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# USING THE REGGE-DUAL MODEL OF THE $F_2$ PROTON STRUCTURE FUNCTION AT ALL AVAILABLE ENERGIES

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A dual Regge-model of proton structure function is fitted to the existed experimental data in all Bjorken variable $0 \le x \le 1$ domain at low and moderate momentum transfer $Q^2 < 5$ GeV$^2$ combining the CKMT model with Regge-dual model and taking into account the modified background.

## INTRODUCTION

Two decade ago the precise CLAS data [1] on the proton structure function $F_2$ was successfully described within the framework of FFJLM Regge-dual model [2] which is based on the nonlinear complex Regge trajectories in form of N and Δ isobar resonances dual to the effective bosonic f trajectory in the cross-channel and smooth background based on exotic trajectories dual to the Pomeron. On the other hand the CKMT model [3] for the $F_2$ provided a good description of the $F_2$ data in domain x<0.1.

Here we will try to combine these two approaches, first of all, to adequately take into account the background when describing the already mentioned exact CLAS experiment. For this you need to modify the non-singlet part of $F_2$, which makes the main contribution to the desired background. In the region of low and moderate $Q^2 \le 5$ GeV$^2$, which interests us, there is no need to consider QCD evolution, which greatly simplifies the problem. As a result, an acceptable description of the experimental data was obtained in the entire area $0 \le x \le 1$ and moderate $Q^2$ in the only approximation.

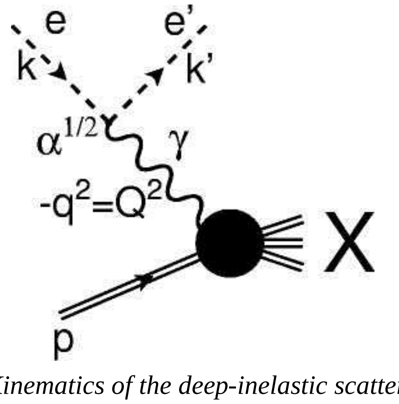


*Fig. 1. Kinematics of the deep-inelastic scattering*

## 1. THE STRUCTURE FUNCTION MODEL CHOOSING

### 1.1. STRUCTURE FUNCTION $F_2$ AT $x < 0.1$

To describe the structure function $F_2$ in the $x < 0.1$ kinematic domain, we will use one of the successful CKMT models [3] based on Regge theory. Currently available experimental data [4] on $F_2$ at low and moderate $Q^2$ can be successfully described by Regge models (including CKMT model) without the need to take into account any evolution within the framework of QCD.

In the model the nucleon structure function consists of singlet and non-singlet terms:

$$F_2(x,Q^2) = F_S(x,Q^2) + F_{NS}(x,Q^2), \qquad (1)$$

$$Q^2 = -q^2,\ x=Q^2/2pq, \qquad (2)$$

(Fig. 1), where for the singlet term, corresponding to the Pomeron contribution:

$$F_S(x,Q^2) = Ax^{-\Delta(Q^2)}(1-x)^{n(Q^2)+4}\left(\frac{Q^2}{Q^2+a}\right)^{1+\Delta(Q^2)}, \qquad (3)$$

$$\Delta(Q^2) = \Delta_0\left(1+\frac{2Q^2}{Q^2+d}\right). \qquad (4)$$

The non-singlet term, which corresponds to the secondary reggeons (f, $A_2$), reads as:

$$F_{NS}(x,Q^2) = Bx^{1-\alpha_R}(1-x)^{n(Q^2)}\left(\frac{Q^2}{Q^2-b}\right)^{\alpha_R}, \qquad (5)$$

$$n(Q^2) = \frac{3}{2}\cdot\left(1+\frac{Q^2}{Q^2+c}\right). \qquad (6)$$

The fit of $F_2(x,Q^2)$ for low x<0.1 and $Q^2 < 5$ GeV$^2$ gives a very good description of the experimental data [2] with small number of free parameters.

### 1.2. STRUCTURE FUNCTION $F_2$ AT $x > 0.1$

At the turn of the ages new data on inclusive *ep*-cross-section in the resonance region (x>0.1) and $Q^2 < 4.7$ GeV$^2$, measured at JLab (CEBAF, USA) with the CLAS detector [1] were obtained. In the other hand the model combining parton-hadron and resonance-Regge duality was developed [2], fitting the $F_2$ proton structure function in the resonance domain. The model is based on idea of reggeization of resonances both in s- and t-channels.

