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MATHEMATICAL MODELING OF SURFACES OF PLASMA FACILITIES MAGNETIC WINDINGS

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Among the many problems associated with the creation of plasma facilities, the problem of obtaining mathematical models of the geometry of the magnetic system elements can be singled out. Such elements include magnetic surfaces, among which the most complex is the helical winding. The article presents methods and algorithms for obtaining a solid-state geometric model of the helical winding surface using kinematic modeling methods.

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INTRODUCTION

The most important property of the magnetic configuration of the system for containing high-temperature plasma, is the presence of magnetic surfaces, which are formed by the trajectories of the magnetic field lines with multiple rotations around the main axis of the torus. Obtaining three-dimensional geometric models of magnetic surfaces is the most important labor-intensive design task. In this article this task was solved for the design of a magnetic system with a toroidal two-way helical winding (HW) and geometric characteristics similar to the geometry of the Uragan-2M torsatron.

The magnetic system of the facility consists of a number of magnetic windings, among which the most complex is the helical winding (HW) [1]. It is two-way and consists of 40 conductors. The HW conductors are formed into 2 poles, 20 conductors in each. The HW poles consist of two half-poles of 10 conductors each. The HW conductors are located on a toroidal surface with a large radius R=1700 mm and a small radius r=450 mm.

The geometric model of the HW surface can be obtained as a result of the motion in space of a drawing curve, the shape of which can change or remain constant during the motion [2].

When modeling the HW, it is necessary to make the following assumption that at each moment of time the drawing curve is in the same plane. Therefore, to obtain a kinematic surface, it is necessary to specify a family of plane curves and the law of change of the plane in space. That is, it is necessary to specify the shape of the drawing curve in the plane and the law of change of such a curve in space.

Let $\varphi_u(\vartheta)$ – a family of HW plane curves of the, depends on the parameter U, and belongs to a fixed plane R_1^2, $T_u : R_1^2 \to R^3$ – a family of reflections of this plane in space. Then the surface can be given by a composition of reflections

$$\varphi(u,\vartheta) = T_u \varphi_u(\vartheta),$$

where u, ϑ – curvilinear coordinates of the surface.

The choice of a kinematic model is appropriate when the motion of the drawing curve is described quite simply or when the modeled surface itself is described as kinematic.

Curves formed from the drawing curve $\varphi_u(\vartheta)$ at those values of the parameter u in which it is known, are called generating. If the drawing curve is not known, it is necessary to restore the surface between the generating curves.

The restoration task is performed as follows - in a given class it is necessary to select a surface that satisfies some conditions. If the class under consideration is narrow, then to obtain a certain answer, a minimal set of conditions is sufficient, for example, a set of forming curves.

However, if the class of the considered surface is quite wide, then for a unique definition of the surface it is necessary to set additional conditions.

The kinematic nature of the surface is manifested here in the fact that the basic curves are not just arbitrary curves in space, but are obtained from each other according to a certain law that can be used to restore the surface. The advantage of the kinematic method of defining a surface compared to other algorithms is that in this case, significantly fewer intermediate sections are often required to describe a surface of complex shape due to additional plane control capabilities.

1. METHODS AND ALGORITHMS FOR MODELING THE MAGNETIC WINDINGS SURFACES

The helical winding of the facility is presented in the form of current-carrying conductors formed into poles. The poles are placed on a toroidal surface. When current is passed through them, a magnetic configuration is formed that holds the plasma.

Input data: the law of winding (winding line) of the pole on a toroidal surface; the frame of the generating curves; boundary conditions; the law of interpolation of the surface between the generating curves.

Let us introduce the coordinate system (Fig 1).

The law of winding in a toroidal coordinate system

$$\varphi = \frac{1}{m_b}\left(\theta - \alpha\sin\theta - \beta\sin 2\theta\right) + \frac{2\pi}{l}k,$$

where k = 0,1,...,m-1 – pole number; l – number of poles; $\alpha = \frac{r}{2R_0}$, $\beta = \left(\frac{r}{2R_0}\right)^2$ – modulation coefficients; m_b – number of revolutions (number of field periods).

