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ON THE WAVE GENERATION IN A MAGNETOACTIVE WAVEGUIDE

E.V. Poklonskiy, V.M. Kuklin
V.N. Karazin Kharkov National University, Kharkiv, Ukraine
E-mails: v.m.kuklin@karazin.ua; evpoklonsky@karazin.ua

The conditions for generation in the superradiance regime and the formation of a waveguide field taking into account reflection from the ends are discussed. In different generation modes, as shown in this work, both the waveguide field, determined by reflection effects and the geometry of the system, and the field of the total radiation of core particles, that is, the superradiance field, can dominate.

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INTRODUCTION

Let us consider the excitation of an electromagnetic TE wave in a cylindrical open waveguide by electrons rotating in a constant magnetic field. Electron injection occurs at the left end of the waveguide. Then they drift along the system and, upon reaching the right end of the waveguide, are removed, and the same number of electrons are injected at the left end of the waveguide. The transverse velocity of electrons significantly exceeds the longitudinal velocity, which is small and comparable to the group velocity of the wave near the cutoff frequency. Rotating electron oscillators are capable of exciting natural oscillations of the waveguide in the transparency region. The traditional description of the generation process involves the excitation of an initially specified waveguide field of small amplitude, which is formed due to reflections in the waveguide. In this case, the particles in the active zone do not interact with each other, but interact only with the waveguide field.

A super-radiation regime is also possible, when particles interact only with each other, while the existence of a waveguide field in a completely open waveguide without reflection from its ends is ignored. Each particle, an electron (oscillator) rotating in a magnetic field, emits waves that are transparent to the waveguide and which propagate in two directions. Thus, first the total field of radiating particles is formed, essentially a superradiance field. Moreover, it is important to note that in the superradiation field it is impossible to identify integral waves that propagate in the volume of the waveguide in two directions and form a standing wave – the waveguide field.

In an open system, without taking into account reflection effects, when describing the excitation of a cyclotron wave, one can consider only the total field of interacting electron oscillators (this field is actually a superradiance field). The possibility of generating superradiance in this regime was pointed out in the work of A.G. Zagorodniy and P.I. Fomina [1].

However, the numerical simulation of field generation in such a waveguide, carried out in [2, 3], drew attention to the two-stage nature of the process. This work shows that the total radiation field of particles in the active zone of the waveguide is first generated, that is, in fact, a superradiance field. Then, due to the effects of multiple reflections of the field, two integral waves traveling in two directions can be formed. This is the waveguide field, which is a standing wave.

Below we will discuss the influence of reflection effects, as well as the influence of particle injection and drift in the core on the ratio of the amplitudes of the particle superradiance field and the waveguide field. This relationship significantly influences the choice by the system of either the traditional waveguide field generation mode, or the generation of the superradiation field, which was discussed in [1].

1. SYSTEM OF EQUATIONS

Excitation of a waveguide field. Let us present the equations that describe the generation of the field of a TE wave, the electric vector of which is perpendicular to the axis of the waveguide. The structure of the waveguide field is formed by the boundary conditions in the transparency zone and, generally speaking, its spatial form weakly depends on the active zone filled with rotating electrons-oscillators. The traditional description of the process of excitation of a waveguide field assumes that the interaction of particles with each other in the active zone is neglected. Particles interact only with the waveguide field. The equations for the longitudinal component of the magnetic field and electromagnetic waves propagating in both directions have the form [2–6]

$$\frac{d\mathrm{B}_{\pm}}{d\tau}+\theta\cdot\mathrm{B}_{\pm}=\frac{i}{2N}\sum_{j=1}^{N}a_j J_1'(a_j)\,\mathrm{e}^{-2\pi i\zeta_j}\,e^{\mp 2\pi i Z_j}\ , \quad (1)$$

where the total value of the magnetic field of the waves is equal to

$$\mathrm{B}_W(Z_j)=\frac{1}{2}[\mathrm{B}_+\,\mathrm{e}^{2\pi i Z_j}+\mathrm{B}_-\,\mathrm{e}^{2\pi i Z_j}], \quad (2)$$

and $B_{\pm}=|B_{\pm}|\exp\{i\varphi_{\pm}\}$. Equations of motion for electrons rotating in a constant magnetic field

