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HYDRODYNAMIC STABILITY OF A WATER VAPOR NANOBUBBLE IN WATER UNDER NORMAL CONDITIONS

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The hydrodynamic stability of a water vapor nanobubble in water under normal conditions is investigated. The hydrodynamic equation describing the motion of the bubble boundary taking into account the bubble charge and the dependence of the surface tension coefficient on the radius, as well as the heat balance equation of the nanobubble are used for the study. Taking into account the nanobubble charge leads to a competition between capillary and electrostatic forces, which affects the hydrodynamic stability of the nanobubble. The main equations are written down, and an analysis of the stability of small disturbances is carried out. The characteristic equation is obtained and the stability conditions of the nanobubble are determined. It is shown that the radius of a stationary nanobubble is equal to the Tolman radius, since at this radius its energy is minimal. The value of the surface charge density and the charge of a vapor nanobubble in water are determined.

The formation and existence of surface nanosized bubbles have been studied experimentally and theoretically for over 20 years [1]. For this purpose, atomic force microscopy, rapid cryofixation, neutron reflectometry, and direct optical visualization were used for experimental research. The results of the studies showed that nanosized bubbles are formed in the form of bridges adhering to solid surfaces. However, the question of the existence of vapor nanobubbles in water under normal conditions remains unclear. From a theoretical point of view, the formation and existence of nanosized vapor bubbles is described by the theory of homogeneous nucleation, where the nucleation condition is determined by the dependence of the difference in the Gibbs free energies of both media (hereinafter referred to as the Gibbs free energy) on the size of the nucleus [2 - 4]. The basic idea of the nucleation process is that for a nanoscale or microscopic nucleus of a new phase to emerge, the requirement for a minimum Gibbs free energy must be met, the value of which can be represented as the sum of the volume and surface energy.

Historically, studies of nanobubble stability have been divided into two groups, which include surface and volumetric nanobubbles [5]. However, the question of whether the stability of surface and volumetric nanobubbles has the same physical nature has not yet been clarified [6]. In addition, despite a number of research results, volumetric nanobubbles are still a new field, and there is still no physical justification for their existence and stability.

To clarify this situation, the most accessible and illustrative example of the nucleation of vapor nanobubbles under normal conditions can be water. In this case, not only the process of their nucleation is important, but also their further resistance to destruction over time.

In this case, when studying the stability of vapor nanobubbles, it is necessary to take into account such a previously unconsidered phenomenon as the dependence of the coefficient of tension of the surface layer of a water nanobubble on its size. Taking into account such a dependence can significantly change the conclusions of works [7–9], which consider issues of the theory of growth of vapor bubbles in metastable liquids and alkali metals.

Therefore, it is necessary to continue the study of the hydrodynamic stability of a steam bubble in water, taking into account that the obtained result can also be used for metallic media in the liquid phase.

In this paper, the hydrodynamic stability of a water vapor nanobubble is investigated taking into account the linear dependence of the surface tension coefficient on the radius at small values of its radius.

INITIAL CONDITIONS AND INITIAL DATA

It is known that the equilibrium state of a vapor bubble in water is always unstable [7–9]. This conclusion is based on the fact that the equation of motion of the bubble boundary used the coefficient of surface tension of water, the value of which does not depend on the size of the bubble. However, in the case of nano- and micro-sized bubbles, as shown in [10], the coefficient of surface tension is not a constant, and for small radii of the bubble it depends linearly on its radius. Following [10], the dependence of the coefficient of surface tension $\sigma(r)$ on the radius r can be represented as:

$$\sigma(r) = \frac{\sigma_\infty r}{r+R_*}, \qquad (1)$$

where σ_∞ is the coefficient of surface tension of the flat boundary of the "water – air" system (the bubble radius tends to ∞); $R_* -$ is the bubble size characteristic of each type of liquid, above which the coefficient of surface tension tends to σ_∞. Hereinafter, R_* will be called the Tolman radius. Its value is individual for each liquid. For example, for a nanosized air bubble $R_* \approx 1.6 \cdot 10^{-8}$ m [11].

