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THREE DIFFERENT SPATIAL POSITIONS OF FAST MAGNETOSONIC WAVE COMPONENT TURNING POINTS

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Two different approaches to defining the cut-off in a magnetoactive plasmas are well-known. In one approach, a physicist searches the cut-off frequency for uniform plasma, that is for fixed values of plasma parameters, such as the plasma composition, plasma particle densities and external static magnetic field. This cut-off frequency separates those frequencies, at which the electromagnetic wave is propagative, from those, at which it is evanescent one. In the other approach, a physicist searches the cut-off plasma particle density for the fixed frequency. The density is observed at the cut-off coordinate called in some text-books as “turning point” or “transition point”. This coordinate separates the space, within which the wave is propagative, from the space, within which the wave is evanescent one. The conditions for determining the turning points depend not only on the values of plasma parameters themselves but on the plasma parameter gradients as well. These conditions are determined for the case of the electromagnetic waves in ion cyclotron frequency range. Their deviation from the condition applied for the first approach is discussed.

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INTRODUCTION

Plasma is considered in Cartesian coordinates. The z-axis is directed along the external static magnetic field, $\vec{B}_0||\vec{z}$. The plasma is assumed to be nonuniform along the x-axis. Fourier analysis is applied, e.g., $H_z(\vec{r},t) = H_z(x)\exp[ik_z z + ik_y y - i\omega t]$. (Here k_z and k_y are toroidal and poloidal wavenumbers, respectively, ω is the wave angular frequency). Cold collisionless plasma dielectric tensor is applied (Stix notations [1]),

$$\hat{\varepsilon} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}. \quad (1)$$

Within the ion cyclotron frequency range, $\omega_{ci} \le \omega << \|\omega_{ce}\|$, the tensor components read

$$S = 1 - \sum_i \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2},\ D = \sum_i \frac{\omega_{pi}^2 \omega}{\omega_{ci}(\omega^2 - \omega_{ci}^2)},\ P = 1 - \frac{\omega_{pe}^2}{\omega^2}. \quad (2)$$

In (2), $\omega_{c\alpha}$ and $\omega_{p\alpha}$ are cyclotron and plasma frequencies of plasma particle species α, respectively: $\alpha = i$ for ions, and $a = e$ for electrons. Since $|P| \gg |S|, |D|$, toroidal component of the fast magnetosonic wave (FMSW) electric field can be neglected, $E_z = 0$. This results in linear proportionality of two pairs of the wave field components:

$$-N_z E_y(x) = H_x(x), \quad N_z E_x(x) = H_y(x). \quad (3)$$

In (3), $N_z = k_z/k_0$ is toroidal refractive index, $k_0 = \omega/c$ is vacuum wavenumber. One should keep in mind that there are cases when one cannot neglect E_z, as, for example, non-cyclotron Landau damping of the fast wave and the fast wave current drive related to that.

Stix T.H. [1] mentions Allis W.P. [2] as a person who was the first to apply the term “cut-off” to the certain values of parameters, for which refractive index squared goes to zero. There are two approaches to defining the cut-off. One approach is as follows. A physicist searches the cut-off frequency for fixed values of plasma parameters. “In a given plasma, a cut-off separates frequencies, at which the corresponding mode propagates, from the frequencies, at which it is evanescent” [3]. In this approach, the cut-off frequency for the FMSWs can be found as the solution of the equation [1]:

$$N_x^2 \equiv N_\perp^2 - N_y^2 = 0. \quad (4)$$

In (4), $N_\perp^2 = (S + D - N_z^2)(S - D - N_z^2)/(S - N_z^2)$. Equation (4) is equivalent to $D^2 = T(S - N_z^2)$ with $T = S - N_y^2 - N_z^2$, and $N_y = k_y/k_0$ as poloidal refractive index.

