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FEATURES OF GENERATION OF SPONTANEOUS MAGNETIC FIELDS IN FULLY IONIZED PLASMA

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In this paper, magnetic field generation in a fully ionized plasma is discussed both in the presence and absence of an external constant magnetic field. Considering the dependence of the transfer coefficients on the magnetic field, Braginsky's equations are applied to describe collective processes in plasma. Criteria for the formation of instabilities and, consequently, the generation of magnetic fields are obtained, taking into account convective heat transfer and thermomagnetic phenomena. We derive expressions for the growth rate of perturbations in the magnetic field and the temperature in the short-wave range.

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INTRODUCTION

The advent of lasers in the 1960s led to the emergence of a rapidly developing field in plasma physics – laser thermonuclear fusion. However, in various experiments on the interaction of laser radiation with matter, after a very short time ($\sim 10^{-9}$ s), huge magnetic fields were spontaneously excited in the plasma, sometimes reaching $\sim 10^2$ T [1, 2]. In studies of thermonuclear energy, the generation of magnetic fields can have a significant effect on the hydrodynamics of target compression, thereby worsening the regime of plasma compression and heating [3].

A large number of mechanisms are known to explain the origin of strong magnetic fields in laser plasma. A recent review [4] presents some mechanisms underlying the generation of a magnetic field in a laser plasma, as well as the latest experimental and theoretical developments in this field. In Ref. [2], the excitation of strong magnetic fields was associated with the thermoelectric power arising in a plasma with a crossed distribution of density and temperature inhomogeneities. This mechanism is similar to the operation of the Biermann battery [5], which is characterized by the fact that the generation of the field begins with a zero value. The Biermann battery effect is caused by the presence of an imbalance in the distribution of charges due to gradients in the plasma properties. When there is a temperature or pressure gradient in the plasma, the electrons and ions in the plasma experience different forces, leading to a net electric current. This electric current, in turn, generates a magnetic field. This effect is important in astrophysics, where it is thought to play a role in the generation of magnetic fields in the early universe and in the vicinity of black holes. In addition, the Biermann battery effect also takes place in turbulent plasma since the role of the pressure gradient is played by the turbulent stress gradient. In this case, the Biermann effect is often referred to as the cross-helicity effect [6, 7]. Thermal energy transfer and plasma hydrodynamics could be significantly impacted by magnetic fields. One way magnetic fields can be spontaneously generated is through the thermomagnetic instability driven by parallel equilibrium densities and temperature gradients. If an equilibrium temperature gradient is perturbed ∇T_1, a magnetic field is generated due to the Biermann battery $\partial\vec{B}_1/\partial t \sim [\nabla N \times \nabla T_1]$, where N and T represent the electron density and temperature, respectively. The magnetic field perturbation $\vec{B}_1$ strengthens the temperature perturbation due to the magnetic field's effect on thermal conductivity.

The most probable mechanism for the generation of magnetic fields during spherically symmetric target laser heating is magneto-thermal (or thermomagnetic) instability, which occurs when the density and temperature gradients are parallel. In several papers [8-11], this instability was analyzed for nonmagnetized plasma. In the paper [12], it is shown that plasmas that are hot T =10 keV and have a density $10^{20}\,\mathrm{cm}^{-3}$ of become magnetized with a magnetic field strength of only 0.03 T. In magnetized plasma, the transverse heat conductivity causes the instability to be driven by density and temperature gradients that are opposite in direction. The growth rate of this instability in magnetized plasma is significantly lower than in unmagnetized plasma. In plasma that is subjected to a very strong magnetic field, heat convection is more prominent than heat conduction, and the Hall effect provides additional stabilization for the instability. One of the types of thermomagnetic instability is the mechanism of the generation of magnetic fields in a plasma with a nonuniform temperature. This instability is realized in a hot plasma, where thermomagnetic processes convert part of the heat flux into magnetic field energy. For astrophysical conditions, thermomagnetic instability (TM) has been used to explain the generation of strong magnetic fields in the surface layers of neutron stars [13], as well as in hot massive stars [14], and in plasma when radiative processes play an important role in heat transfer [15]. This type of thermomagnetic instability is due to the action of the feedback between the Nernst effect and “skew” heat fluxes (the Rigi-Leduc effect).

In the papers mentioned above, the mechanisms of generation of magnetic fields in a high-temperature plasma were considered when “skew” heat fluxes became significant in heat transfer. In this case, the scales of the generated fields are small compared to the characteristic scales of plasma inhomogeneity in temperature and density. This brings up the question of whether magnetic fields can be generated in inhomogeneous low-temperature plasma across the entire range of perturbation lengths while taking into account convective plasma flows.