Nonlinear complex Regge trajectories replace individual resonance contribution. The resulting scattering amplitude is a complex function of the Mandelstam variables and $Q^2$. There are large number of resonances in the $\gamma^*$p system, but only a few of them can be good identified. Therefore, instead of identifying each individual resonance, one considers some „effective” resonance contributions. The model uses the approaches whereby taken into account the off-massshell continuation of the dual amplitude with nonlinear complex Regge trajectories.

The $F_2$ structure function related to the photoproduction cross section by

$$F_2\left(x,Q^2\right)=\frac{Q^2(1-x)}{4\pi\alpha_{em}\left(1+\frac{4m^2x^2}{Q^2}\right)}\sigma_t^{\gamma^* p}\left(s,Q^2\right), \qquad (7)$$

where the total cross section is the imaginary part of the forward Compton scattering

$$\sigma_T^{\gamma^* p}\left(W,Q^2\right)=\mathrm{Im}\left[A\left(W^2,t=0,Q^2\right)\right]. \quad (8)$$

The c.m. energy of the $\gamma^*$p-system, the negative squared photon virtuality $Q^2$ and the Bjorken variable x are related by

$$s=W^2=Q^2(1-x)/x+m^2, \qquad (9)$$

*m* is proton mass. In the two-component picture of strong interaction the direct-channel resonances are dual to cross-channel Regge exchange and the smooth background corresponds in the s-channel is dual to the Pomeron exchange in the t-channel. As explained in [5], the background corresponds in the dual model to a pole term with an exotic trajectory that does not produce any resonance.

The used trajectories contains the lightest threshold $s_0 = (m_\pi+m_P)^2$ , produces the imaginary part and the heaves thresholds producing the real part:

$$\alpha(s)=\alpha_0+\alpha_1 s+\alpha_2\left(\sqrt{s_0}-\sqrt{s_0-s}\right). \qquad (10)$$

The exotic trajectories has taken in the form:

$$\alpha_E(s)=\alpha_{E0}+\alpha_{E1}\left(\sqrt{s_E}-\sqrt{s_E-s}\right). \qquad (11)$$

The model [5] was fitted to the JLab data [1] taking into account only the contribution from three prominent resonances, namely Δ(1232), $N_1$ (1520), and $N_1$ (1680), corresponding to three baryon trajectories with one resonance on each.

To perform best fit [2] two background term were taken into account. As a result:

$$\mathrm{Im}\,A\left(s,Q^2\right)=norm\left[\frac{f_\Delta I_\Delta}{(1-R_{N_1})^2+I_{N_1}^2}+\frac{f_{N_1}I_{N_1}}{(1-R_{N_1})^2+I_{N_1}^2}+\right.$$

$$\left.+\frac{f_{N_1}I_{N_1}}{(1-R_{N_1})^2+I_{N_1}^2}+background\right], \qquad (12)$$

$$f_\Delta=\left(\frac{\left|\vec{q}\right|}{\left|\vec{q}\right|_{Q=0}}\frac{Q_0^{'2}}{Q^2+Q_0^{'2}}\right)^{2J-1=2}\times$$

$$\times\left(\left|G_+(0)\right|^2\left(\frac{Q_0^2}{Q^2+Q_0^{'2}}\right)^3+\left|G_-(0)\right|^2\left(\frac{Q_0^2}{Q^2+Q_0^2}\right)^5\right), \quad (13)$$

where

$$\left|G_0\right|^2=C^2\left(\frac{Q_{02}^2}{Q^2+Q_0^2}\right)^{2a}\frac{q_0^2}{|\vec{q}|^2}\times$$

$$\left(\frac{|\vec{q}|}{|\vec{q}|Q=0}\frac{Q_0^{'2}}{Q^2+Q_0^{'2}}\right)^{2J-1}\left(\frac{Q_0^2}{Q^2+Q_0^2}\right) \qquad (14)$$

for normal transitions ($1/2^+\to3/2^-$, $5/2^+$ , $7/2^-$, …) and

$$\left|G_0\right|^2=C^2\left(\frac{Q_0^2}{Q^2+Q_0^2}\right)^{2a}\times$$

$$\times\left(\frac{q_0^2}{\left|q\right|^2}\frac{Q_2}{Q^2+Q_0^{'2}}\right)^{2J+1}\left(\frac{Q_0^2}{Q^2+Q_0^2}\right)^{m_0} \qquad (15)$$