The other approach relates to the situation when a physicist searches the cut-off density in a nonuniform plasma for the fixed frequency. This density relates to the coordinate, which separates the space, within which the corresponding mode propagates, from the space, within which the mode is evanescent one. Following Fedoryuk M.V. [4] this coordinate is called hereinafter as turning point. The conditions for determining the turning point coordinates appear to depend also on the values of plasma parameter gradients. The conditions naturally are different from (4). Moreover, they are different for different components of the wave field: one condition is derived below for E_x and H_y, the other – for E_y and H_x, and the third – for H_z.

The general approach to deriving these conditions is known from the theory of ordinary differential equations [4]. It is in brief as follows. A second order ordinary differential equation can be represented in the form

$$a(x)f'' + b(x)f' + c(x)f = 0. \quad (5)$$

In (5), all the coefficients: $a(x)$, $b(x)$ and $c(x)$, – are real when the eq. (5) describes FMSW propagation neglecting dissipations. The role of dissipations is physically clear that is why they are neglected in the present paper.

In the turning point $x = x_0$, the roots of the characteristic equation

$$a(x)\Lambda^2 + b(x)\Lambda + c(x) = 0, \qquad (6)$$

should coincide. The condition for determining x_0 consists in equating to zero of the discriminant $D(x) = b^2 - 4ac$ of the characteristic equation (6). The discriminant has opposite signs on the opposite sides of x_0. On one side $D(x)>0$, and both roots of the eq. (6) are real. At this side the wave is evanescent since infinite ("exponential") increase of the wave amplitude (e.g. $H_z(x)$) with going away from x_0 has not physical reasons. On the opposite side $D(x) < 0$, and both roots of eq. (6) are complex. This means that the wave is propagative, its amplitude (e.g. $H_z(x)$) oscillates with x coordinate.

1. EXPLICIT FORM OF THE DIFFERENTIAL EQUATIONS AND CONDITIONS FOR DETERMINING THE TURNING POINTS

Maxwell`s equations provide the following three second-order ordinary differential equations for the amplitudes of the FMSW components:

$$0 = -N_y^2(S - N_z^2)E''_x + \qquad (7)$$
$$\left[\frac{N_y^2(S-N_z^2)\left(2DD' - N_yD'' + N_y^2S'\right)}{D^2 - N_yD' + N_y^2(S-N_z^2)} - 2N_y^2S'\right]E'_x +$$
$$\left\{\begin{array}{c} -N_yS'D + N_yTD' + N_y^2D^2 - N_y^2(S-N_z^2)T - \\ \left[D^2 - N_yD' + N_y^2(S-N_z^2)\right]\left[\frac{N_y^2S' - N_yD(S-N_z^2)}{D^2 - N_yD' + N_y^2(S-N_z^2)}\right]' \end{array}\right\}E_x,$$

$$\left(\frac{S-N_z^2}{T}E_y{}'\right)' + \left[S - N_z^2 - \frac{D^2}{T} + N_y\left(\frac{D}{T}\right)'\right]E_y = 0, \quad (8)$$

$$0 = \left(\frac{S-N_z^2}{(S-N_z^2-D)(S-N_z^2+D)}H_z'\right)' +$$
$$\left[1 - \frac{N_y^2(S-N_z^2)}{(S-N_z^2-D)(S-N_z^2+D)} - \left(\frac{N_yD}{(S-N_z^2-D)(S-N_z^2+D)}\right)'\right]H_z. \quad (9)$$

In (7)-(9), prime denotes the normalized derivative $d/d(k_0x)$.

