INTERACTION OF RELATIVISTIC PARTICLES WITH CRYSTALS AND MATTER

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STUDY OF THE DEPENDENCE OF THE RELATIVE PROBABILITY OF CLOSE COLLISIONS OF NEGATIVELY CHARGED PARTICLES WITH ATOMS OF A BENT CRYSTAL ON CRYSTAL ORIENTATION

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The study establishes the dependence of the probability of close collisions between high-energy negatively charged particles and atoms of a bent crystal on the crystal orientation. Special attention is given to comparing this probability in the cases of planar channeling and above-barrier motion. Analytical calculations are performed for a simplified model potential of the crystal atomic planes, while numerical simulations are carried out for a realistic potential.

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INTRODUCTION

When high-energy charged particles pass through oriented crystal, correlations in scattering on neighboring crystal atoms may occur. These correlations arise when the angle between the particle momentum and a crystallographic axis or plane is smaller than or comparable to the critical planar channeling angle θ_c [1]. In this case, the impact parameter during scattering on neighboring atoms of a string or a plane changes only slightly, and scattering on neighboring atoms occurs coherently [2]. Such coherent scattering allows the direction of high-energy charged particles to be deflected as they pass through a crystal, which becomes most pronounced when particles pass through bent crystals [3]. The effect of particle deflection while passing through bent crystals can be used to implement slow extraction of a fraction of charged particles from circular accelerators, as proposed, in particular, in [4]. In this context, it is important to know not only the deflection angles achievable using bent crystals but also the dependence of the probability of close collisions (PCC) of particles with crystal atoms on the orientation of the bent crystal. Such an investigation was carried out in [5, 6] for the case of positively charged particles, and in [7], a comparison of PCC for axial and planar orientations of a bent crystal was performed for negatively charged particles. In this work, we will study how the PCC changes with increasing angle between the particle momentum and the atomic plane of a bent crystal. We will be interested not in the absolute value of the PCC, but in its ratio to the PCC value for different crystal orientations.

1. ORIENTATIONAL DEPENDENCE OF THE PCC FOR NEGATIVELY CHARGED PARTICLES IN A STRAIGHT CRYSTAL

To analytically determine the dependence of the PCC on the angle between the momentum of a high-energy charged particle and a crystallographic atomic plane for a straight crystal, we will employ the following model form for the particle's potential energy in the field of crystallographic atomic planes:

$$U(x) = -\frac{U_0}{2}\left[1+\cos\left(2\pi\frac{x}{d_p}\right)\right], \qquad (1)$$

where U_0 is the depth of potential wells formed by atomic planes, d_p is the distance between adjacent atomic planes, the *x*-axis is perpendicular to the atomic planes.

The particle trajectory in such a field can be determined from the equation of motion

$$\frac{d^2x}{dt^2} = -\frac{c^2}{E}\frac{\partial U(x)}{\partial x}, \qquad (2)$$

where E denotes the particle's energy. Since the potential energy (1) differs from the potential energy of positively charged particles (see equation (1) in [6]) only by the sign of the constant term and the fact that for negatively charged particles, the atomic planes have coordinates $x_n = nd_p$, we can avoid solving the equation of motion again and instead use the result obtained in [6] (see equation (5) in [6]).

Therefore, for negatively charged particles, if the angle between the particle momentum and the crystalline atomic plane does not exceed several dozen of θ_c, the PCC using the model potential energy (1) can be found as

$$P = \frac{\sigma N v d_p}{\sqrt{2\pi r_T^2}}\int_{T_{in}}^{T_{out}}\sum_n \exp\left(-\frac{\left(x(t)-nd_p\right)^2}{2r_T^2}\right)dt, \qquad (3)$$

where σ is the cross-section for close collisions with a single atom in an amorphous target, N is the average concentration of atoms in the crystal, v is the speed of particles, and r_T^2 is the rms atomic thermal vibration amplitude in one direction.

Assuming the crystal is infinite along the *x*-direction, the summation over *n* can be performed analytically, resulting in

$$P = \sigma N v \int_{T_{in}}^{T_{out}} \vartheta_3\left(\pi \frac{x(t)}{d_p}, \exp\left(-\frac{2\pi^2 r_T^2}{d_p^2}\right)\right) dt, \quad (4)$$

where $\vartheta_3(z,q)$ is the third-kind theta function [8].