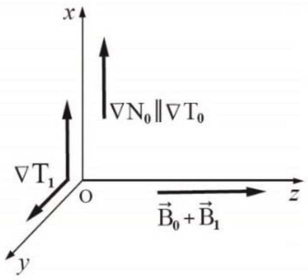

Fig. 1. The geometry of the problem

The concept of magnetic field generation through use of convective motions of charged particles (electrons, ions) in an inhomogeneous plasma was first proposed in Ref. [16]. Based on the idea [16], in the work [17] studied the generation of a magnetic field in a weakly ionized, spatially inhomogeneous plasma with collinear density and temperature gradients. As shown in Ref. [17], there is no generation of magnetic fields in such a plasma due to the dissipation of thermomagnetic disturbances by the effect of plasma heat conduction. In a recent paper [18], the processes of spontaneous generation of magnetic fields by Rayleigh-Bénard convection in a thin plasma layer in a constant gravitational field were considered. Estimates of the excited magnetic field are obtained, which coincide in order of magnitude with the characteristic values of the magnetic fields of neutron stars.

The purpose of this study is to study the influence of thermoforce, the Nernst effect, and the Rigi-Leduc effect on the development of thermomagnetic instabilities in fully ionized plasma. Through analysis, we have determined the criteria and necessary conditions for 1) excitation of a magnetic field in a uniformly magnetized plasma 2) generation of a magnetic field in both non-magnetized and magnetized temperature-inhomogeneous plasma and 3) generation of a magnetic field with collinear gradients of density and temperature in non-magnetized and magnetized plasma.

The results of this study may have practical applications for both laboratory and astrophysically fully ionized plasma.

STATEMENT OF THE PROBLEM

Consider a fully ionized plasma with equilibrium gradients in temperature and density maintained by external sources (laser radiation, a beam of charged particles). A uniform magnetic field $\vec{B}_0$ is acting on the plasma, and there is no unperturbed hydrodynamic motion. Assume that the perturbation evolution process is so fast that the ions are "immobile" and "cold". For simplicity, consider the plane geometry of the problem, where the unperturbed temperature T_0 and density N_0 depend on a single Cartesian coordinate x. Since the temperature and density gradients are collinear with each other, this automatically leads to the fulfillment of condition $[\nabla N_0 \times \nabla T_0] = 0$. In this case, there is no source of excitation of the magnetic field by the baroclinic effect: $\partial B/\partial t \sim [\nabla N_0 \times \nabla T_0] = 0$. Let us start studying the problem of generating magnetic fields in a plasma with collinear density and temperature gradients immersed in a uniform magnetic field $\vec{B}_0 = B_0 \vec{e}_z$ ($\vec{e}_z$ is a unit vector along the z axis) orthogonal to the gradients. The motion of electrons and heat transfer processes in a fully ionized plasma are described by the Braginskii equations [19].

$$\frac{dV_\alpha}{dt} = -\frac{e}{m}\left(E_\alpha + \frac{1}{c}\left[\vec{V}\times\vec{B}\right]_\alpha\right) - \frac{1}{mN}\frac{\partial P}{\partial x_\alpha} -$$
$$-\frac{1}{mN}\frac{\partial \Pi_{\alpha\beta}}{\partial x_\beta} + \frac{1}{mN}\left(\vec{R}_v + \vec{R}_T\right)_\alpha, \; \alpha = (x, y, z), \tag{1}$$

$$\frac{3}{2}\frac{dT}{dt} + P\mathrm{d}iv\vec{V} = -\mathrm{d}iv\vec{q} - \Pi_{\alpha\beta}\frac{\partial V_\alpha}{\partial x_\beta} + Q, \tag{2}$$

$$\frac{\partial N}{\partial t} + \mathrm{d}ivN\vec{V} = 0, \tag{3}$$

$$P = NT, \tag{4}$$

where operator *d/dt* is the substantial derivative given by $d/dt = \partial/\partial t + \vec{V}\nabla$; *P*, *N*, V_α denote the pressure, density, temperature, and velocities of electrons; $\Pi_{\alpha\beta}$ is the viscous stress tensor describing the friction in the electron gas. The expression for the viscous stress tensor $\Pi_{\alpha\beta}$ for a magnetized plasma has a rather complicated form, which is given by Braginskii [19]. Due to the assumptions listed below, we do not present the viscous stress tensor's form here. $\vec{R}_v$ is the friction force associated with collisions of electrons with ions, $\vec{R}_T$ is the thermal force due to the electron temperature gradient. $Q = Q_0+Q_e$ is the total specific heat release in the electron gas from external energy sources Q_0 and due to collisions of electrons with ions Q_e:

$$Q_e = -\frac{3mN}{M\tau_{ei}}T - \frac{\vec{j}\vec{R}_v}{eN} - \frac{\vec{j}\vec{R}_T}{eN},$$

where τ_{ei} is the time between the collisions of electrons with ions; $\vec{j} = -eN\vec{V}$ is the current density associated with the speed of the electrons. In the general case of arbitrary magnetic fields, the expressions for the friction force $\vec{R}_v$ and thermal force $\vec{R}_T$ take the following form:

$$\frac{\vec{R}_v}{mN} = -\alpha_\parallel \vec{V}_\parallel - \alpha_\perp \vec{V}_\perp + \alpha_\wedge[\vec{h}\times\vec{V}], \tag{5}$$

$$\frac{\vec{R}_T}{mN} = -\beta_\parallel^{uT}\nabla_\parallel T - \beta_\perp^{uT}\nabla_\perp T - \beta_\wedge^{uT}[\vec{h}\times\nabla T]. \tag{6}$$

The thermal electron flow similarly consists of two parts $q = q_v+q_T$:

$$q_v = -\beta_\parallel^{Tu}V_\parallel + \beta_\perp^{Tu}V_\perp + \beta_\wedge^{Tu}[h\times V], \tag{7}$$

$$q_T = -\kappa_\parallel\nabla_\parallel T - \kappa_\perp\nabla_\perp T - \kappa_\wedge[h\times\nabla T]. \tag{8}$$

Here $\vec{h} = \vec{B}/B$ is the unit vector along the magnetic field. The transfer coefficients in expressions (5)-(8) have the form $\alpha_\parallel = \dfrac{\alpha_0}{\tau_{ei}}, \alpha_\perp = \dfrac{1}{\tau_{ei}}\left(1 - \dfrac{\alpha_1'\xi^2 + \alpha_0'}{\Delta}\right),$

$$\alpha_{\wedge} = \frac{1}{\tau_{ei}} \frac{\xi(\alpha_1''\xi^2 + \alpha_0'')}{\Delta},$$

$$\beta_{\parallel}^{uT} = \frac{\beta_0}{m}, \beta_{\perp}^{uT} = \frac{\beta_1'\xi^2 + \beta_0'}{m\Delta}, \beta_{\wedge}^{uT} = \frac{\xi(\beta_1''\xi^2 + \beta_0'')}{m\Delta},$$

$$\beta_{\parallel}^{Tu} = NT\beta_0, \beta_{\perp}^{Tu} = NT\frac{\beta_1'\xi^2 + \beta_0'}{\Delta},$$

$$\beta_{\wedge}^{Tu} = NT\frac{\xi(\beta_1''\xi^2 + \beta_0'')}{\Delta},$$

$$\kappa_{\parallel} = \frac{NT\tau_e}{m}\gamma_0, \kappa_{\perp} = \frac{NT\tau_e}{m}\frac{\gamma_1'\xi^2 + \gamma_0'}{\Delta},$$

$$\kappa_{\wedge} = \frac{NT\tau_e}{m}\frac{\xi(\gamma_1''\xi^2 + \gamma_0'')}{\Delta}, \ \Delta = \xi^4 + \delta_1\xi^2 + \delta_0 .$$

Next, we use the numerical values of the coefficients for positively charged ions at Z = 1 (hydrogen plasma) of the following form [19]: $\alpha_0 = 0.5129$, $\beta_0 = 0.7110$, $\gamma_0 = 3.1616$, $\delta_0 = 3.77$, $\delta_1 = 14.79$, $\alpha'_1 = 6.416$, $\alpha'_0 = 1.837$, $\alpha'_1 = 1.704$, $\alpha''_0 = 0.7796$, $\beta'_1 = 5.101$, $\beta'_0 = 2.681$, $\beta''_1 = 3/2$, $\beta''_0 = 3.053$, $\gamma'_1 = 4.664$, $\gamma'_0 = 11.92$, $\gamma''_1 = 5/2$, $\gamma''_0 = 21.67$.

The symbols P and $\perp$ represent their use along and across the magnetic field, respectively. We supplement the equations (1)-(4) with Faraday's law rot $\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$, Ampere's law rot $\vec{B} = -\frac{4\pi eN}{c}\vec{V}$, and magnetic field solenoidality equation div $\vec{B} = 0$.