Conditions for determining the turning points in a nonuniform plasma for the waves with $k_y \neq 0$ read

$$0 = N_y\left[\frac{(S-N_z^2)\left(2DD' - N_yD'' + N_y^2S'\right)}{D^2 - N_yD' + N_y^2(S-N_z^2)} - 2S'\right]^2 +$$
$$4(S-N_z^2)\times \qquad (10)$$
$$\left(\begin{array}{c} -S'D + TD' + N_yD^2 - N_y(S-N_z^2)T - \\ \left[D^2 - N_yD' + N_y^2(S-N_z^2)\right]\left[\frac{N_yS' - D(S-N_z^2)}{D^2 - N_yD' + N_y^2(S-N_z^2)}\right]' \end{array}\right),$$

– for the pair of the wave field components E_x and H_y,

$$\left(\frac{N_y^2S'}{T^2}\right)^2 - 4\left(\frac{S-N_z^2}{T}\right)\left[S - N_z^2 - \frac{D^2}{T} + N_y\left(\frac{D}{T}\right)'\right] = 0, \quad (11)$$

– for the pair of the wave field components E_y and H_x. And finally, the condition for determining the turning point for the FMSW toroidal magnetic field H_z reads

$$\left[\left(\frac{S-N_z^2}{(S-N_z^2-D)(S-N_z^2+D)}\right)'\right]^2 - \qquad (12)$$
$$4\frac{S-N_z^2}{(S-N_z^2-D)(S-N_z^2+D)}\times$$
$$\left[1 - \frac{N_y^2(S-N_z^2)}{(S-N_z^2-D)(S-N_z^2+D)} - \left(\frac{N_yD}{(S-N_z^2-D)(S-N_z^2+D)}\right)'\right] = 0.$$

The conditions (10)-(12) turn to eq. (4) in the case of uniform plasma, i.e. if to put prime equal to zero, $d / d(k_0x) = 0$. Equation (11) is the less cumbersome among the conditions (10)-(12) which makes it the most suitable one for analyzing whether plasma nonuniformity is significant or not in determining the turning points. If to assume that $N_y^2 \geq 2|(1 - N_z^2)(\omega/\omega_{ci} - 2)|$, and introduce L as the length scale of the plasma parameter variation, $S' \sim (k_0L)S$, then the scale should be sufficiently large to make it possible to neglect the nonuniformity, so that

$$L \gg \omega/(\omega_{ci}k_y). \qquad (13)$$

Conditions for determining the turning points in a nonuniform plasma for the waves with $k_y = 0$ are much simpler than (10)-(12). Specifically, the condition for the pair of the wave field components E_x and H_y reads in this case:

$$\frac{S-N_z^2}{D}\left(\frac{D}{S-N_z^2}\right)'' - \left(\frac{S-N_z^2}{D}\right)^2\left(\frac{D}{S-N_z^2}\right)'^2$$
$$-\frac{\left(S-N_z^2\right)^2 - D^2}{S-N_z^2} = 0. \qquad (14)$$

It is important to underline that the pair of the wave field components E_y and H_x with $k_y = 0$ is the only one, for which the condition for determining the turning point does not contain any spatial derivative from the plasma parameters. In other words, it is the only one which does not change its form as compared with the condition (4).

And finally, the condition for determining the turning point for the wave toroidal magnetic field H_z with $k_y = 0$ reads:

$$\left(\frac{\left(S-N_z^2\right)^2 - D^2}{S-N_z^2}\right)'^2 - 4\left(\frac{\left(S-N_z^2\right)^2 - D^2}{S-N_z^2}\right)^3 = 0. \qquad (15)$$