Eq. (4) makes it possible to determine the PCC for different initial conditions of particle incidence on an oriented crystal. For example, if the particle enters the crystal at $x = 0$, the following dependence of the PCC on angle $\theta_{x,0}$ between the momentum of particles impinging on the crystal and the atomic planes is obtained:

$$P(\theta_{x,0}) = \sigma N \int_0^L \vartheta_3\left(\mathrm{am}\left(\frac{\pi l \theta_{x,0}}{d_p}, \frac{\theta_c}{\theta_{x,0}}\right), \exp\left(-\frac{2\pi^2 r_T^2}{d_p^2}\right)\right) dl \,. \quad (5)$$

We can see that the difference from the case of positively charged particles lies in the type of theta function (the third one instead of the fourth [6]).

The obtained dependence is shown in Fig. 1, where the PCC is normalized to the value of $P(\theta_{x,0})$ at $\theta_{x,0} \gg \theta_c$ (the probability of close collisions in the orientation of the crystal at which coherent effects in scattering are absent).

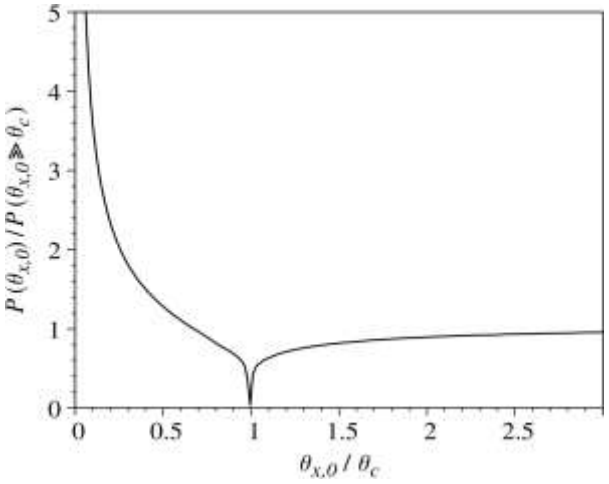


Fig. 1. The dependence of the PCC on $\theta_{x,0}$ for negatively charged particles with potential energy (1)

Small values of $\theta_{x,0}$ corresponds to the planar channeling at which the PCC is high as particles are moving in a potential well in the center of which is the atomic plane. At the same time, when $\theta_{x,0} = \theta_c$, the particle "hangs over" between the atomic planes, leading to a sharp drop in the probability of close collisions. In Fig. 1, we also observe that, in the case of over-barrier motion ($\theta_{x,0} > \theta_c$), the probability of close collisions is significantly lower than in the case of planar channeling ($\theta_{x,0} < \theta_c$).

Let us now analyze how Fig. 1 will change if the particles move not in the field (1) but in a more realistic planar potential within the Doyle-Turner approximation [9]. To do this we will use numerical simulation of the motion of negatively charged particles in the field of atomic strings. The simulation was carried out for π^- mesons with a kinetic energy of 400 GeV moving in a silicon crystal near the ⟨110⟩ axis. The thickness of the crystal was 2 mm. The angle between the initial direction of the particle momentum and the plane of the crystal bending, $\theta_{y,in}$, during the simulation was 1 mrad, which significantly exceeds the critical planar channeling angle in the $(1\bar{1}0)$ atomic plane, $\theta_c \approx 10.3$ μrad.

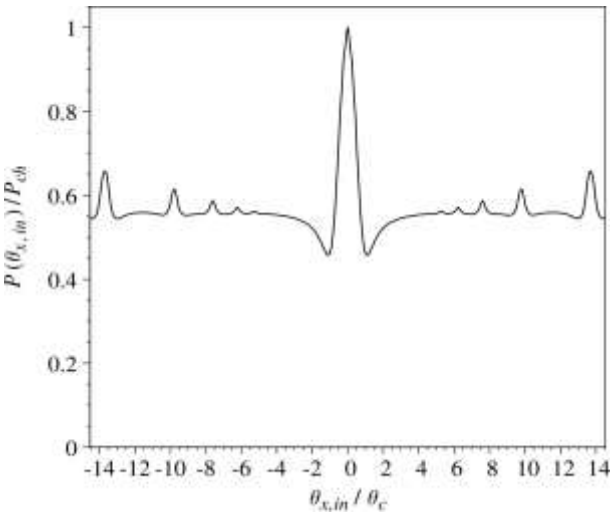


Fig. 2. The dependence of the PCC of π^- mesons on $\theta_{x,in}$ during planar channeling and over-barrier motion in an straight crystal

Fig. 2 shows the dependence of the PCC on the initial value of the angle $\theta_x = v_x / c$, obtained as a result of simulating the motion of π^- mesons in the crystal, taking into account incoherent scattering by thermal vibrations of atoms and by electrons. This probability is normalized to its value at $\theta_{x,in} = 0$, which is denoted as P_{ch}.