In addition, it is known that air nanobubbles in water have a charge [4]. We will assume that in the case of filling the bubble with water vapor, the charge remains of this order. The charge value can reach values of $q \approx 10^{-15} \dots 10^{-14}$ C [4]. Due to the repulsion of negative charges on the surface of the bubble, the charged sphere will be subject to expanding pressure. This pressure is oriented against the capillary compression pressure, and its value is determined by the expression [4, 12]:

$$p_q(r) = -\frac{q^2}{32\pi\varepsilon\varepsilon_0 r^4}, \qquad (1a)$$

where q – charge of the nanobubble sphere; $\varepsilon = 81$– is the permittivity of water; ε_0 – is the electric constant; r – is the radius of the bubble.

Thus, the conclusions of works [7–9] should be clarified in connection with the fact of the inconstancy of the surface tension coefficient at nanosized radii of the bubble.

Therefore, this article presents a study of the stability of a water vapor bubble, which has a negative charge, and its surface tension coefficient in the region of small radii of the bubble is proportional to the radius.

In studying this issue, we will retain the notation and sequence of calculations used in work [7].

The unknown variables of the problem of hydrodynamic stability of a water vapor bubble are the water temperature – $T(t)$; $a(t)$ – the radius of the vapor bubble; $p(t)$ and $p_V(t)$ – the water pressure and the vapor pressure in the bubble, respectively. In this case, we assume that water is incompressible, i.e., the density of water is considered constant – $\rho_l(t) = \rho_l^0$.

Let $T_0 = T(0), a_0 = a(0), p_0 = p(0)$ and $p_{V0} = p_V(0)$ denote the equilibrium initial water temperature, bubble radius, water pressure and water vapor pressure in the bubble (here and below, the index 0 determines the equilibrium values). The term equilibrium parameters of the "water – vapor bubble" system indicates that the conditions of mechanical equilibrium, taking into account the electrostatic forces of repulsion of like charges, and thermodynamic equilibrium must be met:

$$p_{V0} = p_0 + \frac{2\sigma(a_0)}{a_0} - \frac{q^2}{32\pi\varepsilon\varepsilon_0 a_0^4}, \qquad (2)$$

$$T_0 = T_s(p_{V0}), \qquad (3)$$

where $T_s(p_{V0})$ – equilibrium initial temperature of phase transitions at pressure p_{V0}, which is valid under the condition that the vapor density ρ_V^0 is small compared to the density of water ρ_l^0: $\rho_V^0 \ll \rho_l^0$.

From equality (2) it follows that the pressure in the bubble p_{V0} is greater than the pressure in water p_0. Therefore, water in equilibrium with the vapor bubble is always superheated relative to the equilibrium temperature $T_s(p_0)$ outside the bubble ($T_0 > T_s(p_0)$).

SYSTEM OF INITIAL EQUATIONS

To describe the change in the parameters of a steam bubble over time, we use the equation describing the change in the radius of a bubble in incompressible water, the equation of heat and mass transfer, and the equation of heat diffusion in water [7, 8]:

$$\rho_l^0\left(a\ddot{a} + \frac{3}{2}\dot{a}^2 + \frac{4\nu_l^{(\mu)}\dot{a}}{a}\right) = p_V - p_0 - \frac{2\sigma(a)}{a} +$$
$$+\frac{q^2}{32\pi\varepsilon\varepsilon_0 a^4}, \qquad (4)$$

$$\dot{p}_V = -\frac{3\gamma p_V}{a}\dot{a} + \frac{3(\gamma-1)c_{pV}T_a}{l}\frac{\lambda_l}{a}\left(\frac{\partial T_l}{\partial r}\right)_a, \qquad (5)$$

$$\rho_l^0 c_l\left(\frac{\partial T_l}{\partial t} + w\frac{\partial T_l}{\partial r}\right) = \frac{\lambda_l}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T_l}{\partial r}\right), \qquad (6)$$

where $\nu_l^{(\mu)}$, λ_l – coefficients of kinematic viscosity and thermal conductivity of water; γ, c_{pV}, l – adiabatic index, heat capacity of steam at constant pressure, specific heat of vaporization; w and c_l – radial velocity and heat capacity of water. A dot or two dots above a symbol denote the first or second partial derivative of this symbol with respect to time. The index a for temperature and its derivative with respect to radius denote their values on the surface of the bubble.