2. NUMERICAL ANALYSIS OF THE RELATIONS FOR DETERMINING THE TURNING POINTS. LINEAR DENSITY PROFILE

The linear dependence of the plasma particle density, $n_e(x) = n_0x/\lambda$, as well as uniform external static magnetic field are suggested at the first step. Hereinafter, variation of the external magnetic field is neglected during making numerical analysis. In this case, the cut-off position $x_{uniform}$, calculated from the condition (4), corresponds to the fixed value of the plasma particle density and naturally linearly increases with the reverse density gradient, as it is shown in Fig. 1 by solid line for the comparison. Turning points $x_{cut\text{-}off}$ for E_x and H_y are presented by dashed curve, for E_y and H_x – by dotted curve, and for H_z – by dash-dotted curve. Turning points $x_{cut\text{-}off}$ for E_x and H_y are calculated as the solution of eq. (10), for E_y and H_x – of eq. (11), and for H_z – of eq. (12). Alfven refractive index squared is applied in Figs. 1-5: $N_A^2(x) \equiv \omega_{pi}^2 / \omega_{ci}^2$. The following plasma parameters are chosen: $\omega = 3.34\ \omega_{ci}$, $B_0 = 2.0\ T$, poloidal wave index $m = +5$, toroidal wave index $l = 1$, plasma minor radius $\alpha = 0.5\ m$, and plasma major radius $R = 2.12\ m$. Only one ion species is considered, which is reasonable if wave propagation in the plasma edge rather than its absorption is studied (see, e.g., [5]). For these plasma parameters, the condition (13) says that the plasma nonuniformity can be neglected in determining the position of the turning points of the wave fields if the length scale L of the plasma particle density variation is much larger than 0.3 *m*. The latter length is of the order of the plasma minor radius a and is much larger than the decay length Λ of the plasma particle density in the scrape-off layer of ASDEX Upgrade, Λ = 0.01801 *m* [6]. This estimation favors the necessity to take the plasma nonuniformity into account when determining the turning points in medium-sized tokamaks.

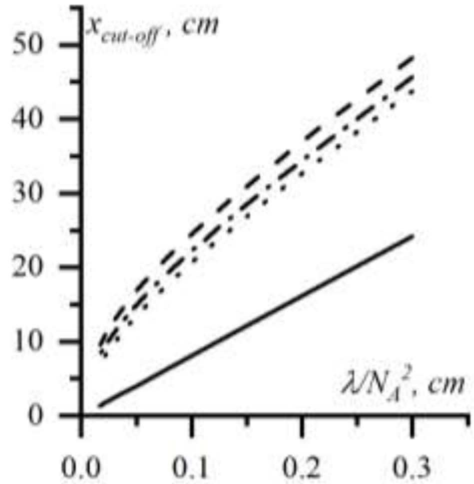

Fig. 1. Turning points vs reverse plasma density gradient. Cut-off coordinates in the limiting case of uniform plasma (solid line), and turning points in the case of the density linear variation for E_x and H_y (dashed curve), E_y and H_x (dotted curve), and H_z (dash-dotted curve). ω = 3.34 ω_{ci}, *m* = +5, *l* =1, *a* = 50 *cm*, *R* = 212 *cm*

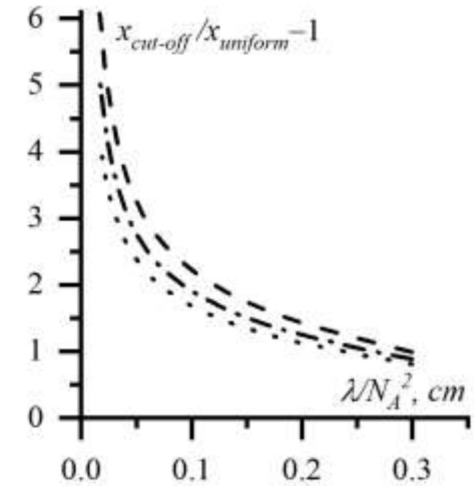

Fig. 2. Relative difference between the turning points and cut-off coordinates vs reverse plasma density gradient. The relative difference in the case of the density linear variation is presented for E_x and H_y by the dashed curve, for E_y and H_x – by the dotted curve, and for H_z – by the dash-dotted curve. ω = 3.34 ω_{ci}, *m* = +5, *l* =1, *a* = 50 *cm*, *R* = 212 *cm*

The smaller is the plasma density gradient, the smaller should be difference between the turning point positions for nonuniform and uniform cases. This is not obvious from the Fig. 1. That is why relative differences $(x_{cut\text{-}off} / x_{uniform} - 1)$ rather than absolute values of the turning point coordinates are presented for the FMSW with positive wave index *m*=+5 in Fig. 2. The relative differences for the components E_x and H_y are presented by dashed curve, for E_y and H_x – by dotted curve, and for H_z – by dash-dotted curve, similarly to Fig. 1.