Let us present the main assumptions that restrict the considered physical model of the generation of magnetic fields in an inhomogeneous magnetized plasma:

1) we employ a hydrodynamic description of electron motions, which assumes that the lengths of perturbations λ of all plasma parameters and the characteristic spatial scales of inhomogeneities L are much larger than the mean free path of an electron l: $\lambda, L >> l$;

2) we neglect viscosity forces in comparison to friction forces because
$R_V // |\partial \Pi_{\alpha\beta}/\partial x_\beta| \cong L^2/NT\tau^2_{ei} \sim L^2/l^2 >> 1$;

3) it is assumed that the fraction of Joule heating is small compared to the fraction of heat release associated with collisions of electrons with ions:

$$C_s >> V_e(\omega_{Be}\tau_e)^{1/2},\ C_s >> V_e(\omega_{Be}\tau_e)^{-1/2},$$

where $C_s = (T/M)^{1/2}$ is the velocity of ionic sound, $\omega_{Be} = eB/mc$ is the Larmor frequency of rotation of the electron;

4) we also neglect the relative contribution of the work of the thermal force $\vec{j}\vec{R}_T/(eN)$ in the electron heat balance equation compared to the heat release process associated with collisions of electrons with ions: $ml/M\tau_e >> V_e$, $ml/M\tau_e >> V_e(\omega_{Be}\tau_e)^{-2}$;

5) the density profile is not disturbed by electron velocity perturbations;

6) the frequency of electron-electron collisions ($\nu_{ee} \cong \tau_e^{-1}$) is approximately the same as the frequency of electron-ion collisions ($\nu_{ei} \cong \tau_{ei}^{-1}$).

Given the above assumptions and applying operation **rot** to equation (1) by using Maxwell's equations and the equation of state (4), we can derive an equation for magnetic field induction $\vec{B}$. As a result of applying Ampere's law to the heat balance equation (2), we obtain a self-consistent system of equations for the magnetic field $\vec{B}$ and temperature T.

Let us choose the configuration of the problem so that the plane of electron motion is aligned with the plane xy (Fig. 1). Next, we neglect terms of the second order of smallness when analyzing linear stability, so we represent the expansion of physical values as follows:
$N = N_0(x)$, $T = T_0(x)+T_1(x,y)$, $B = e_z(B_0+B_1(x,y))$.
Let's represent low-intensity perturbations as wave motions that have no impact on the plasma equilibrium properties. Then, in the short-wavelength limit $L>>\lambda$, we perform the perturbations of the magnetic field and temperature in the form of harmonic oscillations:

$$(B_1, T_1) = (A_{B_1}, A_{T_1})\exp\left(ik_y + i\int k_x(x')dx' - i\omega t\right) . \quad (9)$$

Neglecting the derivatives of the function $k_x(x)$, i.e., limiting ourselves to the zeroth approximation of geometric optics, we obtain the desired dispersion equation. This equation is very cumbersome, so let's consider special cases.

EXCITATION OF A MAGNETIC FIELD IN A HOMOGENEOUS MAGNETIZED PLASMA

In this case, the dispersion equation takes the following form:

$$\omega^2 + i\omega(\gamma_T + \gamma_B) - \gamma_T\gamma_B - \frac{2}{3}\gamma_\beta^2(\omega_{Be}\tau)^2\frac{k^2r_d^2}{1+k^2r_d^2} = 0, \quad (10)$$

where $r_d = c/\omega_{pe}$ is the Debye radius of electrons; $\omega_{pe} = (4\pi e^2N/m)^{1/2}$ is the Langmuir frequency of the electrons, $\xi = \omega_{Be}\tau_e$ is the Hall parameter; $\gamma_\beta = ((\beta''_1\xi^2+\beta''_0)/\Delta)(T_0k^2/m)^{1/2}$ is the characteristic oscillation frequency due to the influence of the thermal force and the external magnetic field; $\gamma_T = (2/3\chi^{(0)}_{\perp}k^2+m\nu_{ei}/M) = \gamma_\chi + \gamma_\nu$; γ_χ is part of the damping decrement associated with the thermal conductivity of the electron gas; γ_ν is part of the damping decrement associated with the transfer of thermal energy during electron-ion collisions. The positive root of the quadratic equation (10) allows us to find an expression for the growth rate $\Gamma = \mathrm{Im}\,\omega$ of thermomagnetic disturbances:

$$\Gamma = \frac{2}{3}\frac{\gamma_\beta^2(\omega_{Be}\tau)^2k^2r_d^2}{(\gamma_T+\gamma_B)(1+k^2r_d^2)} - \frac{\gamma_T^2+\gamma_B^2+\gamma_T\gamma_B}{\gamma_T+\gamma_B}. \quad (11)$$