Equations (4) – (6) must be supplemented with boundary conditions.

We write the boundary conditions for equation (6) as follows:

$$T_l = T_a \text{ at } r = a, T_l = T_0 \text{ at } r \to \infty. \qquad (7)$$

For the temperature in the vapor bubble, we assume that the homogeneity condition is satisfied, which corresponds to the equality $T_V = T_a$.

To determine the parameters of the vapor in the bubble, we use the Clapeyron-Mendeleev equation:

$$p_V = \rho_V R T_V, \qquad (8)$$

where $R = 8.3145$ J·K^{-1} mol^{-1} – gas constant.

The dependence of temperature $T_V \equiv T_a$ a on vapor pressure at subcritical states can be determined from the Clapeyron-Clausius equation [7, 8]:

$$\frac{dT_V}{dp_V} = \frac{T_V}{\rho_V^0 l}. \qquad (9)$$

From (8), (9) we can obtain the dependence of temperature on pressure:

$$T_V = T_s(p_V) = \frac{T_*}{\ln(p_*/p_V)}, \qquad (10)$$

where T_* and p_* – constants determined by the properties of water.

OBTAINING A CHARACTERISTIC EQUATION FOR THE «WATER – STEAM BUBBLE» SYSTEM

Let us consider small deviations of the system parameters (radius, pressure and temperature of the steam bubble) from their initial equilibrium values:

$$a(t) = a_0 + \tilde{a}(t),\ p_V(t) = p_{V0} + \tilde{p}_V(t),$$
$$T_l(r,t) = T_0 + \tilde{T}_l(r,t),\ \rho_V^0 = \rho_{V0}^0 + \tilde{\rho}_V^0, \qquad (11)$$

where the condition of small deviations is ensured by the fulfillment of the inequalities $|\tilde{A}| \ll A_0$, A_0 denotes the equilibrium values of the corresponding parameters.

Linearization of equations (4) – (6) yields the following system of linear equations with respect to the perturbed quantities:

$$\rho_l^0\left(a_0\ddot{\tilde{a}} + \frac{4\nu_l^{(\mu)}\dot{\tilde{a}}}{a_0}\right) = \tilde{p}_V + \frac{2\sigma_\infty\tilde{a}}{(a_0+R_*)^2} - \frac{q^2\tilde{a}}{8\pi\varepsilon\varepsilon_0 a_0^5}, \quad (12)$$

$$\dot{\tilde{p}}_V = -\frac{3\gamma p_{V0}}{a_0}\ddot{\alpha} + \frac{3(\gamma-1)c_{pV}T_a}{l}, \quad (13)$$

$$\rho_l^0 c_l \frac{\partial \tilde{T}_l}{\partial t} = \frac{\lambda_l}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \tilde{T}_l}{\partial r}\right), \quad (14)$$

The boundary conditions take the form:

$$\tilde{T}_l = \tilde{T}_a \text{ at } r = a_0, \tilde{T}_l = 0 \text{ at } r \to \infty. \quad (15)$$

From (9), after differentiation and letting the integration constant tend to zero, we obtain the relationship between the disturbed temperature and the vapor pressure inside the vapor bubble:

$$\frac{\tilde{T}_V}{T_0} = \frac{\tilde{p}_V}{\rho_{V0}^0 l}. \quad (16)$$

Further in the calculations we omit the sign "~".

We will look for solutions to the system of equations (12) - (14), taking into account (15), (16) in the form:

$$a(t) = A_a e^{\lambda t}, p_V(t) = A_p e^{\lambda t}, T_l(r,t) = A_T(r)e^{\lambda t}, \quad (17)$$

where $A_a, A_V, A_T(r)$ – amplitudes of change in radius, steam pressure and water temperature, respectively, λ is the stability index, determining the change in the corresponding quantity by e times during time λ^{-1}. If $\lambda > 0$ the disturbance increases, $\lambda < 0$ the disturbance dies out, $\lambda = 0$ the disturbance is stable.

