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MODELING OF SUPERRADIANCE MODES AND RESONATOR FIELD FORMATION

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It is shown that the resulting total field of moving active elements-oscillators in the volume of the resonator is capable of forming a resonator field due to reflections from the ends. At a high injection rate of particles with a random phase, the conditions for their phase synchronization are suppressed and generation is disrupted. In the case of noticeable reflection and fulfillment of the conditions for synchronization of emitters at a relatively high injection rate, the resonator field significantly exceeds the total field of emitters and a traditional mode of resonator field generation is formed. In the case of an open resonator with small reflection coefficients from the ends, the amplitudes of the total radiation of the oscillators exceed the amplitudes of the resonator field and the superradiance regime is realized. The zones where the breakdown of generation is observed, generation of the resonator field and generation under superradiance conditions are shown on the plane "injection velocity – reflection coefficient".

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INTRODUCTION

The nature of excitation of oscillations in limited systems – waveguides and resonators – due to interaction with flows of charged particles and oscillators has been considered in a large number of different works (see, for example, [1 - 5]). Traditionally, the problems of generation and amplification were solved in the paradigm of the interaction of the field determined by the geometry of the waveguide or resonator with active particles according to the scheme proposed in [6], moreover, under the condition of the absence of direct interaction between them.

On the other hand, each particle or oscillator in the active zone radiates and it is possible to find the total field of these emitters. In the absence of a waveguide and a resonator, a system of such emitters under certain conditions is capable of generating radiation with a noticeable share of coherence and such radiation is usually called superluminescence or superradiance.

However, even in a waveguide and in resonators, such a total field of emitters arising due to their spatial or phase synchronization can also be quite intense and demonstrate noticeable coherence. In [7], it is shown that the intensity of the intrinsic field of particles in the modes of their arising synchronization is comparable to the intensity of traditional waveguide or resonator generation.

Since the total field of the emitters (charged particles and oscillators) of the active zone in waveguide and resonator systems is always present and can be quite significant[1], it is rational to study the influence of this field on the formation of the waveguide or resonator field. This process of formation of the waveguide and resonator fields occurs due to the effects of reflection from the ends of the system and the interaction of the reflected waves with particles in the active zone. In certain cases, when the synchronization of the emitters is weakened by the fast injection of particle emitters and a short residence time in the waveguide or resonator, a breakdown of generation can be observed. When the conditions for synchronization of particles and oscillators are met and the reflection of the field from the ends of the system increases, it is possible to observe the formation of a waveguide or resonator field that exceeds the total eigenfield of the active particles. At low reflection coefficients, the eigenfield of the particles can dominate, which corresponds to the superradiance mode.

The aim of the work is to identify zones on the "injection velocity-reflection coefficient" plane where the conditions for generation in superradiance modes are met, where the generation of the resonator field is realized, and where the breakdown of generation occurs.

1. PARTICLE OWN FIELDS AND RESONATOR FIELD

Let us consider an oscillator whose charge (electron) moves along the OX axis, $\vec{r} = (x(t), 0, z_0)$, where $x(t) = i \cdot a \cdot \exp\{-i\omega t + i\psi\}$, while $\mathrm{Re}\, x = a \cdot \sin(\omega_0 t - \psi)$. The velocity and current can be written as $dx/dt = a \cdot \omega_0 \cdot \exp\{-i\omega t + i\psi\}$ and $J_x = -e dx/dt = -e \cdot a \cdot \omega_0 \cdot \exp\{-i\omega t + i\psi\}$. The equation describing the excitation of the field by the oscillator current

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 D_x}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial J_x}{\partial t} = \frac{4\pi}{c^2} \cdot e \cdot a \cdot \omega_0^2 \cdot i \cdot \exp\{-i\omega t + i\psi\} \cdot \delta(z - z_0). \quad (1)$$

The field of such an oscillator can be written as [7]

$$E_x = 2\pi e a \omega_0 M \cdot c^{-1} \exp\{-i\omega t + i\psi\} \cdot [\exp\{ik_0(z - z_0) \cdot U(z - z_0) + \exp\{-ik(z - z_0) \cdot U(z - z_0)\}, \quad (2)$$

where $U(z) = 1$ at $z \geq 0$ and $U(z) = 0$ at $z < 0$. For one particle in such a volume of unit cross-section and