The *xyz* coordinates are determined from the relation that connects the global and toroidal coordinate systems

$$x = (R_0 + r\cos\theta)\cos\varphi;$$

$$y = (R_0 + r\cos\theta)\sin\varphi;$$

$$z = r\sin\theta.$$

The framework of the generating curves is determined by the normal cross-section to the winding line at point M during movement along (Fig. 1).

We define an array of points belonging to the pole surface in this cross-section (Fig. 2).

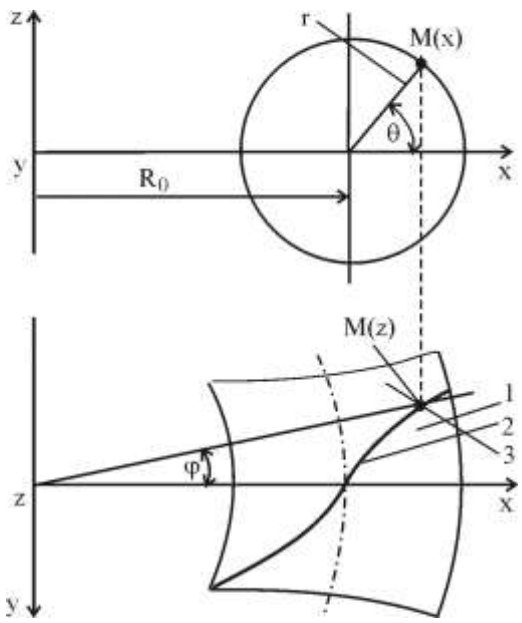


Fig. 1. Coordinate system: r – small radius of the torus; R_0 – large radius of the torus; φ, θ – toroidal coordinates; xyz – Cartesian coordinate system; 1 – torus surface; 2 – winding line; 3 – normal cross-section to the winding line at point M

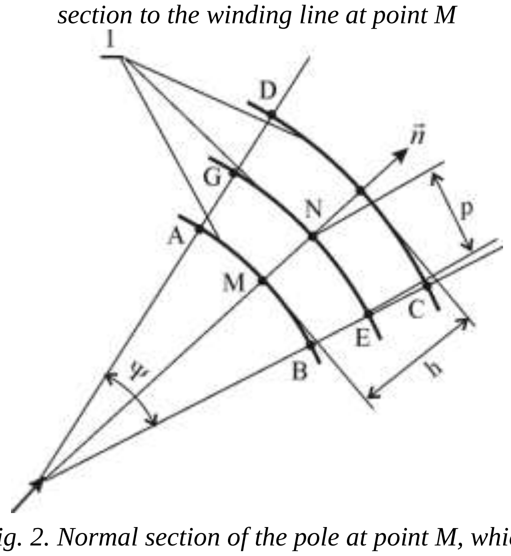


Fig. 2. Normal section of the pole at point M, which belongs to the winding line: 1 – line of intersection of the normal plane and torus surfaces; h – pole height; Ψ – angular width of the pole; 2p – width of the pole in the average cross-section

To do this, on the normal $\vec{n}$ to the torus surface at point M, we set aside a segment of size h, which is equal to the height of the pole. On the line of intersection of the normal plane with the torus surface, which has a radius of the meridional section $r + \frac{h}{2}$, to the left and right of point N we set aside segments of size ρ and fix points E and G. Through the obtained points at an angle $\psi = \text{const}$, we draw lines that intersect at points A, B, C, D with the lines of torus surfaces with radii and (r + h). We represent the coordinates of the points of the array components A, B, C, D in the Cartesian coordinate system xyz (see Fig. 1).

A set of these arrays, obtained in different meridional sections (φ = var), forms a HW raster.

