δ_1. At the step itself, the fast energy fluctuations decrease on the growing interval of the function Δ_1 ($\partial\Delta_1/\partial\tau > 0$). On the decreasing interval of the function Δ_1 ($\partial\Delta_1/\partial\tau < 0$), the amplitudes of these oscillation increase. Jumps from one step to another occur at the minima of the function Δ_1.

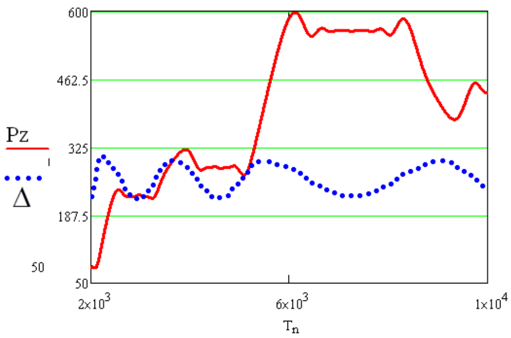

Fig. 3. Time evolution of longitudinal momentum when wave-vector is practically coincided with direction of external magnetic field ($\vec{k} \parallel \vec{H}_0$)

One should pay attention to the following fact. Using the change of variables (5), the condition for resonances (8) was obtained, and when using transformation (6), the condition for resonances has the form (10). These different terms are used to describe the same process. Which one should you use? To answer this question, let us note that the process of resonant interaction is characterized not only by the resonant condition. It is determined by the equation of particle motion under these conditions. Taking into account the interdependence of the particle motion and resonance conditions, in particular, leads to the fact that the width of the resonance and the distance between the resonances are of nonlinear nature. This is the basis of the concept of nonlinear resonance. Therefore, to answer the question of which of the conditions (8) or (10) to use, we must understand what we want to see, what we want to understand. Let's give an example. Condition (8) is simpler, but among the new features it only describes well the appearance of steps. The dynamics characteristics of the steps themselves are hidden in the equations of motion. Moreover, the equations for energy and momenta are proportional to the Bessel functions, the arguments of which are proportional to the transverse momenta of the particles ($J_n(\mu)$, $\mu = k_x p_\perp / \omega_H$). Therefore, at large momenta the sign of the Bessel function quickly changes. At the same time, it is difficult to study in detail the process of energy exchange between particles and a wave. When using transformation (6) and condition (10), it is immediately possible to find out the main features of the dynamics of particles at the steps. In addition, the equations for energy and momentum contain Bessel functions with a different argument ($J_n(\mu)$, $\mu = k_x p_\perp / \omega_H \gamma$). Therefore, it is more convenient to study them at high energies. Thus both conditions (8) and (10) are useful. At low energy values, it is more convenient to use the condition (8) since there is a lot of experience in this area and many known results. At higher energies it is more convenient to use the condition (10). It is clear that the concept of large and small energies is very arbitrary. The efficiency of particle acceleration was studied using numerical methods for various wave and particle parameters. In this case, it is worth considering two polarizations. One of them has two components of the electric field (Ex,Ez) of the wave, the other has only one (Ey) component. The main conclusion of these studies can be formulated as follows: the most effective acceleration scheme is close to autoresonant when the wave has two components Ez and Ex. This conclusion can be illustrated by Figs. 3 and 4. These figures show the time dynamics for the wave propagating almost strictly along the external magnetic field (see Fig. 3) and the particle dynamics when the wave propagates perpendicular to the external magnetic field (see Fig. 4). It can be seen that for the wave propagating perpendicularly, the efficiency of particle acceleration is much lower. In Figs. 3 and 4, the function Δ is related to the function Δ_1 by: $\Delta = \Delta_1 \cdot 300 + 250$ (see Fig. 3) and $\Delta = \Delta_1 \cdot 30 + 40$ (see Fig. 4). Parameters: $\varepsilon = 2$, $H_0 \approx 1$.

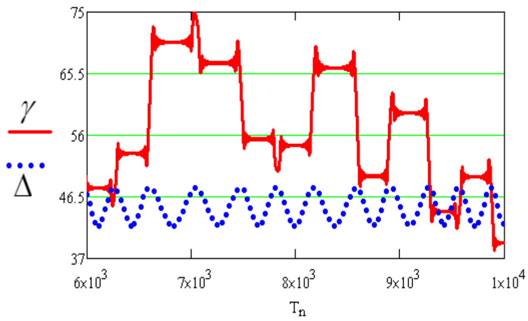

Fig. 4. Time evolution of energy when wave-vector is practically perpendicular to direction external magnetic field ($\vec{k} \perp \vec{H}_0$)

3. ACCELERATION OF CHARGED PARTICLES IN A VACUUM WITHOUT AN EXTERNAL MAGNETIC FIELD

To study particle acceleration in a vacuum without an external magnetic field, one can use the same equations as in the study of cyclotron resonances (Eqs. (2)). Only in these equations the external magnetic field strength should be set to zero. There were several works describing the rigorous solution of the problem of particle dynamics in a wave field in a vacuum [8–12]. These studies did not relate to the description of resonance and, together with the Lawson-Woodward theorem [6, 7], did not contribute to the further study of the acceleration of particles by an electromagnetic wave in a vacuum.