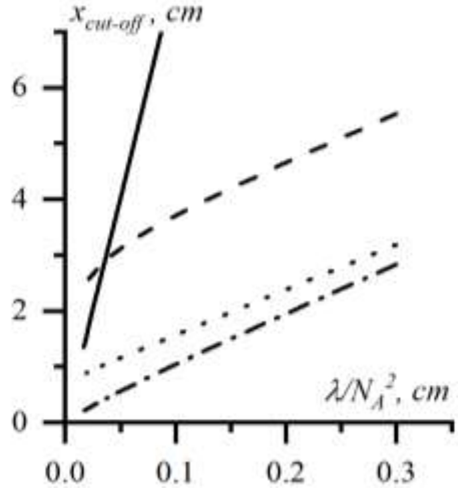

Fig. 3. The same as in Fig. 1, but form m = -5

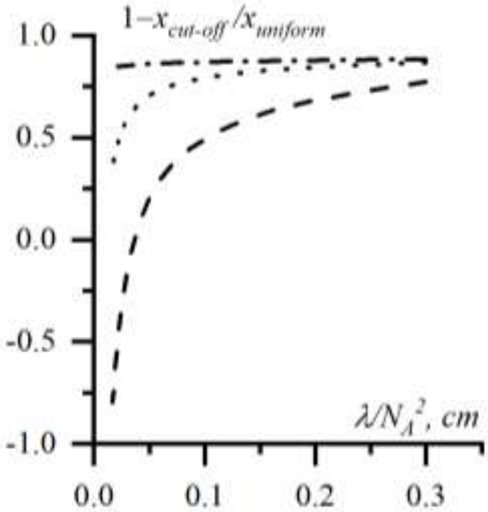

Fig. 4. The same as in Fig. 2, but for m = -5

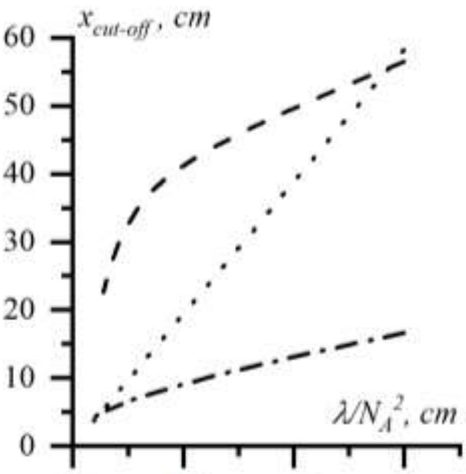

Fig. 5. The same as in Fig. 1, but for m=0 *and l*=15

Table 1

Position of the turning points (in cm) for the wave field components E_x and H_y. The figures given in italic correspond to the small values of the wave indices, such that $k^2_y + k^2_z < k^2_0$

$l\backslash m$	-5	-4	-3	-2	-1	0	1	2	3	4	5
1	214.81	214.35	213.62	212.61	211.8	211.68	208.9	207.88	207.27	206.82	206.46
2	217.37	216.85	215.89	213.88	212.37	213.00	208.92	207.89	207.27	206.82	206.46
3	218.59	218.59	218.56	218.46	218.25	231.56	208.94	207.9	207.27	206.82	206.47
4	216.79	216.80	216.78	216.7	216.58	227.96	208.98	207.91	207.28	206.82	206.47
5	215.73	215.74	215.73	215.67	215.64	225.81	209.02	207.92	207.28	206.83	206.47
6	214.95	214.97	214.96	214.94	214.97	224.23	209.06	207.93	207.29	206.83	206.47
7	214.32	214.35	214.36	214.36	214.46	222.97	209.10	207.95	207.29	206.83	206.47
8	213.80	213.84	213.86	213.88	214.04	221.91	209.14	207.96	207.30	206.84	206.47
9	213.36	213.39	213.43	213.48	213.68	220.99	209.17	207.98	207.31	206.84	206.48
10	212.96	213.01	213.05	213.12	213.36	220.18	209.2	207.99	207.32	206.84	206.48