The algorithm for obtaining the HW kinematic surface is quite laborious [3, 4]. The task can be significantly simplified if you use a raster representation of the surface. In this case, the model of the HW surface is represented as a grid consisting of characteristic intersecting lines belonging to the surface. Such lines are meridional cross-sections and segments connecting characteristic points when breaking closed lines limiting the sections. The intersection points of closed curves limiting the section and segments form raster nodes, and a set of such points on the modeled surface is a raster (Fig. 3). If the distance between the raster nodes is small, then the raster points describe the HW surface quite accurately.

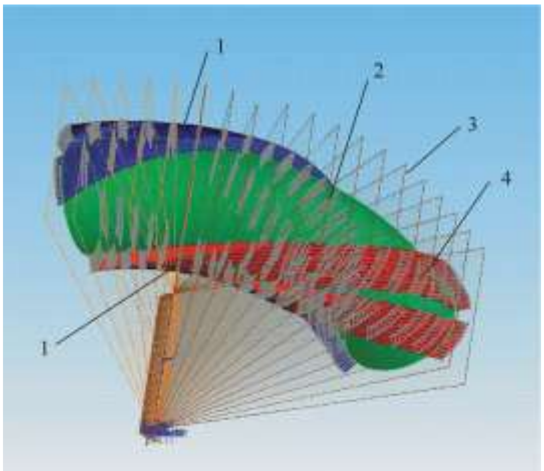


Fig. 3. Raster representation of the HW surface: 1 – HW poles; 2 – surface of the torus; 3 – meridional section; 4 – segments connecting characteristic points of closed lines

When solving practical problems, for example, calculating the stress-strain state of a magnetic system [5, 6], there is a need to have access to the surface points lying between the nodal points of the raster. To restore the coordinates of points on the HW surface, approximation by Bezier splines is used.

To restore the surface with a Bezier spline, twelve more points are added to each grid cell (to the existing four raster points) so that there are sixteen reference nodes in each cell (Fig. 4,a). Based on these sixteen reference nodes, an approximating Bezier surface for the cell is constructed.

$$\overline{K}(u,\vartheta) = \sum_{i=0}^{3}\sum_{j=0}^{3} c_3^i c_3^j \Delta u^i (1-\Delta u)^{3-i} \Delta\vartheta^j (1-\Delta\vartheta)^{3-j}\, \overline{K}_{ij},$$

where $\Delta u = u - u_0$, $\Delta\vartheta = \vartheta - \vartheta_0$, $\tilde{n}_k^j$ – binomial coefficients; k_{ij} – radii vectors of reference nodes.

By choosing reference nodes, it is possible to achieve continuity and smoothness of the surface at the junctions of cells, as well as to satisfy boundary conditions at the edges of the surface. In geometric modeling of the HW surface, a simplified algorithm for surface approximation is provided, which retains the advantages of Bezier splines, but requires a significantly smaller number of reference nodes. These nodes are added only near the edges of the surface or near the fracture line. With this approach, the surface will be smooth everywhere except for the fracture line. The fracture lines look like lines on which the surface smoothness is violated.

Let the studied HW surface have curvilinear coordinates (u,ϑ) so that if the surface is covered with a mesh $u=i$, $\vartheta=j$ (i, j – integers), then the boundary of the region on the plane (u,ϑ) will be mesh-polygonal. Let us consider the approximation of a cell of such a mesh. Let us assume that no corner point of the cell belongs to the boundary of the region or the fracture line.

Let's introduce a 4×4 grid of cell vertices into the vicinity of the cell (Fig. 4,b).