Unlike the analytical results shown in Fig. 1, the simulation included averaging the probability of close collisions over the initial coordinates of the particles in the crystal. Due to this averaging, the probability does not tend to zero when $\theta_{x,in} = \theta_c$. Incoherent scattering causes the trajectories of channeling negatively charged particles with small oscillation amplitude and high oscillation frequency to become unstable due to rapid dechanneling. Such dechanneling, together with averaging over the initial position, results in a less sharp peak at $\theta_{x,in} = 0$. In Fig. 2, we observe a maximum associated with planar channeling in the vertical $(1\bar{1}0)$ plane at $\theta_{x,in} = 0$ and several local maxima associated with planar channeling in skew planes with higher Miller indices. Each maximum is accompanied by two local minima, related to the "hanging over" of π^- mesons between the atomic planes.

2. ORIENTATIONAL DEPENDENCE OF THE PCC FOR NEGATIVELY CHARGED PARTICLES IN A BENT CRYSTAL

To study how the dependence of the PCC of negatively charged particles on $\theta_{x,in}$ changes during their motion in a bent crystal, numerical simulation was carried out for π^- mesons with a kinetic energy of 400 GeV near the ⟨110⟩ axis of a bent silicon crystal with a thickness $L = 2$ mm and a bending radius $R = 20$ m. The angle $\theta_{y,in}$ was 1 mrad, as in the case of the straight crystal. In the simulation we took into account incoherent scattering by thermal vibrations of atoms and by atomic electrons.

Fig. 3 shows the dependence of the PCC of π^- mesons on $\theta_{x,in}$ under conditions of planar channeling and over-barrier motion in a bent crystal. In Fig. 3, the probability is normalized to the value P_{ch} in the straight crystal.

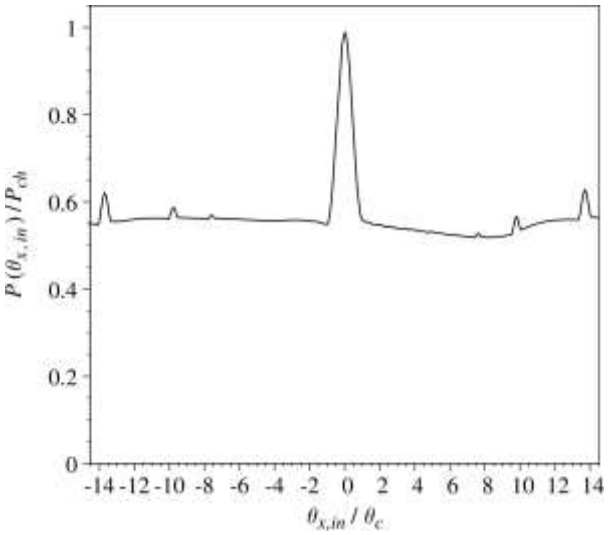


Fig. 3. The dependence of the PCC of π^- mesons on $\theta_{x,in}$ under conditions of planar channeling and over-barrier motion in a bent crystal

From Fig. 3, we can see that the maximum at $\theta_{x,in} = 0$, caused by planar channeling in the bent $(1\bar{1}0)$ planes, as well as the local maxima caused by channeling in planes with higher Miller indices, are lower than in the straight crystal. This is because the depth of the potential well formed by the atomic planes is smaller in a bent crystal compared to a straight one. In a bent crystal, not all particles with $\theta_{x,in} = 0$ enter the planar channeling regime; some become over-barrier particles upon entering the bent crystal. As shown in Fig. 1, the PCC for over-barrier particles is lower than for channeling particles, and therefore the maxima in Fig. 3 are lower than in the straight crystal case.

Moreover, in Fig. 3, almost no local minima in probability are observed at $\theta_{x,in} = \pm\theta_c$ near the maximum at $\theta_{x,in} = 0$, or near other local maxima. This is due to the fact that, because of the crystal bending and account of incoherent scattering, the "hanging over" effect between atomic planes is possible only over short segments of the π^- meson trajectories.

Unlike the case of a straight crystal, the probability of close collisions of negatively charged particles in a bent crystal is asymmetric with respect to the point $\theta_{x,in} = 0$. In a bent crystal, due to the presence of bending, asymmetry in the (*x*,*y*) plane occurs only along the *x*-direction, since the *y*-axis is perpendicular to the bending plane of the crystal (Fig. 4).