[1]The reason for neglecting the radiation of the active zone particles themselves was possibly related to the idea that its intensity was low.

resonator length b, M is numerically equal to one. The equation of motion for an oscillating electron has the form

$$\frac{d}{dt}v_i(1-\frac{|v_i|^2}{c^2})^{1/2}+\omega_0{}^2x_i=-\frac{e}{m}E_x(z_i,t),\ \frac{dx_i}{dt}=v_i, \quad (3)$$

where $x_i(t)=i\cdot a_i\cdot\exp\{-i\omega t+i\psi\}=iA\cdot\exp\{-i\omega t\}$,

$v_i=\omega\cdot a_i\cdot\exp\{-i\omega t+i\psi\}=\omega A\cdot\exp\{-i\omega t\}$.

Using these notations, we write the equation of motion as

$$\frac{d\,\mathrm{A}_j}{d\tau}=\frac{i\alpha}{2}\cdot\left|\mathrm{A}_j\right|^2\mathrm{A}_j-\mathrm{E}(Z_j,\tau),\quad \frac{dZ_j}{d\tau}=V. \quad (4)$$

Here the following notations are accepted:

$\mathrm{A}=A/a_0$, $Z=kx/2\pi$, $V=kv/2\pi\gamma$, $\gamma=\gamma_0^2/\delta$,

$\gamma_0^2=\omega_{pe}^2/4=\dfrac{\pi ne^2}{m}$, $\gamma t=\tau$, $\delta=\dfrac{c}{b}$, $\Theta=\delta/\gamma_0$ – the ratio of the attenuation caused by the energy output from the resonator to the increment of oscillation generation in the system; $E_0=\dfrac{2m\cdot\gamma_0\cdot\omega\cdot a_0}{e}$, $\mathrm{E}=E/E_0$, $\alpha=\dfrac{3\omega}{4\gamma_0}(ka_0)^2$ – taking into account relativism (nonlinearity of the oscillators); E is the electric field strength acting on the oscillators; $b=\dfrac{2\pi}{k}=\lambda$ – the length of the resonator; A_j, Z_j – the complex (normalized to the oscillation amplitude) dimensionless amplitude of the oscillator and its coordinate; $j=1,2,...,N$, N – the number of oscillators in the waveguide; V – the constant speed with which the particles move along the waveguide, while the particles in the waveguide are constantly renewed. The oscillator oscillation frequency is close to the frequency of the waves propagating in the resonator, for which the resonator medium is transparent.

The radiation field of the all particles. The radiation of individual oscillators is summed up, which is equivalent to the so-called superradiance in the absence of a resonator [7, 8].

$$\mathrm{E}_{sr}(Z)=\frac{1}{\Theta\cdot N}\sum_{s=1}^{N}\mathrm{A}_s\cdot e^{i2\pi|Z-Z_s|}. \quad (5)$$

Resonator field. The traditional description of generation in a spatially limited resonator is based on the fact that all oscillators in its volume (active zone) interact only with the resonator field, and do not interact with each other. For a resonator, its resonator field, generally speaking, is determined by its geometry and is a standing wave, which consists of two waves traveling in different directions. For the amplitudes of these traveling waves of the resonator field, under the condition $|\partial\mathrm{E}_\pm/\partial\tau| \ll \Theta|\mathrm{E}_\pm|$, we obtain the expression in the units chosen above [7],

$$\mathrm{E}_{wg+}=\frac{1}{\Theta N}\sum_s\mathrm{A}_s\cdot e^{-i2\pi Z_s},$$
$$\mathrm{E}_{wg-}=\frac{1}{\Theta N}\sum_s\mathrm{A}_s\cdot e^{i2\pi Z_s}, \quad (6)$$

where the right-hand sides of these expressions determine the efficiency of the interaction of oscillator particles with these waves. The sum of these waves is the resonator field in the absence of interaction between the particles of the active zone, for example,

$$\mathrm{E}_{wg}(Z)=0.5\cdot(\mathrm{E}_{wg+}\cdot e^{2\pi iZ}+\mathrm{E}_{wg-}\cdot e^{-2\pi iZ}). \quad (7)$$