for the anomalous ones ($1/2^+\to1/2^-$, $3/2^+$ , $5/2^-$, …),

$$\left|\vec{q}\right|=\frac{\sqrt{\left(M^2-m^2-Q^2\right)^2+4M^2Q^2}}{2M}, \qquad (16)$$

$$q_0=\frac{M^2-m^2-Q^2}{2M}, \qquad (17)$$

M is the resonance mass. The form-factors at $Q^2$ =0 are related to the helicity photoprodiction amplitudes $A_{1/2}$ and $A_{3/2}$ by

$$\left|G_{+,-}(0)\right|=\frac{1}{\sqrt{4\pi\alpha_{em}}}\sqrt{\frac{M}{M-m}}\left|A_{1/2,3/2}\right|. \quad (18)$$

*R* and *I* denote the real and imaginary parts of the relevant trajectory specified by the subscript. Similar expressions have the $f_N^+$ and $f_N^-$ .

The background presented as:

$$background=\sum_{i=1}^{2}\frac{f_{EiI}I_{Ei}}{\left(n_{Ei}^{\min}-R_{Ei}\right)^2+I_{Ei}^2}, \qquad (19)$$

$$f_{Ei}=G_E\left(\frac{Q_{Ei}^2}{Q^2+Q_{Ei}^2}\right)^8. \qquad (20)$$

Here $n_{Ei}^{\min}$ is the lowest integer larger than the Max(Re$\alpha_{Ei}$) (to make sure there are no resonances on the exotic trajectory).

The model constructed in this way, has 23 free parameters: each resonances is characterized by three (the intercept is kept fixed) coefficients describing the relevant Regge trajectory plus two helicity photoproduction amplitudes (18). The resulting fit to the CLAS data is excellent ($\chi^2_{dof}$=1.3).

### 1.3. STRUCTURE FUNCTION FOR ALL $0 \le x \le 1$

In paper [2] the background was chosen in a form similar to that for resonances (sum of square-root thresholds), but with parameters that do not produce resonance peaks, just a smooth background. Asymptotically, as x ~1/x →0 such a background contribution increases too fast compared with the data.

To improve the quality of the fit in this model, it was necessary to double the number of parameters (to 10) responsible for the contribution of exotic states from the introduction of two different scales $Q^2_{E1}$ and $Q^2_{E2}$. Therefore we replace the failed background (19) and (20) by advanced ones in modified form of non-singlet contribution (5) and (6) as:

$$background$$
$$=(B_0-B_1\cdot Q^2)(1-x)^{n(Q^2)}x^{(1-\alpha_R)}\left(\frac{Q^2}{Q^2+c}\right)^{\alpha_R}, \qquad (21)$$

where

$$n\left(Q^2\right)=n_0\cdot\left(1+\frac{Q_0^2}{Q^2+b}\right). \qquad (22)$$

As a result, instead of two separate expressions: the non-singlet contribution in the SCMT model for the description of structure functions $F_2$ in the domain x<0.1 and the background contribution for the description of $F_2$ in the resonance region x>0.1 we have the single expression that simultaneously has the properties of both the first and the second ones with total number of 17 free parameters to describe the structural function in whole region $0 \le x \le 1$ instead of 7 parameters in the SKMT model [3] and 23 in the FFJL model [2]. I this version we obtained qualitative agreement with the experiment $\chi 2$/dof=3.80. It is worth noting that in comparison with a similar fit performed in [6], where only one term in the background mode is taken into account, a description with $\chi 2$/dof=9.6 is obtained.

The results of the data description of the structure function $F_2$ in the entire range $0.0 \le x \le 1.0$ are presented in Table. In Fig. 2. examples of the fitting result for $Q^2$ = 0.65; 1.525, and 4.225 $GeV^2$ are shown.