$$2\pi\frac{d\zeta_i}{d\tau}=\eta_i(1-\alpha)+J_1(a_i)[1-\frac{1}{a_i^2}]\cdot \\ \cdot\mathrm{Re}\{\exp(2\pi i\zeta_i)\cdot\mathrm{B}_W(Z_i)\}, \quad (3)$$

$$da_i/d\tau=\mathrm{Re}\{i\cdot J_1'(a_i)\cdot\exp(2\pi i\zeta_i)\cdot\mathrm{B}_W(Z_i)\} \quad (4)$$

should be supplemented with two more equations that take into account the longitudinal motion of the oscillators:

$$\frac{d\eta_i}{d\tau} = 2\pi \frac{d^2 Z_i}{d\tau^2} = \mathrm{Re}\{iR \cdot a_i \cdot J_1^{'}(a_i) \mathrm{e}^{2\pi i \zeta_i} \cdot \mathrm{B}_W(Z_i)\}, \quad (5)$$

$$2\pi dZ_i / d\tau = \eta_i, \quad (6)$$

where $\delta_e^2 = \dfrac{4e^2 \cdot \omega_B \cdot \mathrm{N}_{b0} \cdot J_{m-n}^2(k_{ms} \cdot r_C)}{m_e \cdot c \cdot k_{ms}^2 \cdot r_w \cdot J_m^2(x_{ms}) \cdot \left(1 - \dfrac{m^2}{x_{ms}^2}\right) \cdot D_\omega}$,

$$\tau = \delta_e t, \quad D_\omega = \partial D / \partial \omega = \frac{\partial}{\partial \omega} \left(\frac{\omega^2 - (k_z^2 + k_{ms}^2) c^2}{\omega^2 - k_z^2 c^2} \right) \Bigg|_{D=0},$$

$$\mathrm{E}_e = \frac{e \cdot b \cdot J_{m+n}(k_{ms} \cdot r_C)}{m_e \cdot c \cdot \delta_e}, \quad R_e = \frac{k_z^2 \cdot \omega_B}{k_{ms}^2 \cdot \delta_e}, \quad \omega_B = eB / m_e c,$$

$a = k_{ms} r_B = k_{ms} v_\Phi / \omega_B$, $Z_j = k z_j / 2\pi$ is the position of the center of rotation of the electron along the axis of the waveguide, and N_{b0} is the number of particles of the unperturbed beam per unit length. Here is b the wave amplitude, and the longitudinal component of the wave's magnetic field has the form

$$B_z = b \cdot J_m(k_{ms} r) \cdot \exp\{-i\omega t + i k_z z + i m \vartheta\}$$

in a cylindrical coordinate system (r, ϑ, z); m – is an integer; $J_m(x)$ and $J_m^{'}(x) = dJ_m(x)/dx$ – is the Bessel function and its derivative. The requirement that the tangential component of the field vanish at the waveguide boundary determines the values of the transverse wave number $k_\perp = k_{ms} = x_{ms} / r_w$, and x_{ms} – s – is the root of the equation $dJ_m(x)/dx = 0$.

Superradiance mode. Another approach to describing the process of field generation by the same system of oscillators in this waveguide assumes that each such oscillator emits the same wave in both directions [7]. Because the oscillator is capable of emitting only waves located in the zone of transparency of the medium (or system – in this case, a waveguide). In this case, the field of this radiation will act on all particles of the ensemble of oscillators. In other words, all oscillators interact with each other. This mode can be considered a superradiation mode, if, of course, the phase synchronization of the oscillators occurs in the future. The case of radiation of an individual particle (out of a total number of particles equal to) should be considered as follows. The equation for the field that an individual particle emits can be written as

$$v_g \frac{\partial B_j}{\partial z} = i \frac{a_j}{N} J_n^{'}(a_j) \mathrm{e}^{-2\pi i \zeta_j} \mathrm{e}^{-ik_z z_j} \delta(z - z_j), \quad (7)$$

the solution of which is $B_j = C + \lambda \cdot \theta(z - z_j)$, where C is a constant that should be determined. Since for the wave emitted by the oscillator the equation is valid $D(\omega, k) = 0$, the roots of which are