2. FORMATION OF THE RESONATOR FIELD

We will pay attention to the fact that the total field of oscillator particles always exists in the resonator, and the process of the appearance of the resonator field is associated with the formation of reflected waves at the boundaries. The total field of oscillators in the absence of reflection from the ends of the resonator (5) at a speed is shown in Fig. 1. In this case, there are no reflected waves and there is no resonator field.

For the number of oscillators $N=2500$, the amplitudes of which are equal to unity, and the phases are randomly distributed, the integral field $|\mathrm{E}|$ (5) (which can be considered as spontaneous in the classical case [9]) is approximately $1/\sqrt{N}$ times smaller than the maximum possible value of the field in the case when all the phases of the oscillators are close to the phase of the total field at the point where the oscillator is located. That is, for the fields of spontaneous and absolutely coherent induced radiation, the relation $(1/N)^{1/2}:1$ is satisfied. For the squares of the amplitudes $|\mathrm{E}|^2$, this relation takes the form $1/N$. It is not difficult to estimate the last relation, it is enough to take the square of the modulus of the right and left parts of the first equation (4) for and average over fast oscillations. In Fig. 1, the maximum field amplitude is 0.22, which is an order of magnitude greater than the average amplitude of the spontaneous field and five times less than the maximum possible value of the field in the case when all oscillators are synchronized in phase.

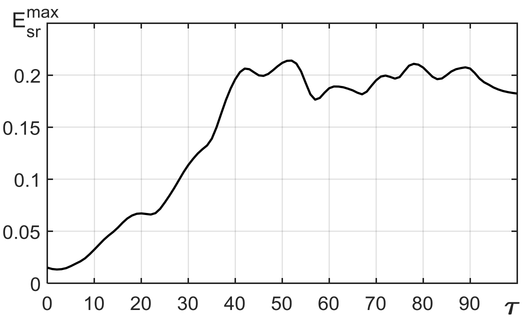

Fig. 1. Time dependence of the maximum amplitude modulus of the oscillator field for the resonator volume for the particle velocity $V = 0.15$, the number of particles N=2500 and in the absence of reflection from both ends of the resonator

When reflecting from the ends of the resonator, reflected waves arise, traveling in the opposite direction from the end Z = 0, a reflected wave traveling to the right with an amplitude at the boundary (Z = 0), from the end Z = 1, a wave traveling to the left with an amplitude at the other boundary (Z = 1). Generally speaking, these waves change when passing through the active zone under the influence of the fields of moving oscillators. Therefore, it is necessary to take into account the effect of oscillators in the active zone on these reflected waves. And then the resonator field will consist of two waves traveling in different directions, where a term is added to the amplitude of the reflected waves at the boundary, qualitatively taking into account the influence of oscillators in the resonator volume on these waves

$$\mathrm{E}_{w+}=\mathrm{E}_{ref+}+0.5\mathrm{E}_{wg+},\quad \mathrm{E}_{w-}=\mathrm{E}_{ref-}+0.5\mathrm{E}_{wg-}. \quad (8)$$

In essence, and perform the role of boundary and initial conditions for each step of calculations. Now the condition for the formation of waves reflected from the ends of the resonator must take into account these corrections. To determine the amplitudes of the traveling waves of the reflected field, we use the balance conditions of the incident/reflected field at the boundaries:

$$\mathrm{E}_{ref+}=-r_L\cdot\left(\mathrm{E}_{sr-}+0.5\mathrm{E}_{wg-}+\mathrm{E}_{ref-}\right),\quad Z=0,$$
$$\mathrm{E}_{ref-}=-r_R\cdot\left(\mathrm{E}_{sr+}+0.5\mathrm{E}_{wg+}+\mathrm{E}_{ref+}\right),\quad Z=1. \quad (9)$$

Here r_L, r_R are the reflection coefficients at the left and right edges of the waveguide.