The system of equations (12) – (14) after substituting (17) takes the form:

$$\left(a_0\rho_l^0\lambda^2 + \frac{4\nu_l^{(\mu)}\rho_l^0}{a_0}\lambda - \frac{2\sigma_\infty}{(a_0+R_*)^2} + \frac{q^2}{8\pi\varepsilon\varepsilon_0 a_0^5}\right)A_a = A_p, \quad (18)$$

$$\lambda A_p = -\frac{3\gamma p_{V0}}{a_0}\lambda A_a + \frac{3(\gamma-1)c_{pV}T_0\lambda_l}{l a_0}\left(\frac{\partial A_T(r)}{\partial r}\right)_{a_0}, \quad (19)$$

$$k^2 A_T(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial A_T(r)}{\partial r}\right), \quad (20)$$

where $k^2 = \frac{\rho_l^0 c_l \lambda}{\lambda_l}$.

The solution to equation (20) is the function [8]:

$$A_T(r) = \frac{T_0}{\rho_{V0}^0 l}A_p\frac{a_0}{r}\exp\big(k(a_0 - r)\big). \quad (21)$$

From (21) we determine the derivative $\left(\frac{\partial A_T(r)}{\partial r}\right)_{a_0}$:

$$\left(\frac{\partial A_T(r)}{\partial r}\right)_{a_0} = -\frac{T_0}{\rho_{V0}^0 l}A_p\left(\frac{1+a_0k}{a_0}\right). \quad (22)$$

From equations (18), (19) taking into account (22), a system of equations follows that determines the relationship between A_p and A_a:

$$\left(a_0^2\rho_l^0\lambda^2 + 4\nu_l^{(\mu)}\rho_l^0\lambda - \frac{2\sigma_\infty a_0}{(a_0+R_*)^2} + \frac{q^2}{8\pi\varepsilon\varepsilon_0 a_0^4}\right)A_a - a_0A_p = 0, \quad (23)$$

$$\frac{3\gamma p_{V0}}{a_0}\lambda A_a + \left(\lambda + \beta\nu_l^{(T)}\left(\frac{1+a_0k}{a_0^2}\right)\right)A_p = 0, \quad (24)$$

where $\beta = 3(\gamma-1)\frac{\rho_l^0 c_l}{\rho_{V0}^0 c_{pV}}\left(\frac{c_{pV}T_0}{l}\right)^2$, $\lambda_l = \nu_l^{(T)}\rho_l^0 c_l$, $\nu_l^{(T)}$ – coefficient of thermal diffusivity of water.

Equating the determinant of this system of equations to zero, we find the characteristic equation:

$$\left(\lambda + \beta\nu_l^{(T)}\left(\frac{1+a_0k}{a_0^2}\right)\right)\left(a_0^2\rho_l^0\lambda^2 + 4\nu_l^{(\mu)}\rho_l^0\lambda - \frac{2\sigma_\infty a_0}{(a_0+R_*)^2} + \frac{q^2}{8\pi\varepsilon\varepsilon_0 a_0^4}\right) + 3\gamma p_{V0}\lambda = 0. \quad (25)$$

Equation (25) differs from that obtained in [7, 8] by the presence of a term with the critical size of the vapor bubble R_* and a term that takes into account the bubble charge. This fact changes the dynamics of vapor bubbles at small radii, since at $a_0 \ll R_*$ the capillary term tends to a constant value proportional to a_0/R_*^2. Otherwise, when$a_0 \gg R_*$, the capillary term is proportional to $1/a_0$, which corresponds to the theory proposed in [7, 8]. The presence of a bubble charge also changes the dynamics of bubble formation, since there is a competition between capillary and electrostatic forces.