Table 2

Position of the turning points (in cm) for the wave field components E_y and H_x

$l\backslash m$	-5	-4	-3	-2	-1	0	1	2	3	4	5
1	216.0	215.65	214.97	213.88	213.61	212.05	212.17	210.45	209.462	208.77	208.24
2	218.6	218.24	217.43	215.19	214.28	212.01	212.12	210.43	209.456	208.77	208.24
3	222.01	222.43	222.95	223.64	224.64	211.95	212.04	210.41	209.445	208.76	208.23
4	220.18	220.58	221.07	221.68	222.33	211.86	211.92	210.38	209.428	208.75	208.22
5	219.07	219.45	219.9	220.41	220.75	211.75	211.77	210.33	209.406	208.74	208.21
6	218.24	218.59	219.0	219.41	219.48	211.62	211.60	210.27	209.379	208.72	208.20
7	217.56	217.89	218.24	218.55	218.39	211.46	211.43	210.21	209.345	208.7	208.19
8	216.97	217.27	217.57	217.78	217.44	211.27	211.26	210.13	209.305	208.68	208.17
9	216.44	216.72	216.97	217.08	216.58	211.06	211.09	210.05	209.26	208.65	208.15
10	215.97	216.21	216.41	216.44	215.81	210.83	210.93	209.96	209.209	208.62	208.13

Table 3

Position of the turning points (in cm) for the wave field components H_z

$l\backslash m$	-5	-4	-3	-2	-1	0	1	2	3	4	5
1	222.45	222.81	223.3	224.04	210.44	209.02	208.27	207.73	207.32	206.97	206.68
2	219.24	225.40	225.88	226.57	210.17	208.96	208.24	207.72	207.30	206.96	206.67
3	215.03	214.41	213.37	211.57	209.88	208.87	208.19	207.69	207.29	206.95	206.66
4	213.29	212.72	211.9	210.75	209.61	208.76	208.14	207.65	207.26	206.93	206.65
5	212.28	211.78	211.09	210.23	209.36	208.64	208.07	207.61	207.23	206.91	206.63
6	211.57	211.11	210.53	209.84	209.13	208.51	207.99	207.56	207.2	206.88	206.61
7	211.01	210.6	210.1	209.52	208.93	208.38	207.91	207.50	207.16	206.85	206.59
8	210.55	210.18	209.74	209.25	208.74	208.25	207.82	207.44	207.11	206.82	206.56
9	210.17	209.83	209.44	209.05	208.56	208.13	207.73	207.38	207.06	206.78	206.53
10	209.84	209.53	209.18	208.79	208.39	208.00	207.64	207.31	207.01	206.74	206.5

The curves in Fig. 2 clearly demonstrate the decrease of the relative differences with the decrease of the plasma particle density gradient.

Equation (13) is applied above for determining the condition for neglecting the plasma nonuniformity in calculating the turning point position. The turning points of the wave fields E_y and H_x (dotted curve) are the closest to those calculated from eq. (4) or in other words calculated just with neglecting the plasma nonuniformity. This confirms the correctness of applying just eq. (11) for deriving eq. (13): if one cannot neglect the nonuniformity for determining the turning point position for this pair of the wave fields, the more it is impossible to neglect the nonuniformity while determining the turning point positions for other wave fields.