Consider the case when the inertia of the electrons is small $kr_d = 1$. Then, under the condition $\gamma_\chi >> \gamma_B$ or $2/3\chi^{(0)}_{\perp}/(\tilde{a}^{(0)}_{\perp}r^2_d) >> 1$ the necessary condition for the development of instability, leading to the generation of magnetic fields, if the following inequality is satisfied

$$\left(\frac{3}{2}\right)^3\frac{(\omega_{Be}\tau)^2}{(4.66)^2}\frac{\nu_e^2r_d^2m}{T_0} = \frac{(3/2)^3}{(4.66)^2}\frac{B_0^2}{4\pi P_0} > 1. \quad (12)$$

Inequality (12) can be written in terms of plasma beta $\beta = 8\pi N_0T_0/B_0^2$: $2(3/2)^3/(4.66)^2 > \beta$ or

$$0.31 > \beta. \quad (13)$$

Plasma beta is a dimensionless parameter that represents the ratio of plasma pressure to magnetic pressure. A high plasma beta means that the plasma pressure is dominant, while a low plasma beta means that the magnetic pressure is dominant. Condition (13) shows that instability is possible at low values of plasma beta. Examples of low plasma beta are observed in tokamaks, as well as in astrophysical plasmas such as the earth's magnetosphere, solar wind plasma, etc. For tokamak plasma (thermonuclear experiments), the electron density ranges from 10^{19} to 10^{20} particles per m^{-3}, the temperature is about 10^8 K, and the magnetic field varies from 2 to 5 T. In this case, the plasma beta value lies in the range of 0.01 to 0.1 and satisfies inequality (5). The magnetic field of the earth is strong enough to confine the plasma particles, so the magnetic pressure is dominant over the thermal pressure. The beta value in the earth's magnetosphere is typically less than one (around 10^{-3} to 10^{-5}) for electron density, which varies around 10^{10} to 10^{12} particles per m^{-3}, temperature ranges of 1160 to 2320 K, and magnetic field ranges around $3\cdot10^{-5}$ to $6\cdot10^{-5}$ T. The solar wind is a stream of charged particles that travels from the sun and interacts with the earth's magnetic field. Because the solar wind's magnetic pressure is much higher than its thermal pressure, the beta value is very low (around 10^{-6}). Parameter plasmas of solar wind have an electron density varying from around 10^7 particles per m^{-3}. The temperature ranges around 10^5 K, and the magnetic field ranges around 10^{-6} T. If another condition $\gamma_\chi = \gamma_B$ or $2/3\chi^{(0)}_\perp/(\alpha_\perp^{(0)}r^2_d) = 1$ is met, the perturbation growth rate is

$$\Gamma \approx \frac{2}{3}\frac{\gamma_\beta^2(\omega_{Be}\tau)^2}{\alpha_\perp^{(0)}} - \alpha_\perp^{(0)}k^2r_d^2 \ . \quad (14)$$

It follows from expression (6) that the generation of a magnetic field is possible if the inequality is satisfied:

$$\frac{3}{2}\frac{4\pi P_0}{B_0^2} > 1 \quad \text{or} \quad \beta > \frac{4}{3}. \quad (15)$$

As can be seen from condition (15), for the development of instability, it is necessary that the equilibrium pressure of the plasma be greater than the magnetic pressure. The solar corona is an example of a high beta plasma $\beta>>1$. The temperature of the solar corona is millions of degrees ($\simeq 10^6$ K), which creates a high thermal pressure that dominates over the magnetic pressure. The characteristics of the plasma of the solar corona are as follows: the electron density varies from 10^{14} to 10^{15} particles per m^{-3}, the temperature ranges from 10^6 to $3\cdot10^6$ K, and the magnetic field is about 10^{-4} to $4\cdot10^{-4}$ T.

GENERATION OF A MAGNETIC FIELD IN A TEMPERATURE-INHOMOGENEOUS PLASMA

In this section, we consider the mechanisms of the generation of magnetic fields in a temperature-inhomogeneous plasma in which the electron density concentration is constant $N_0 = const$ in the space occupied by the plasma. One of the possible mechanisms to generate a magnetic field in a temperature-inhomogeneous plasma is the Nernst effect. We get the oscillation frequency ω_0 and the growth rate Γ of thermomagnetic disturbances:

$$\omega_0 = \frac{\frac{3}{2}\cdot 0.51\nu_{ei}r_d^2k_xK_T}{1+k^2r_d^2},$$

$$\Gamma = 0.81\frac{T_0\tau}{m}\left(\frac{1}{T_0}\frac{d^2T_0}{dx^2}+3K_T^2\right) - \frac{0.51\nu_{ei}r_d^2k^2}{1+k^2r_d^2}. \quad (16)$$