$$k_{z1,2} = \pm \mathrm{Re}\, D(1 + i \,\mathrm{Im}\, D / \mathrm{Re}\, D) \approx \pm\left(\frac{\omega^2}{c^2} - k_\perp^2\right)^{1/2} (1 + i0),$$

then for a wave propagating in the direction $z > z_j$, the wave number $k_z = k_{z1} > 0$ and the value of the constant C should be chosen equal to zero in order to avoid unlimited growth of the field at infinity. For a wave propagating in the direction $z < z_j$, the wave number $k_z = k_{z2} < 0$, value of the constant should be chosen equal $-\lambda$ to for the same reasons. The field amplitude in this case

$$B_j(Z) = i \frac{a_j}{N V_g} J_n^{'}(a_j) \mathrm{e}^{-2\pi i \zeta_j} \cdot \mathrm{e}^{2\pi i |Z - Z_j|}. \quad (8)$$

Let us pay attention to the fact that the direction of the strength vector of the longitudinal component of the magnetic field in this case does not depend on the direction of wave propagation. This is due to their suppression of the rotating electron's own magnetic field when emitting waves in both directions. Obviously, for a system of N oscillators, the equation for the field can be written in the form

$$B(Z) = i \frac{1}{2N\vartheta} \sum_{j=1}^{N} a_j J_n^{'}(a_j) \mathrm{e}^{-2\pi i \zeta_j} \cdot \mathrm{e}^{2\pi i \cdot |Z - Z_j|}, \quad (9)$$

where $\vartheta = 2V_g N_{0b} / M = 2v_g / d \cdot \delta$ is the ratio of the maximum increment δ to the attenuation decrement due to radiation from the ends of the system $2v_g / d$.

2. FIELD GENERATION SUPER-RADIATION IN OPEN SYSTEM

For an open system, in the absence of field reflection from the ends, one can be convinced that there is no waveguide field, but only a superradiance field, that is, a field that is determined only by the interaction of rotating electrons. To describe the total magnetic field of the waves, you can use the expression

$$\mathrm{B}_{sr}(Z) = i \frac{1}{2N\theta} \sum_{j=1}^{N} a_j J_1^{'}(a_j) \cdot \mathrm{e}^{-2\pi i \zeta_j} \cdot \mathrm{e}^{2\pi i \cdot |Z - Z_j|}. \quad (10)$$

The equations of electron motion will retain the same form, but with the replacement of the expression for the field

$$2\pi \frac{d\zeta_i}{d\tau} = \eta_i(1 - \alpha) + J_1(a_i)\left[1 - \frac{1}{a_i^2}\right] \cdot \cdot \mathrm{Re}\{\exp(2\pi i \zeta_i) \cdot \mathrm{B}_{sr}(Z_i)\}, \quad (11)$$

$$da_i / d\tau = \mathrm{Re}\{i \cdot J_1^{'}(a_i) \cdot \exp(2\pi i \zeta_i) \cdot \mathrm{B}_{sr}(Z_i)\}, \quad (12)$$

$$\frac{d\eta_i}{d\tau} = 2\pi \frac{d^2 Z_i}{d\tau^2} = \mathrm{Re}\{iR \cdot a_i \cdot J_1^{'}(a_i) \mathrm{e}^{2\pi i \zeta_i} \cdot \mathrm{B}_{sr}(Z_i)\}.$$
(13)

The total radiation field of all particles $\mathrm{B}_{sr}(Z)$ is found according to formula (10). Particles move along the waveguide at a speed of $\eta_i(\tau)$.

When particles are removed outside the resonator (0<Z<1), they are replaced by new ones with an initial coordinate Z=0, an initial velocity η_0, an initial amplitude a_0, and a random phase. Calculations were carried out with fixed parameters: $N = 900$, $\alpha = 0.5$, $\theta = 1$, $R = 0.1$, $a_0 = 1$.

The field in the waveguide is a function of time and coordinates. Next, the time dependences are plotted for the maximum field modules in the volume of the waveguide:

$$\mathrm{B}_{sr}^{\max}(\tau)=\max_{Z}\mathrm{B}_{sr}(\tau,Z)\,,\quad \mathrm{B}_{w}^{\max}(\tau)=\max_{Z}\mathrm{B}_{w}(\tau,Z)\,.$$

Fig. 1 shows the time dependence of the maximum field of particles (in the volume of the waveguide) at a fixed initial speed $\eta_0=0.05$.