STABILITY ANALYSIS OF THE WATER – STEAM BUBBLE SYSTEM UNDER NORMAL CONDITIONS

We multiply equation (25) by a_0^4 and introduce a new variable $\delta^2 = a_0^2\,\lambda$. Then we obtain the following equation for determining the dependence of the instability parameter δ^2 on the initial radius of the vapor bubble a_0:

$$\left(\delta^2 + \left(\nu_l^{(T)}\right)^{\frac{1}{2}}\cdot\delta + \beta\nu_l^{(T)}\right)\left(\frac{2\sigma_\infty a_0^3}{(a_0+R_*)^2} - \frac{q^2}{8\pi\varepsilon\varepsilon_0 a_0^2} - \rho_l^0\delta^4 - 4\nu_l^{(\mu)}\rho_l^0\delta^2\right) = 3\gamma p_{V0}\delta^2 a_0^2. \quad (26)$$

Let us determine the characteristic parameters of the water – steam bubble system under normal conditions (water pressure – 0.1 MPa, water temperature – 20 °C), using reference data [13–15]. Table shows the characteristic parameters of the water – steam bubble system under normal conditions.

Characteristic parameters of the water – steam bubble system under normal conditions

Parameter designation	Water		Steam	
Density, kg/m^3	ρ_l^0	998.2	ρ_{V0}^0	1.729·10^{-2}
Thermal conductivity coefficient, W/(m•K)	λ_l	0.603	λ_V	1.824·10^{-2}
Kinematic viscosity, m^{2}s^{-1}	$\nu_l^{(\mu)}$	10^{-6}	-	-
Heat capacity, J/(kg•K)	c_l	4183	c_{pV}*	1.877·10^3
Specific heat of vaporization, J/kg	-	-	l	2.453·10^6
Adiabatic index	-	-	γ	1.33
Surface tension coefficient (flat boundary), N/m	σ_∞	0.073	-	-

*at constant pressure.

In equation (26), the charge of the nanobubble is uncertain. In [4, 12], the charge of the nanobubble is determined by the value $q = 10^{-17}$ ... 10^{-13} C with the radius of the nanobubble being – 2 ... 10^3 nm.

For the data in Table and the renormalized values $a_0 = b \cdot 10^{-9}$, $R_* = R \cdot 10^{-9}$, $\delta = x \cdot 10^{-3}$ we obtain the characteristic equation for x with numerical coefficients:

$$(D(b,R) - x^4 - 4x^2)(x^2 + 10x + 1) - 3.99x^2b^2 10^{-4} = 0, \quad (27)$$

where $D(b,R) = 10^2 b^2 [14.6 \cdot b(b+R)^{-2} - 2.46]$.

STATIONARY VAPOR NANOBUBBLES STABILITY OF EQUILIBRIUM NANOBUBBLES WITH TOLMAN RADIUS

Under the condition $D(b,R) = 0$, the nanobubble will be stable, since in this case the solutions of equation (27), which is a 6th degree polynomial, will be six numbers. The first two solutions correspond to stationary states of the bubble ($x_{1,2}^2 = 0 \rightarrow \lambda_{1,2} = 0$). The numerically found solutions $x_{3,4}$ give positive values $x_{3,4}^2 > 0$, which corresponds to the growth of the bubble radius. But since no energy comes from outside to the water – steam bubble system under normal conditions, the growth of the bubble radius is physically unjustified. Therefore, these solutions are redundant and are not considered further.

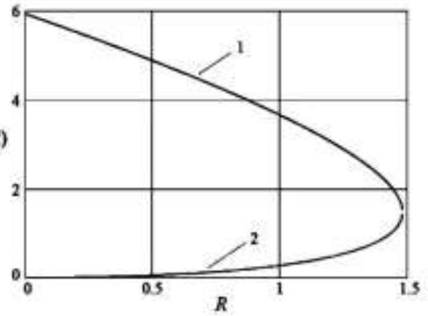
Numerical solutions $x_{5,6}$ give negative values of the stability index $x_{5,6}^2 = -4$ for unperturbed radii $b \leq 2.02$, which corresponds to aperiodic decay of the perturbed radius of the nanobubble. For radii $b > 2.02$ the values of the stability index $\sim x_{5,6}^2$ are complex, the real part of which is $Re(x_{5,6}^2) = -4$, and the imaginary part varies from zero $Im(x_{5,6}^2) = 0$ at $b = 2.02$ to $Im(x_{5,6}^2) = \pm 0.049$ at $b = 50$. This means that perturbations of the nanobubble radius decay, oscillating with a frequency proportional to $Im(x_{5,6}^2)$.