Toroidal wavenumber k_z is present in the FMSW field equations (7)-(9) in even numbered powers. That is why the turning points of the wave fields with opposite values of the toroidal wavenumbers are located in the same positions. In contrast to this, the conditions (10)-(12) strongly depend on the sign of poloidal wavenumbers k_y. This is clearly demonstrated in Figs. 3, and 4 for the FMSW with negative wave index m= - 5. The turning points in this case are situated much closer to the wall than for the FMSW with positive poloidal wave index m=+5. If to go from the wall, the turning point for H_z appears the first, then one meets the turning point for the wave components E_y and H_x, and finally, E_x and H_y experience the transition. Since the turning points in Fig. 3 lie below the solid curve which corresponds to the solution of eq. (4), then relative differences $(1-x_{cut\text{-}off}\,/\,x_{uniform})$ rather than $(x_{cut\text{-}off}\,/\,x_{uniform}-1)$ are presented in Fig. 4. Positions of the turning points for the wave fields with $k_y = 0$ are presented in Fig. 5. The figure does not contain any curves in solid, since the wave components E_y and H_x with $k_y = 0$ have turning points just in the places determined by the condition (4), which determines both the cut-off frequency for given plasma parameters and the turning point for fixed wave frequency. Unlike the wave components E_y and H_x with $k_y = 0$, the turning points of other components of the wave fields E_x, H_y and H_z with $k_y = 0$ appear to be significantly different from the positions determined by the condition (4). Toroidal wave index is chosen for calculations in Fig. 5 as $l = 15$ to provide evanescent nature of the wave fields in rarified plasma.

3. NUMERICAL ANALYSIS OF THE RELATIONS FOR DETERMINING THE TURNING POINTS. EXPONENTIAL DENSITY PROFILE

The numerical analysis presented in Figs. 1-5 is carried out for the linear plasma particle density while the density varies exponentially in a tokamak scrape-off layer [7],

$$n(x) = n_0 \exp[-(x - x_0)/\Lambda]. \qquad (16)$$

In (16), n_0 is plasma particle density at $x = x_0$, Λ is the decay length. The positions of the turning points, calculated for such a plasma particle density profile according to eqs. (10)-(12), are presented in Tables 1-3 for the wave field components E_x and H_y, E_y and H_x, and H_z, respectively. The Tables provide the minor radial positions of the turning points in centimeters. The following plasma parameters are applied: $n_0 = 1.32\cdot10^{18}$ m^{-3}, $x_0 = 2.12$ m, $\Lambda = 0.01801$ m [6], $\omega = 3.34\omega_{ci}$, plasma minor radius a = 0.5 m, and plasma major radius $R = 2.12$ m. For the uniform external static magnetic field $B_0 = 2$ T, Alfven refractive index squared is $N^2_A(x=x_0) = 129$.

Two conclusions follow from the analysis of the data presented in Tables 1-3. First, significant difference in positions of the turning points for the wave fields with opposite values of m should be pointed out. Second, the positions of the turning points are also different for different wave fields. Turning points appear to be situated in the denser (as compared with that predicted by eq. (4) which corresponds to WKB approach) plasma for the wave fields E_x and H_y with positive wave indices, $m > 0$, and in the more rarified plasma – for $m \leq 0$. Very similar situation is observed for the wave fields E_y and H_x: their turning points appear to be situated in the denser plasma for $m \geq 0$, and in the more rarified plasma – for the waves with negative wave indices, $m < 0$. Turning points for the wave field H_z are situated in the denser plasma for almost all the studied values of the wave indices; the waves with negative poloidal wave indices and small toroidal wave indices are exceptions. If to assume that the plasma particle density at the last closed magnetic surface is about 10^{20} m^{-3}, then its coordinate is ≈ 204.0 cm. In other words, all the turning points presented in Tables 1-3 are situated within the scrape-off layer where the plasma particle density can be described by eq. (16).