Fitting parameters of Regge-dual model

| | Resonances parameters | | | |
|---|---|---|---|---|
| | Parameter | Unit | Value | Error |
| $N_1^*$ | $\alpha_0$ | – | -0.8377 | Fixed |
| | $\alpha_1$ | $GeV^{-2}$ | 0.9916 | 0.0019 |
| | $\alpha_2$ | $GeV^{-1}$ | 0.0843 | 0.0018 |
| | A(1/2) | $GeV^{-1/2}$ | 0.3249 | 0.0055 |
| | A(3/2) | $GeV^{-1/2}$ | 0.0 | Fixed |
| $N_2^*$ | $\alpha_0$ | – | -0.3700 | Fixed |
| | $\alpha_1$ | $GeV^{-2}$ | 0.6784 | 0.0067 |
| | $\alpha_2$ | $GeV^{-1}$ | 0.7667 | 0.0110 |
| | A(1/2) | $GeV^{-1/2}$ | 0.5949 | 0.0126 |
| | A(3/2) | $GeV^{-1-2}$ | 0.0 | Fixed |
| **Δ** | $\alpha_0$ | – | 0.0038 | Fixed |
| | $\alpha_1$ | $GeV^{-2}$ | 0.8803 | 0.011 |
| | $\alpha_2$ | $GeV^{-1}$ | 0.1621 | 0.0013 |
| | A(1/2) | $GeV^{-1/2}$ | 0 | Fixed |
| | A(3/2) | $GeV^{-1/2}$ | -0.6068 | 0.0027 |

Pomeron parameters.
All dimensional parameters are givev in $GeV^2$

| Parameter | Value | Error |
|---|---|---|
| *A* | 0.0941 | 0.0020 |
| *a* | 0.1981 | 0.0047 |
| $\Delta_0$ | 0.1085 | 0.0012 |
| *b* | 0.045 | Fixed |
| *c* | 0.045 | Fixed |
| *b* | 1.404 | 0.226 |

Background parameters

| Parameter | Value | Error |
|---|---|---|
| $B_0$ | 2.535 | 0.088 |
| $B_1$ | 0.1930 | 0.0143 |
| $n_0$ | 2.2067 | 0.023 |
| $Q_0^2$ | 1.322 | 0.059 |
| *d* | 1.117 | Fixed |
| $\alpha_R$ | 0.400 | Fixed |
| $\chi^2$/dof = 8924.43 / (2372-17) = 3.80 | | |

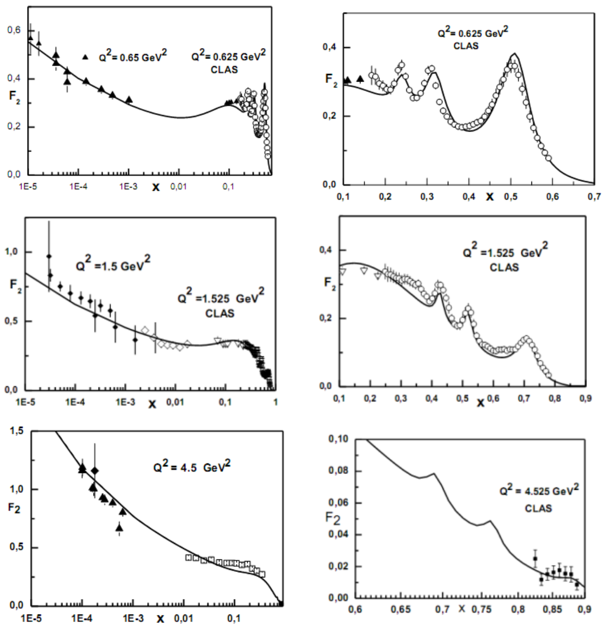


*Fig. 2. The result of fitting the structure function $F_2$ as a function of x. Left column: the entire x region, right column: enlarged image from the CLAS region of the data*

## CONCLUSIONS

Previous results describing the resonance structure of the JLab data of deep inelastic scattering of electrons on protons are extended to all *x* and moderate $Q^2$ < 5 $GeV^2$ by combining with the CKMT approach for $F_2$ and taking into account the modified background. A qualitative description of the existing experimental data for small *x* was obtained together with the exact data of JLab with the reproduction of the data of three isobaric resonances.

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## ВИКОРИСТАННЯ РЕДЖЕ-ДУАЛЬНОЇ МОДЕЛІ СТРУКТУРНОЇ ФУНКЦІЇ ПРОТОНА $F_2$ У КІНЕМАТИЧНО ДОСТУПНІЙ ОБЛАСТІ

***О. Лендєл***

Дуальна Редже-модель протонної структурної функції $F_2$ використана для підгонки до існуючих експериментальних даних у всій області змінної Бйоркена $0 \le x \le 1$ при малій і помірній передачі імпульсу $Q^2 < 5$ GeV$^2$, поєднуючи модель CKMT з моделлю, що реалізує партон-адронну дуальність, з урахуванням модифікованого фона.