The equality $D(b,R) = 0$ defines the relationship between the nanobubble radius b and the critical size of the vapor nanobubble, equal to the Tolman radius R:

$$b_{1,2}(R) = \frac{5.935 - 2R}{2} \pm \frac{1}{2}\sqrt{5.935}\sqrt{5.935 - 4 \cdot R}\,. \quad (28)$$

From (28) it follows that for $R = 1.48375$ the relation $b_1 = b_2 = 1.48375$ is fulfilled. The equality of the radii b_1, b_2 of the vapor nanobubble to the Tolman radius R will be considered an equilibrium state, and the bubble itself will be considered as an equilibrium one.

Thus, $D(b,R) = 0$ the perturbations of the nanobubble radius tend to zero, i.e. it is stable at $R = b_1 = b_2 = 1.48375$.



Dependence of the radius of a vapor nanobubble $b(R)$ on the Tolman radius R.
Curve 1 – $b_1(R)$, curve 2 – $b_2(R)$

Figure shows the dependence of the nanobubble radius *b* on the Tolman radius *R*.

Expression (28) defines the dependence of the nanobubble radius on the Tolman radius. Below we will show that in the equilibrium state, when $D(b,R) = 0$, the nanobubble energy corresponds to the minimum energy.

Using (28), we determine the radius of stationary vapor nanobubbles in water. From (28) it follows that for a given value of the Tolman radius *R,* the existence of stationary vapor nanobubbles of two radii is possible: $b_1(R)$ and $b_2(R)$. However, the presence of two nanobubbles of different radii must correspond to the minimum energy, which is proportional to the sum of the surfaces of the nanobubbles. For two nanobubbles, the energy is proportional to the sum of its areas, i.e. $W_2 = 4\pi(b_1(R))^2 + 4\pi(b_2(R))^2$. On the other hand, the energy of a nanobubble with the Tolman radius R is proportional to $W_T = 4\pi R^2$ Calculations show that the energy of two nanobubbles with the Tolman radius R is always less than the energy of two nanobubbles with different radii: $b_1(R)$ and $b_2(R)$, i.e. the inequality $W_2 \geq 2 \cdot W_T$ is always satisfied. It follows from this that the formation of two nanobubbles with the Tolman radius is energetically more favorable instead of two nanobubbles with different radii. Consequently, in real conditions, the radius of a stationary nanobubble is equal to the Tolman radius *R*.

This conclusion can be used to experimentally determine the Tolman radius when registering stationary vapor nanobubbles in water under normal conditions.

STABILITY OF STATIONARY NANOBUBBLES WITH A RADIUS OTHER THAN EQUILIBRIUM

If the initial value of the nanobubble radius *b* differs from the equilibrium value b_1 by a large amount, then the stability of the perturbations of the radius of such bubbles is described by equation (27). Due to the complexity of the analytical study of this equation, numerical methods for finding its roots are used.

An analysis of the four roots shows that for $b \leq 0.15$ their squares are negative, and for $b > 0.16$ they have a negative real part equal to -2.

Positive values of the squares of the two roots are given by redundant solutions and are not considered.

A small deviation of the nanobubble radius *b* from the equilibrium value b_1 leads to the appearance of a small parameter $D(b,R)$. Assuming the deviation in the form $b = b_1 + \delta b$, $b_1 = b_2 = R$, where $|\delta b| \ll b_1$, we write equation (27) as:

$$(61.5 \cdot \delta b^2 + x^4 + 4x^2)(x^2 + 10x + 1) + 3.99x^2b^2 10^{-4} = 0. \quad (29)$$

Equation (29) describes the change in the radius of a nanobubble if the initial value of the radius differs from the equilibrium value by a small value ±*δb.* Numerical calculation methods were used to analyze equation (29). It follows from the form of equation (29) that the solutions do not depend on the sign of the deviation of the nanobubble radius from the equilibrium value. Numerical calculations (29) show that the radius perturbations are stable. This follows from the fact that the squares of the four roots of this equation have negative values. Positive values of the squares of two roots are given by redundant solutions and are not considered.