Table 4

Positions of the turning points (in cm) calculated according to eq. (4). The figures given in italic correspond to the small values of the wave indices, such that $k^2_y + k^2_z < k^2_0$

$l\backslash m$	0	1	2	3	4	5
1	220	–	–	–	214	213
2	222	–	216	215	214	213
3	219	217	215	214	213	213
4	217	216	215	214	213	212
5	216	215	214	213	213	212
6	215	214	214	213	212	212
7	214	214	213	213	212	212
8	214	213	213	212	212	211
9	213	213	213	212	212	211
10	213	213	212	212	211	211

They often apply just WKB approach for determining the cut-off position (see, e.g., [8-10]). It is important to compare the positions of the turning points presented in Tables 1-3 with the cut-off positions calculated in WKB approach (according to eq. (4)). The latter data is presented in Table 4. Since eq. (4) is symmetric in m then Table 4 does not contain any data for negative values of m. The turning point positions calculated from eqs. (10)-(12) (that is from the explicit conditions) appear to significantly differ from those calculated from eq. (4), which corresponds to WKB approach. For example, the difference of the positions of the turning points presented in Tables 3 and 4 for $l = 1$ and $m = 5$ is 6.168 cm which is more than three times larger than the applied decay length. These two positions correspond to the plasma particle densities which differ by the factor of ≈ 30.7 which cannot be considered as a negligible one.

CONCLUSIONS

In a nonuniform plasma, turning points have different positions for three sets of the fast magnetosonic wave field components: the first set consists of the wave components E_x and H_y, the second – of E_y and H_x, and the third is H_z.

The condition (4) is well-known [1, 3] as that for determining the cut-off frequency of FMSWs for given uniform plasma parameters such as plasma contents, values of external static magnetic field and plasma particle densities. Generally speaking, in a nonuniform plasma, the condition is applicable for determining the turning points of the FMSW field components E_y and H_x with $k_y = 0$ only. The positions of the turning points for other FMSW components cannot be determined from this condition in a nonuniform plasma, especially in a tokamak scrape-off layer, even in the case of the modes with $k_y = 0$. The condition (14) rather than (4) should be applied for determining the positions of the turning points of the FMSW components E_x and H_y, and the condition (15) – for the component H_z with $k_y = 0$.

The condition (4) cannot be applied for determining the turning points of the FMSW components even for the field components E_y and H_x with $k_y \neq 0$. The conditions (10)-(12) should be applied in this case instead of eq. (4).

The plasma nonuniformity causes different influence on the turning point positions for the waves with different wavenumbers. The influence is symmetric in respect of toroidal wave indices, and has no symmetry in respect of poloidal wave indices. For the waves with positive poloidal wavenumbers, $k_y > 0$, the turning points are situated deeper in plasma than that predicted by eq. (4) (in WKB approach).

The turning points of the wave fields E_y and H_x are the nearest to the plasma wall for almost all the studied values of the wave indices. The turning points of the waves with negative poloidal wave indices and small toroidal wave indices, for which $k^2_y + k^2_z < k^2_0$, are the exclusions.

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ТРИ РІЗНІ ПРОСТОРОВІ ПОЛОЖЕННЯ ТОЧОК ПОВОРОТУ КОМПОНЕНТІВ ПОЛЯ ШВИДКИХ МАГНІТОЗВУКОВИХ ХВИЛЬ

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Загально відомі два різні підходи до визначення відсічки в магнітоактивній плазмі. В одному підході шукають частоту відсічки в однорідній плазмі, тобто для фіксованих значень параметрів плазми, таких як склад плазми, густина частинок плазми та зовнішнє стале магнітне поле. Ця частота відсічки відокремлює ті частоти, на яких електромагнітна хвиля поширюється, від тих частот, на яких вона просторово загасає. В іншому підході шукають густину відсічки для заданої частоти. Цю густину плазми спостерігають у точці відсічки, яку ще називають у підручниках «точкою повороту». Ця точка відокремлює простір, у якому хвиля поширюється, від простору, в якому хвиля просторово загасає. Координати точок повороту залежать не лише від значень самих параметрів плазми, але й від градієнтів параметрів плазми. Умови для визначення цих координат здобуто для випадку електромагнітних хвиль у діапазоні йонної циклотронної частоти. Проаналізовано їхнє відхилення від умови, придатної для застосування в першому підході.