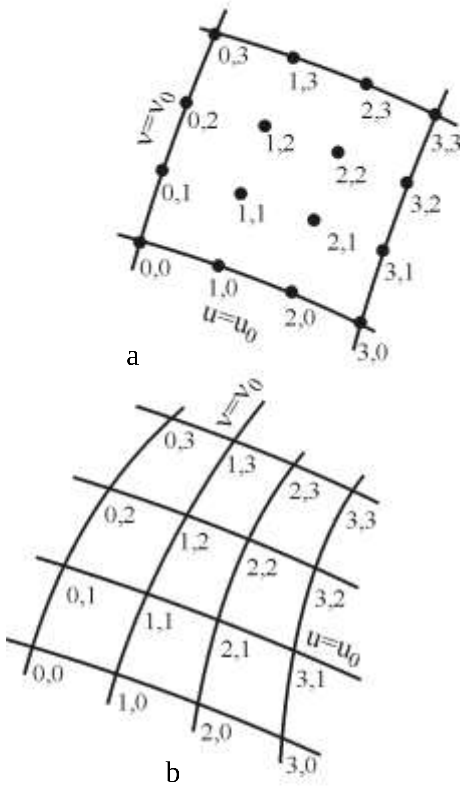


Fig. 4. Bézier surface reconstruction: 16 reference nodes (a); 4×4 mesh of cell vertices (b)

The approximating surface for a cell is described by the formula

$$\overline{K}(u,\vartheta)=\sum_{i=0}^{3}\sum_{j=0}^{3}R_i(u-u_0)R_j(\vartheta-\vartheta_0)\overline{K}_{ij},$$

where $R_\ell(t)$, $0\le\ell\le3$ polynoms:

$$R_0(t)=-t(t-\ell)^2/2;$$
$$R_1(t)=(3t^3-5t^2+2)/2;$$
$$R_2(t)=(3t^3-4t-\ell)/2;$$
$$R_3(t)=t(t-\ell)/2.$$

The surface has the property of continuity and smoothness at the cell joints. It is obtained from four biquadratic segments of the surface by the Overhauser method.

Let us consider the properties of the approximating HW surface. To do this, we introduce the notation. The surface cell corresponds to a square on the $(u,\vartheta)-$ plane, with $u_0\le u\le u_0+1$, $\vartheta_0\le\vartheta\le1$. The cell has coordinates $(u_0+0.5,\ \vartheta_0+0.5)$. Nodes are denoted by pairs of coordinates (u,ϑ), for example (u_0,ϑ_0+1). The reference nodes are all sixteen nodes used in its parameterization.

The continuity of the surface at the junction of cells is determined by the fact that the junction line is completely given by the superposition of the same four nodes that are the reference for both adjacent cells of the HW surface. This is confirmed by the fact that if the cells $(i+0.5,\ j+0.5)$ and $(i-0.5,\ j+0.5)$ border, then the coordinates of the point, as the contact line when approximating one and the other cell, are described by the same relation

$$\overline{K}(\vartheta)=\sum_{k=0}^{3}R_k(\vartheta-j)\overline{K}_{i,j+k-1}. \qquad (1)$$

The smoothness at the junction of the cells follows from the fact that the parametric derivatives of the surface radius vector at the contact line are completely determined by the superposition of twelve nodes, which are the reference for both cells. Indeed, let the cells $(i+0.5,\ j+0.5)$ and $(i-0.5,\ j+0.5)$ border each other, then the derivative $\partial K/\partial\vartheta$ on the right and on the left is the same according to (1). The derivative $\partial K/\partial\vartheta|_{u=i}$ on the right and on the left is expressed in the same way