Thermomagnetic oscillations are oscillating with the frequency ω_0 and spreading in the direction of the temperature gradient ∇T_0. The generation of the magnetic field occurs due to the development of thermomagnetic (TM) instability with an increment Γ (16). This instability is caused by the Nernst effect, due to which part of the thermal energy is transformed into magnetic energy. An expression similar to (16) for the TM instability increment was obtained in [14, 15] as applied to the thermal generation of the magnetic field in the surface layers of massive stars. For a high-temperature plasma, ohmic dissipation can be neglected since $\nu_{ei} \sim T_0^{-3/2}$. In this case, the perturbation growth rate is determined by the temperature inhomogeneity profile. In the thin crust of massive stars, the temperature dependence on the coordinate can be considered linear. Then the magnetic fields are generated with increment:

$$\Gamma \approx 0.81\cdot\frac{3\tau}{mT_0}\left(\frac{dT_0}{dx}\right)^2 \approx 2.43\frac{\tau T_0}{m}L_T^{-2} \ . \quad (17)$$

Using the values of temperature $T_0 \approx 10^8$ K from expression (17), it is easy to find the growth rate of magnetic disturbances: $\Gamma \approx 10^9 \ldots 10^{11}$ s^{-1}. Then the instability growth time is $t_0 \approx 10^{-11} \ldots 10^{-9}$ s. The given estimates of the growth time of magnetic fields are in good agreement with the results of laboratory experiments.

Next, we get the following dispersion equation for TM disturbances in a temperature-inhomogeneous magnetized plasma. The solutions of this equation in the form of the short-wavelength approximation $kL >> 1$ and the limit of high plasma beta $1<<\beta/(2(kL)^2)<< \beta/2$, have the following form:

$$\omega_0 \approx -\frac{15/4}{4.66}\frac{T_0k_yK_T}{m\omega_{Be}(1+k_y^2r_d^2)},$$

$$\Gamma \approx \frac{\frac{3}{2}\frac{\tau T_0}{m}\left(\frac{1}{T_0}\frac{d^2T_0}{dx^2}+3K_T^2\right)}{(\omega_{Be}\tau)^2(1+k_y^2r_d^2)}. \quad (18)$$

Comparing the TM instability increment in a nonmagnetized plasma (9) with the increment for a magnetized one (10), we get

$$\frac{\Gamma_{\xi\ll1}}{\Gamma_{\xi\gg1}} \approx \frac{2}{3}\cdot 0.81(\omega_{Be}\tau)^2 \gg 1.$$

Thus, TM instability in an unmagnetized plasma has a larger increment than in a magnetized plasma, it means that the perturbations associated with the first instability grow more rapidly than the perturbations associated with the second instability.

Let us now consider the case of a low plasma beta

$$\frac{\beta}{2(kL)^2}, \frac{\beta}{2} \ll 1.$$

Then the solutions take the form:

$$\omega_0 \approx -\frac{3}{2}\frac{\tau T_0}{m(\omega_{Be}\tau)}k_yK_T\ ,\ \Gamma \approx -\gamma_\chi - \frac{3}{2}\frac{T_0k^2}{m\nu_{ei}}(\omega_{Be}\tau)^{-2}. \quad (19)$$

This indicates that TM perturbations are suppressed by the thermal conductivity of the electron gas, resulting in the inability to generate a magnetic field in this case. Therefore, the mechanism of magnetic field generation due to the Nernst effect, which is discussed in this section, is only observable in a plasma that is both high-temperature (high plasma beta) and temperature-inhomogeneous, regardless of whether it is magnetized or not.

MAGNETIC FIELD GENERATION IS CAUSED BY DENSITY AND TEMPERATURE GRADIENTS

In this section, we explore whether it is possible to produce magnetic fields in a magnetized plasma with density and temperature gradients. It was found that in the heat transfer equation, velocity disturbances and associated magnetic field disturbances lead to the appearance of convective and "skew" heat flows, the intensity ratio of which is determined by the dimensionless parameter R_{TM}:

$$\frac{|q_{\wedge}|}{|q_{conv}|} \approx \frac{\frac{2}{3}\chi_{\wedge}^{(0)}\frac{e\tau}{mc}\frac{dT_0}{dx}}{\frac{cT_0}{4\pi e N_0}K_T} = \frac{2}{3}\frac{\gamma_0''}{\delta_0}R_{TM} \approx \frac{2}{3}\cdot 5.75 R_{TM} .$$