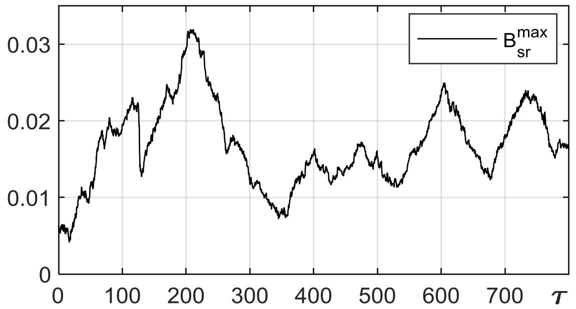

Fig. 1. Time dependence of the particle field maximum at zero reflection coefficients $r_L=0,\ r_R=0$

Attention should be paid to the fact that the generation mode in the superradiance mode is possible only if there are noticeable fluctuations of the electromagnetic field in the system or due to an external initiating field. In our case, a small number of model particles generates significant density fluctuations, which make it possible to exceed the threshold for radiation generation [8].

It is worth noting that in the absence of reflection there is no waveguide field; only the presence of reflection effects allows the formation of a waveguide field.

3. GENERATION OF FIELDS IN A WAVEGUIDE WITH PARTIAL REFLECTION

As already noted, the waveguide field, subject to noticeable reflection, will have the form

$$\mathrm{B}_w(Z,\tau)=\mathrm{B}_+(\tau)e^{2\pi iZ}+\mathrm{B}_-(\tau)e^{-2\pi iZ}\,. \qquad (14)$$

Thus, the total field in the waveguide can be represented as

$$\mathrm{B}(Z,\tau)=\mathrm{B}_+(\tau)e^{2\pi iZ}+\mathrm{B}_-(\tau)e^{-2\pi iZ}+\mathrm{B}_{sr}(Z,\tau)\,. \qquad (15)$$

Here $\mathrm{B}_{sr}(Z,\tau)$ is the total radiation field of rotating electron-oscillators; the field reflected from the left end – $\mathrm{B}_+(\tau)e^{2\pi iZ}$, reflected from the right end – $\mathrm{B}_-(\tau)e^{-2\pi iZ}$, and the amplitudes of these fields are almost equal. It is the total field (15) that acts on the particles. To find the amplitudes of two differently directed waves $\mathrm{B}_\pm$, we will follow the following simplified procedure.

The amplitudes of the reflected waves are found from the conditions of reflection at the ends of the waveguide based on the condition for the amplitudes on the left (Z=0) and right (Z=1) sides of the incident and reflected wave: $\mathrm{B}_{\rightarrow}/\mathrm{B}_{\leftarrow}|_{Z=0,1}=-r_{l,r}$, r_l, r_r, – reflection coefficients at the left and right ends. Total radiation field of particles at the ends of the system

$$\mathrm{B}_{sr}(Z=0)=i\frac{1}{2N\theta}\sum_{j=1}^{N}a_j J_1^{'}(a_j)\mathrm{e}^{-2\pi i\zeta_j}\,\mathrm{e}^{2\pi iZ_j}\,, \qquad (16)$$

$$\mathrm{B}_{sr}(Z=1)=i\frac{1}{2N\theta}\sum_{j=1}^{N}a_j J_1^{'}(a_j)\mathrm{e}^{-2\pi i\zeta_j}\,\mathrm{e}^{-2\pi iZ_j}\,. \qquad (17)$$

Let's consider the fields at the ends of the resonator Z=0 and Z=1. The field incident on the boundary of the Z=0 waveguide is equal to $\mathrm{B}_-(\tau)e^{-2\pi iZ}|_{Z=0}+\mathrm{B}_{sr}(Z=0,\tau)$, and the reflected field is equal to $\mathrm{B}_-(\tau)e^{2\pi iZ}|_{Z=1}$. Similarly for Z=1: incident field $\mathrm{B}_+(\tau)e^{2\pi iZ}|_{Z=1}+\mathrm{B}_{sr}(Z=1,\tau)$, reflected field - $\mathrm{B}_-(\tau)e^{2\pi iZ}|_{Z=1}$. From here we obtain equations for the amplitudes of reflected wave

$$\mathrm{B}_+(\tau)=-r_l\left(\mathrm{B}_-(\tau)+\mathrm{B}_{sr}(Z=0,\tau)\right),$$
$$\mathrm{B}_-(\tau)=-r_r\left(\mathrm{B}_+(\tau)+\mathrm{B}_{sr}(Z=1,\tau)\right),$$

where $r_l;r_r$ are the reflection coefficients, respectively, at the boundaries of the waveguide Z=0 and Z=1.