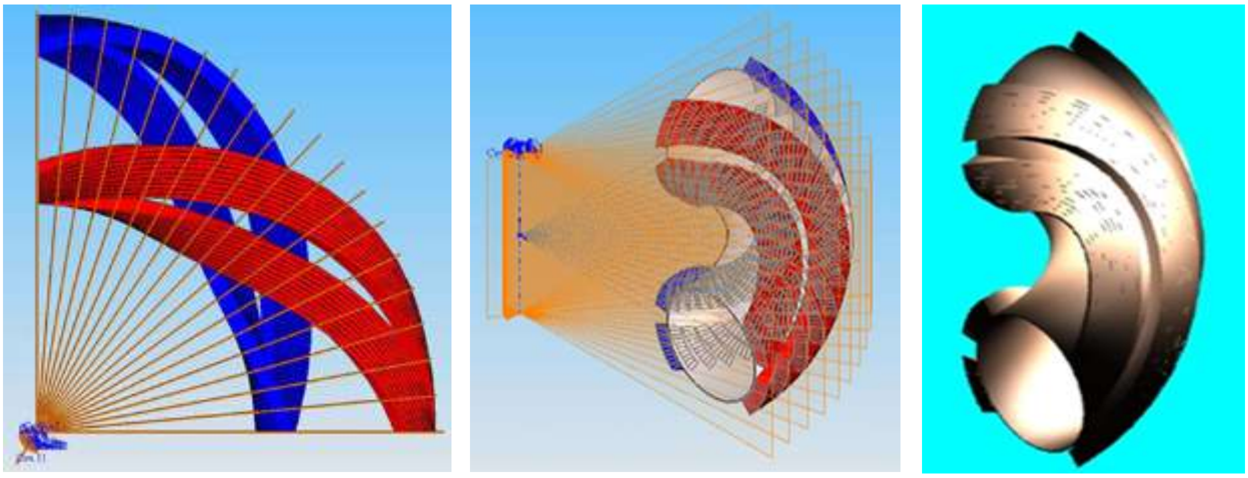
$$\left.\frac{\partial\overline{K}}{\partial u}\right|_{u=i}=\sum_{k=0}^{3}\frac{R_k(\vartheta-j)\overline{K}_{i+1,j+k-1}-\overline{K}_{i-\ell,j+k-1}}{2}.$$

The presented approach is applied only for the internal cells of the HW approximating surface. For the boundary cells, we introduce fictitious nodes instead of the missing real nodes. The coordinates of the fictitious nodes are determined from the conditions of preservation of the continuity and smoothness of the surface and from the boundary conditions at the nodes of the edges (or fracture lines) of the HW.

The dichotomy method in the problems of modeling magnetic system elements. The dichotomy method (half-division method) is a sequential search procedure that belongs to direct methods of one-dimensional optimization. The algorithms use the principle of division in half and allow you to quickly exclude distant links of intersecting objects and focus on close ones. The following algorithms were used: finding the intersection points of two multi-link polylines; two spline curves; surface and straight line.

In geometric modeling of the HW surface, a simplified surface approximation algorithm is provided, which retains the advantages of Bezier splines, but requires a significantly smaller number of reference nodes. These nodes are added only near the edges of the surface or near the fracture line. With this approach, the surface will be smooth everywhere except for the fracture line. The fracture lines look like lines where the surface smoothness is disturbed.

The simulation results are presented at Fig. 5



a *b* *c*

Fig. 5. HW surface shaping: a) by a set of meridional cross-sections; b) raster representation of the surface; c) surface reconstruction by a Bezier spline

CONCLUSIONS

The article presents mathematical models, methods and algorithms that allow obtaining three-dimensional solid-state and surface geometric models of plasma facility helical winding. Kinematic modeling methods, Bezier splines and Overhauser equations are used to determine the coordinates of the points of the surface being modeled. Based on the developed mathematical models, a number of engineering calculations of the system can be performed – calculations of the stress-strain state of the system and heat transfer processes.

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ПОВЕРХОНЬ МАГНІТНИХ ОБМОТОК ПЛАЗМОВИХ УСТАНОВОК

С.О. Мартинов, В.П. Лук'янова, С.І. Прохорець, М.А. Хажмурадов

Серед багатьох проблем, пов'язаних із створенням плазмових установок, можна виділити проблему отримання математичних моделей геометрії елементів магнітної системи. До таких елементів відносяться магнітні поверхні, серед яких найбільш складною є гвинтова обмотка. Наведено методи та алгоритми отримання твердотільної геометричної моделі поверхні гвинтової обмотки методами кінематичного моделювання.