One of the solutions to the dispersion equation, which gives an undamped contribution to the instability of TM for a "hot" plasma $1 << R_{TM}/(kL)^2 << R_{TM}$ has the form

$$\omega_0 \approx \frac{\frac{3}{2}\cdot 0.51\nu_{ei}k_x K_T}{1+k^2 r_d^2}, \qquad \Gamma \approx \left[1.82\frac{k_y^2}{k^2}\frac{T_0}{m\nu_{ei}}K_N K_T + \right.$$
$$\left. +0.81\frac{\tau T_0}{m}\left(\frac{1}{T_0}\frac{d^2T_0}{dx^2}+3K_T^2-2K_T K_N\right)\right](1+k^2r_d^2)^{-1} . \quad (20)$$

The first term in (20) corresponds to the instability increment from [8], where it was concluded that magnetic field generation is possible with parallel density and temperature gradients. The second term in (20) is related to the action of the thermal force, or the Nernst effect. Owing to this effect, the generation of a magnetic field is possible in a plasma with a constant electron density [15]: $K_N = 0$. A similar expression for the instability increment (20) was obtained in Ref. [11]. The difference from [11] is only in the numerical coefficients in front of K_T^2 and $K_N K_T$ in the contribution of the thermal force. This is because in the equations for the momentum and energy of the electron gas, we took the effect of temperature variations on the transfer coefficients into account. The time it takes for instability to develop is determined by the magnitude of the density $dN_0/dx = K_N N_0$ and temperature $dT_0/dx = K_T T_0$ gradients and is given by expression (20) as $t_0 = \Gamma^{-1}$. As K_N and K_T increase, the characteristic time for instability development decreases, leading to a faster generation of the magnetic field. Taking into account that $\nu_{ei} \sim N_0 T_0^{-3/2}$, $K_N \sim N_0/L$, $K_T \sim T_0/L$, we can derive the expression for the characteristic time of instability development in the form $t_0 \sim m N_0 L^2/T_0^{5/2}$. This implies that the instability develops efficiently in a plasma that is rarefied, hot, and highly inhomogeneous, characterized by a small value of L. Using the following values for laser plasma: $T_0 \approx 10^7$ K, $N_0 \approx 10^{27}$ m^{-3}, $\tau \approx 10^{-12}$ s, $L \approx 10^{-4}$ m we obtain an estimate for the value of the characteristic instability development time $t_0 \sim 10^{-11}$, which is much shorter than the plasma expansion time $t_p \sim 10^{-9}$ i.e $t_p >> t_0$: The magnitude of the magnetic field generated can be estimated using the expression for the Biermann battery: $B_{max}/t_0 \cong (c/eN_0)\,[\nabla N_0 \times \nabla T_1]_z$, $B_{max} \cong (mc/e\tau)(L/\lambda)(T_1/T_0)$, $L >> \lambda$, $T_0 >> T_1$, $(L/\lambda)(T_1/T_0) \sim 1$.

It is important to note that the plasma should remain unmagnetized, $\omega_{Be}\tau \le 1$ and therefore, the magnitude of the generated magnetic field should not exceed a certain value $B_{max} \le 1$ T.

For the case of "cold" plasma, when heat transfer is dominated by convective heat fluxes $R_{TM}/(kL)^2$, $R_{TM} << 1$we got $\omega_0 \approx 3.16\,(\tau T_0/m)k_x K_T$,

$$\Gamma \approx \gamma_\chi\left[0.93\frac{k_y^2}{k^4}\left(1.47K_N K_T - \frac{2}{3}K_N^2\right) - 1\right]. \quad (21)$$

In the short-wavelength limit $kL >> 1$, oscillating short-wavelength thermomagnetic perturbations decay in a characteristic time $t_d \sim \gamma_\chi^{-1}$.

Finally, we proceed to elucidate the question of the possibility of developing TM instability in a magnetized inhomogeneous plasma with density and temperature gradients. For a high plasma beta, we found

$$\omega_0 \approx \frac{\omega_{Be}k_y K_N r_d^2}{1+k^2 r_d^2},\ \Gamma \approx \frac{5/2}{4.66}\frac{k_y^2}{k^2}\frac{T_0 K_N K_T}{m\nu_{ei}(1+k^2 r_d^2)} . \quad (22)$$