From the above expressions we obtain the values of the amplitudes of the waves that make up the standing wave

$$\mathrm{B}_+=\frac{r_l\left(r_r\,\mathrm{B}_{sr}(Z=1,\tau)-\mathrm{B}_{sr}(Z=0,\tau)\right)}{(1-r_l\cdot r_r)},$$
$$\mathrm{B}_-=\frac{r_r\left(r_l\,\mathrm{B}_{sr}(Z=0,\tau)-\mathrm{B}_{sr}(Z=1,\tau)\right)}{(1-r_l\cdot r_r)}. \qquad (18)$$

4. NUMERICAL SIMULATION OF WAVEGUIDE FIELD FORMATION

When modeling the formation of a waveguide field, system (14)–(18) was solved to find the total field, including the field of particles (superradiance) and the waveguide field. Calculations were carried out with fixed parameters: $N=900$, $\alpha=0.5$, $\theta=1$, $R=0.1$, $a_0=1$ and different initial particle velocities and different reflection coefficients. Figs. 2–4 shows the time dependence of the particle field and the reflected field at a fixed initial velocity $\eta_0=0.05$ and different reflection coefficients.

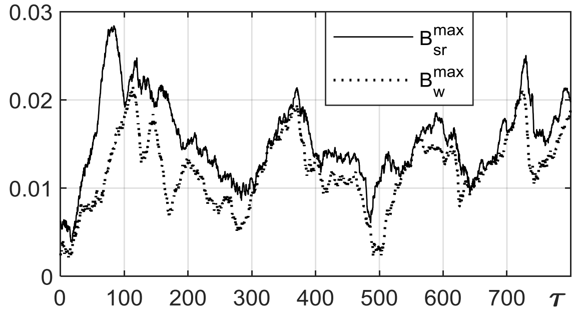

Fig. 2. Time dependence of the particle field and the reflected field at reflection coefficients $r_L=1,\ r_R=0$

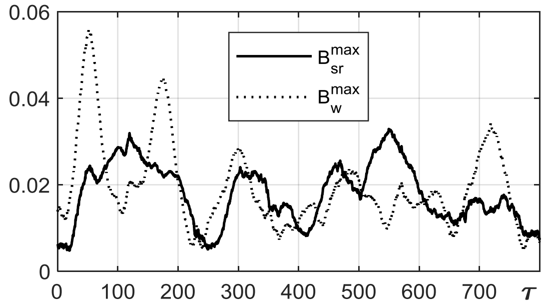

Fig. 3. Time dependence of the maximum of the particle field and the reflected field at reflection coefficients $r_L = 1,\ r_R = 0.4$

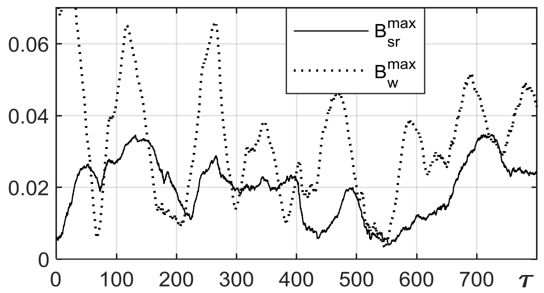

Fig. 4. Time dependence of the maximum of the particle field and the reflected field at reflection coefficients $r_L = 1,\ r_R = 0.8$

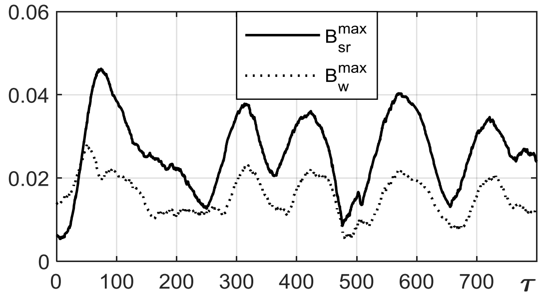

Fig. 5. Time dependence of the maximum of the particle field and the reflected field at the initial velocity $\eta_0 = 0.01$

It is also of interest to consider the influence of the intensity of injection of new particles and the speed of their drift along the length of the waveguide.