Taking into account the second component wave vector $\vec{k}=\{k_x, 0, k_z\}$, as well as the third type of new variables (see (7)), the resonance condition was found for the interaction of a particle with a wave in vacuum [3–5]. It should be noted that in all rigorous solutions only one component of the wave vector was considered $\vec{k}=\{0, 0, k_z\}$. This condition looks as

$$\dot{\Phi}=(1-v_z)\left[1-\frac{\varepsilon}{p_\perp}\sin\Phi\right]=0,$$

where $\Phi=(\tau-z)+\theta$. The first factor on the right-hand part (if it is equal to zero) is the Cherenkov resonance condition. The second factor follow from the Adler equation known in synchronization theory. For this equation to support the Cherenkov resonance, it is necessary that $\varepsilon > p_\perp$. This inequality should be fulfilled at least at the initial moment of time. This is in a good agreement between analytical and numerical results. The time dependence of the particle energy and momentum components corresponded to resonance: $\gamma \approx p_z \approx \varepsilon \cdot \tau;\ p_\perp \approx k_x \cdot \varepsilon \cdot \tau$.

The characteristic time dependence of the longitudinal momentum is presented in Fig. 5.

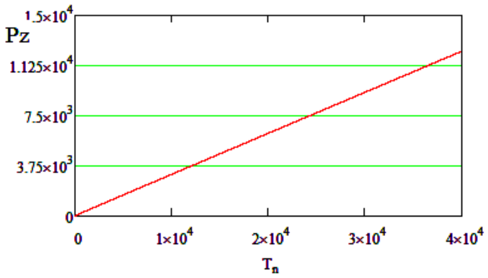

Fig. 5. Dependence of longitudinal momentum on time at $\varepsilon=2$

With an initial energy of 1 MeV at a distance of about 4 cm, the particle acquired an energy of about 6 GeV. In the transverse direction, the particle moved to a distance of 0.4 cm.

4. LAWSON-WOODWARD THEOREM

"It can be shown from Maxwell equations in vacuum that energy gain from optical fields is not possible". This is known as the Lawson–Woodward (LW) theorem. The theorem assumes that : 1. the region of interaction is infinite, 2. the laser fields are in vacuum with no walls or boundaries present, 3. the electron is highly relativistic ($\gamma >> 1$) along the acceleration path, 4. no static electric or magnetic fields are present, and 5. nonlinear effects due to ponderomotive, Lorentz force, and radiation reaction forces are neglected.

It seems that this theorem cannot be general. However, there are references to it. We found papers that indicated that the theorem does not work under the conditions stated above. All our results confirm this conclusion. The theorem is based on the assumption that the particle trajectory is a straight line. We looked at when a theorem can describe reality. To do this, we studied the dependence of the final energy of particles on the initial energies in the field of a wave with intensity . Here are the typical results of this consideration: To do this, we studied the dependence of the final energy of particles on the initial energies in the field of a wave with intensity $\varepsilon=4$. Here are the typical results of this consideration:

$p_z(0)=10,\ p_{z\max}=7820,\ \Delta p=7810;$

$p_z(0)=100,\ p_{z\max}=6179, \Delta p=6089;$

$p_z(0)=200,\ \Delta p=425;$ $\quad p_z(0)=500,\ \Delta p=122;$

$p_z(0)=1000,\ \Delta p=97;$ $\ p_z(0)=2000,\ \Delta p=88.$

Here, with an initial energy of up to 200, particles are captured by the wave field and resonant acceleration occurs. At high energies, the particles are not captured by the wave field, but in all cases the particles acquire some energy. However, as the initial energy increases, the energy exchange decreases. On the one hand, it looks like this is a condition for the validity of the theorem. On the other hand, it can be assumed that this result is due to the relativistic growth of the mass. Let us pay attention to one feature of particle acceleration in cases when particles are captured. These are the cases for the initial energy varied from 10 to 100. With an increase in the initial momentum, not only the energy transferred to the particles from the wave decreases, but the maximum momentum of the particles that they reach in the same time interval decreases (from 7820 with an initial momentum of 10 to 6089 with an initial momentum of 100).

CONCLUSIONS

There are several conditions for the resonant acceleration of charged particles, both in the presence of an external magnetic field and without it. All of them are useful and allow us to see the most important features of the wave-particle interaction from different angles (with different parameters of the process being studied).

1. The efficiency of acceleration depends significantly on the initial conditions. At large initial momentum efficiency decreases. This result can be attributed to the fact that the Lawvson-Woodward theorem is valid. However, in all cases, some average acceleration exists, and the decrease in efficiency can be explained by an increase in the relativistic mass of particles. The theorem appears to be useless.

2. The most effective acceleration of particles occurs under conditions like those of autoresonance and using polarization, which contains two components of the electric field of the wave. The effectiveness of other schemes is significantly less.

3. The concept of resonances should be explained. Using different new variables (for example (5), (6) and (7)) it is possible to obtain different forms of resonance conditions (for example (8) and (10)). In this case, the equations for energy and momenta are different. Only a joint analysis of the resonance conditions and equations for new dependent variables makes it possible to discover new features of the dynamics of the system being studied. Note that this is how the concept of nonlinear cyclotron resonances appeared.

4. An interesting feature of particle acceleration in vacuum has been discovered. Often (examples are given

above) to achieve maximum particle energy it is necessary to greatly reduce the initial energy (in the example from $p_z(0)=100$ *to* $p_z(0)=10$).

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Article received 25.09.2024

ПРО ЕФЕКТИВНІСТЬ ВЗАЄМОДІЇ ХВИЛЯ-ЧАСТИНКА

В.О. Буц, А.Г. Загородній

Розглянуто ефективність прискорення заряджених частинок під час циклотронних резонансів і прискорення частинок у вакуумі без зовнішнього магнітного поля. Показано, що сформульоване в теоремі Лоусона-Вудворда твердження про те, що релятивістські частинки не обмінюються енергією із зовнішньою електромагнітною хвилею у вакуумі, не є досить обґрунтованим.