Thus, perturbations of the radius of a vapor nanobubble in water under normal conditions and at any deviations from the equilibrium value are damped.

Therefore, to determine the equilibrium values of the nanobubble radius, one should rely on the energy principle described above: the formation of nanobubbles with a radius equal to the Tolman radius is energetically favorable.

SURFACE CHARGE DENSITY OF A STATIONARY VAPOR NANOBUBBLE IN WATER

The surface charge density of a vapor nanobubble is determined by the cross-sectional area of the spherical water molecules [14] that cover the surface of the nanobubble:

$$\sigma_0 = \frac{q}{S_0} = \frac{2\cdot 1.6\cdot 10^{-19}\mathrm{C}}{\pi\cdot(1.38\cdot 10^{-10})^2} = 0.53\cdot 10^{1} \approx 5.3\ \frac{\mathrm{C}}{m^2}, \quad (30)$$

where $S_0 = \pi \cdot R_0^2$ – cross-sectional area of a spherical water molecule with radius $R_0 = 1.38 \cdot 10^{-10}$ m [14], $q = 2e$ is the dipole charge of the water molecule.

The charge of an equilibrium vapor nanobubble $q = 4\pi R_*^2 \sigma_0 = 3.5 \cdot 10^{-17}\ C$, which corresponds to the charge of an air nanobubble in water [4, 12].

CONCLUSIONS

The paper studies the hydrodynamic stability of a vapor nanobubble in water under normal conditions. This class of problems pertains to the study of homogeneous nucleation in liquid media under a certain pressure and temperature. It follows from the analysis of previous studies that some effects were not taken into account when considering such problems. One of such unaccounted effects is the specific dependence of the surface tension coefficient on the bubble radius, which was first described by R. Tolman. This dependence is characterized by the fact that at large bubble radii the surface tension coefficient is a constant, and with a decrease in the bubble radius the surface tension coefficient becomes proportional to the radius. In this paper, using water as an example under normal conditions, the stability of a vapor nanobubble in water under normal conditions is studied. The study used a hydrodynamic equation describing the motion of the bubble boundary taking into account the bubble charge and the dependence of the surface tension coefficient on the radius. The nanobubble heat balance equation was also used. The presence of a bubble charge changes the dynamics of bubble formation, since there is a competition between capillary and electrostatic forces.

The main equations for the medium under normal conditions are written, and the stability of small perturbations of the nanobubble radius is analyzed. The characteristic equation is obtained and the stability parameters of the nanobubble are determined. Based on the minimum energy condition of the nanobubble, it is shown that the radius of a stationary nanobubble is equal to the Tolman radius. The surface charge density and the charge of a vapor nanobubble in water are determined.

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ГІДРОДИНАМІЧНА СТІЙКІСТЬ НАНОБУЛЬБАШКИ ПАРУ ВОДИ У ВОДІ ПРИ НОРМАЛЬНИХ УМОВАХ

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Досліджено гідродинамічну стійкість парової нанобульбашки води за нормальних умов. Для дослідження використано гідродинамічне рівняння, що описує рух межі нанобульбашки при врахуванні її заряду та залежності коефіцієнта поверхневого натягу від радіуса та рівняння, що описує тепловий баланс нанобульбашки. Показано, що врахування заряду нанобульбашки призводить до конкуренції капілярних та електростатичних сил, і впливає на гідродинамічну стійкість нанобульбашки. Записано характеристичне рівняння та проведено аналіз стійкості малих збурень. Показано, що радіус стаціонарної нанобульбашки дорівнює радіусу Толмена, тому що при цій величині радіуса її енергія мінімальна. Визначена величина щільності поверхневого заряду і заряд парової нанобульбашки у воді.