In Figs. 5–7 the time dependences of the maximum of the particle field and the reflected field are shown for fixed reflection coefficients and different initial velocities of injected particles.

Let's also repeat Fig. 3, but in a different interpretation.

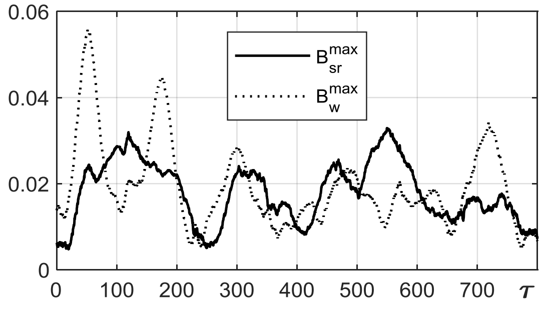

Fig. 6. Time dependence of the maximum of the particle field and the reflected field at the initial velocity $\eta_0 = 0.05$

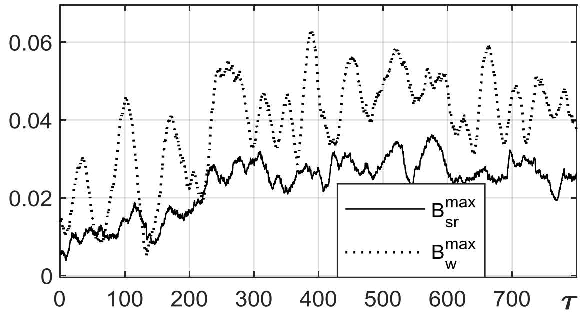

Fig.7. Time dependence of the maximum of the particle field and the reflected field at the initial velocity $\eta_0 = 0.1$

CONCLUSIONS

In this work, the conditions for the generation of TE cyclotron waves in a magnetoactive waveguide by an electron flow, the rotation speed in the cross-sectional plane of which significantly exceeds the longitudinal speed of movement, are studied. It is shown that for completely open gyrotron-type waveguides, a superradiance regime is realized, for which the interaction of rotating electron-oscillators with each other is dominant. As a result of this interaction, the effect of synchronization of emitters and oscillators and the appearance of intense coherent radiation occurs. However, the greatest interest is in the studied conditions for the generation of superradiance and the formation of waveguide fields when reflections from the ends of the system are taken into account. In different generation modes, as shown in this work, both the waveguide field, determined by the geometry of the system, and the field of the total radiation of core particles, that is, the superradiance field, can dominate. A similar flexible approach to describing the generation of an open system–gyrotron, in particular in the superradiance regime, was proposed by A.G. Zagorodny and P.I. Fomin [1].

An increase in reflection coefficients can significantly increase the amplitude of the waveguide field, while the total radiation of the particles remains virtually unchanged. At a fixed level of reflection, an increase in the rate of injection of particles into the system and an increase in the speed of their drift leads to an increase in the amplitude of the waveguide mode, which, with a sufficiently large energy pumping, leads to the dominance of the waveguide mode, and the amplitude of the field of particles decreases slightly. A rational choice of generation modes can explain the high level of output radiation characteristic of the operation of such gyrotron devices, which was subsequently confirmed by numerical modeling of such processes [8].

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ОСОБЛИВОСТІ ГЕНЕРАЦІЇ ХВИЛЬ У МАГНІТОАКТИВНОМУ ХВИЛЕВОДІ

Є.В. Поклонський, В.М. Куклін

Обговорюються умови генерації поля надвипромінювання та формування хвилеводного поля з урахуванням відбивання від торців. У різних режимах генерації, як показано в даній роботі, може домінувати як хвилеводне поле, що визначається ефектами відбивання поля та геометрією системи, так і поле сумарного випромінювання частинок активної зони, тобто поле надвипромінювання.