Comparing expression (21) with expression (14), we conclude that the growth rate of TM perturbations in an unmagnetized hot plasma is larger. Next, for a low plasma beta, we found

$$\omega_0 \approx -\frac{5}{3}\frac{\tau T_0}{m}(\omega_{Be}\tau)k_y K_N ,$$

$$\Gamma \approx \frac{k_y^2}{k^2}\frac{T_0}{m\nu_{ei}}\left(K_N K_T - \frac{7}{3}K_N^2\right) - \frac{2}{3}\cdot 4.66\frac{\tau T_0 k^2}{m(\omega_{Be}\tau)^2} . \quad (23)$$

Therefore, the necessary condition for TM instability in this case takes the form of

$$(\omega_{Be}\tau)^2 > \frac{7}{3}\frac{L_T}{L_N}(\omega_{Be}\tau)^2 + \frac{2}{3}\cdot 4.66(k^2 L_N L_T) \gg 1, \quad (24)$$

$L_N \cong K^{-1}_N$, $L_T \cong K^{-1}_T$, $\lambda^2 = 4\pi^2/k^2$.

On Fig. 2, in the plane (λ^2, ξ^2), the region of development of TM instability in a low-temperature magnetized plasma with parallel density and temperature gradients is shown in gray. The graph is plotted with the following ratio of plasma inhomogeneity scales: $L_N/L_T = 3$.

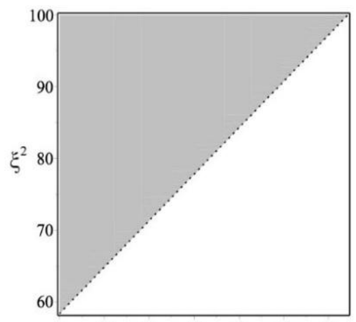

Fig. 2. On the graph, the region of TM instability with an external magnetic field B_0 at $L_N = 3L_T$ is highlighted in gray

Thus, in a strongly magnetized inhomogeneous plasma, the instability development mechanism becomes competitive with the damping mechanism even in the case of short-wavelength perturbations $(k^2 L_N L_T) >>1$.

CONCLUSIONS

In this study, our focus was on investigating the initial phase of magnetic field generation resulting from thermomagnetic (TM) effects in a fully ionized plasma. We derived the fundamental equations that describe the changes in both magnetic and thermal perturbations by utilizing the Braginskii equations, which pertain to an electron gas exhibiting collinear equilibrium density and temperature gradients. Employing the geometrical optics approximation, we successfully derived a dispersion equation for TM disturbances with short wavelengths. By conducting a detailed analysis of this equation, we were able to draw the following significant conclusions:

1) the generation of a magnetic field in a homogeneous magnetized plasma is attributed to the thermal force acting perpendicularly to both the magnetic field vector ($\vec{B}_0$) and the temperature gradient (∇T_1). This phenomenon occurs regardless of whether the plasma has high or low beta β values;

2) the generation of a magnetic field in a temperature-inhomogeneous plasma is observed at high plasma beta, regardless of whether it is magnetized or not;

3) the growth rate of TM perturbations in a temperature-inhomogeneous, non-magnetized plasma is higher than in a magnetized one;

4) spontaneous generation of a magnetic field in a weakly inhomogeneous nonmagnetized plasma with collinear density and temperature gradients is possible only in a high-temperature and low-density plasma $R_{TM} >> 1$;

5) in a weakly inhomogeneous magnetized plasma with collinear density and temperature gradients, a magnetic field is generated at both high and low plasma beta β values;

6) the growth rate of TM perturbations in a weakly inhomogeneous nonmagnetized plasma with collinear density and temperature gradients at $R_{TM} >> 1$ is higher than in a magnetized plasma at the high plasma beta $\beta > 1$ one.

Considering nonlinearity in future studies of magnetic field generation in inhomogeneous plasmas with thermomagnetic instabilities, we can investigate the saturation of the magnetic field generation process and understand the limits or thresholds at which the instabilities stabilize.

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ОСОБЛИВОСТІ ГЕНЕРАЦІЇ СПОНТАННИХ МАГНІТНИХ ПОЛІВ У ПОВНІСТЮ ІОНІЗОВАНІЙ ПЛАЗМІ

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Обговорюється генерація магнітного поля у повністю іонізованій плазмі як за наявності, так і за відсутності зовнішнього постійного магнітного поля. Враховується залежність коефіцієнтів перенесення від магнітного поля. Рівняння Брагінського використовуються для опису колективних процесів у плазмі. Отримано критерії виникнення нестійкостей і, як наслідок, генерації магнітних полів з урахуванням конвективного теплоперенесення та термомагнітних явищ. Отримано вирази для швидкості зростання збурень магнітного поля та температури у короткохвильовому діапазоні.