If the value of $\theta_{x,in}$ exceeds θ_c but does not exceed the crystal bending angle $\alpha \approx L/R = 100$ μrad, then at some point inside the crystal, the π^- mesons will move parallel to the $(1\bar{1}0)$ atomic planes. Near this point, volume reflection [10] will occur, but before that, the π^- mesons will be "hanging over" between atomic planes. This "hanging over" reduces the probability of close collisions in the region $\theta_c < \theta_{x,in} < \alpha$ for particles that have not been captured into planar channeling in skew planes. This result is in good agreement with the results of [11]. As a result of π^- mesons "hanging over" between skew planar channels with high Miller indices, a slight decrease in the probability of close collisions also occurs, but this effect is much smaller.

To compare the PCC at different crystal orientations relative to the beam direction, simulations of the motion of π^- mesons in a bent crystal were performed, using the parameters given above. As a result of the simulation, the PCC was calculated for various values of $(\theta_{x,in}, \theta_{y,in})$, which allowed comparing this probability for the three main mechanisms of charged particle beam deflection.

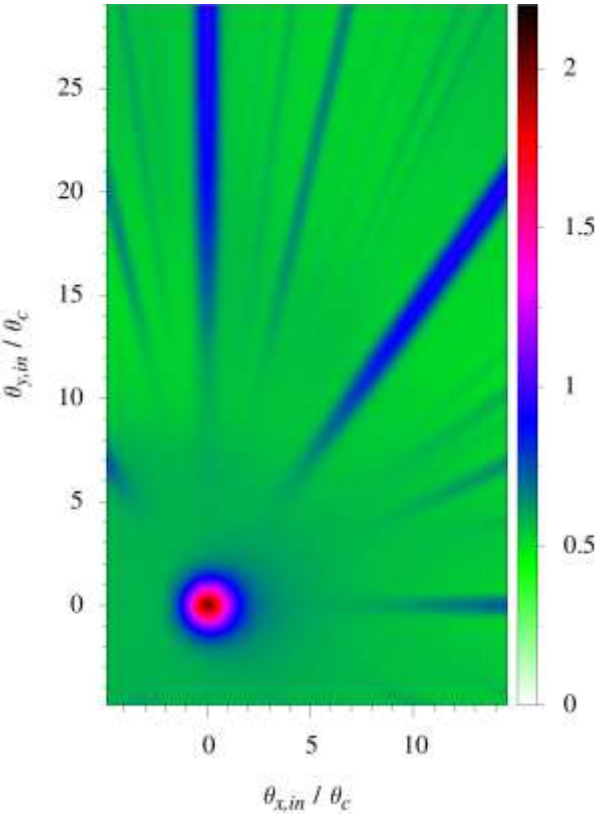


Fig. 4. The orientation dependence of the PCC of negatively charged particles in a bent crystal

Fig. 4 shows the orientation dependence of the PCC of negatively charged particles in a bent crystal. The colors show the probability normalized to the value P_{ch} in a straight crystal. At small $\theta_{x,in}$ and $\theta_{y,in}$ (axial orientation of the crystal), this probability is the highest. Planar channeling in the bent $(1\bar{1}0)$ plane corresponds to $\theta_{x,in} < \theta_c$ and $\theta_{y,in} >> \theta_c$. From Fig. 4, we see that the PCC for this regime is lower than in the case of axial orientation of the crystal. For $\theta_c < \theta_{x,in} < \alpha$ and $\theta_{y,in} >> \theta_c$, the probability of close collisions of negatively charged particles in the case of volume reflection is lower than in the case of planar channeling. Moreover, a large number of local maxima of the PCC are observed, corresponding to planar channeling of negatively charged particles in skew planes.

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ДОСЛІДЖЕННЯ ЗАЛЕЖНОСТІ ВІДНОСНОЇ ЙМОВІРНОСТІ БЛИЗЬКИХ ЗІТКНЕНЬ НЕГАТИВНО ЗАРЯДЖЕНИХ ЧАСТИНОК З АТОМАМИ ВИГНУТОГО КРИСТАЛА ВІД ОРІЄНТАЦІЇ КРИСТАЛА

І.В. Кириллін

Знайдена залежність ймовірності близьких зіткнень високоенергетичних негативно заряджених частинок з атомами вигнутого кристала від орієнтації кристала. Особлива увага приділена порівнянню значення цієї ймовірності при площинному каналюванні та надбар'єрному русі. Аналітичні розрахунки проведені для спрощеного модельного потенціалу атомних площин кристала, а для реалістичного потенціалу проведено чисельне моделювання.