From the solution of system (9) we obtain

$$\mathrm{E}_{ref+}=\frac{r_l\left(r_r(\mathrm{E}_{sr+}+0.5\mathrm{E}_{wg+})-(\mathrm{E}_{sr-}+0.5\mathrm{E}_{wg-})\right)}{(1-r_l\cdot r_r)},$$
$$\mathrm{E}_{ref-}=\frac{r_r\left(r_l(\mathrm{E}_{sr-}+0.5\mathrm{E}_{wg-})-(\mathrm{E}_{sr+}+0.5\mathrm{E}_{wg+})\right)}{(1-r_l\cdot r_r)}. \quad (10)$$

Thus, the total resonator field

$$\mathrm{E}_w(Z)=\mathrm{E}_{w+}\cdot e^{2\pi iZ}+\mathrm{E}_{w-}\cdot e^{-2\pi iZ},$$
$$\mathrm{E}_{w+}=\mathrm{E}_{ref+}+0.5\mathrm{E}_{wg+},\quad \mathrm{E}_{w-}=\mathrm{E}_{ref-}+0.5\mathrm{E}_{wg-} \quad (11)$$

and the total field acting on the particles can be represented as

$$\mathrm{E}(Z,\tau)=\mathrm{E}_{sr}(Z,\tau)+\mathrm{E}_w(Z,\tau). \quad (12)$$

If there is no correction of the reflected waves presented in (5) at all, that is,

$$\mathrm{E}_{w+}=\mathrm{E}_{ref+},\quad \mathrm{E}_{w-}=\mathrm{E}_{ref-}, \quad (13)$$

this means that the effect on the reflected waves of the oscillators in the volume of the active zone occurs only through their total field (2) already at the boundaries of the waveguide, that is, the waveguide field is determined only by the waves formed by the boundary conditions

$$\mathrm{E}_w(Z)=\mathrm{E}_{ref+}\cdot e^{2\pi iZ}+\mathrm{E}_{ref-}\cdot e^{-2\pi iZ}. \quad (14)$$

3. NUMERICAL SOLUTION OF THE PROBLEM

The calculations were performed under the following conditions. The calculations used constant parameters $N = 2500$, $\alpha = \theta = 1$, $r_L = 1$ (total reflection at the left edge). The velocity of the incoming particles and the reflection coefficient at the right edge r_R were changed; particles with a random phase were injected in the region of the left end. After a certain period of establishing generation, a certain stationary mode is formed in the waveguide, on which chaotic oscillations are superimposed due to incoming particles with a random phase. Therefore, after establishing this mode ($\tau > 50$), the values of the squares of the waveguide field and the particle field and their ratio K, averaged over the waveguide volume and time, are calculated:

$$\mathrm{E}_{sr}^{\ 2}=\langle(E_{sr}^{\ 2})_{av}\rangle_t,\quad \mathrm{E}_w^{\ 2}=\langle(E_w^{\ 2})_{av}\rangle_t,$$
$$K=\mathrm{E}_w^{\ 2}/\mathrm{E}_{sr}^{\ 2}. \quad (15)$$

The calculations show that for given reflection coefficients and an increase in the particle input velocity, both the total proper particle field and the waveguide field increase, but the waveguide field increases somewhat faster.

On the other hand, increasing the velocity leads to a decrease in the residence time of the oscillators in the waveguide and a reduction in the time of interaction of the oscillators with each other and with the field in the waveguide. Therefore, with an increase in the velocity, generation is disrupted, the field in the waveguide decreases, the particles do not have time to synchronize due to the short interaction time and, as a consequence, the small amplitude of the field. With an increase in the reflection coefficient, generation is disrupted at a higher injection velocity.

Thus, the following operating modes can be observed:

1 – dominance of the generation of the oscillators' own field ($\mathrm{E}_w^{\ 2}<\mathrm{E}_{sr}^{\ 2}$, $K=\mathrm{E}_w^{\ 2}/\mathrm{E}_{sr}^{\ 2}<1$), the superradiance mode;

2 – dominance of the generation of the resonator field ($\mathrm{E}_w^{\ 2}>\mathrm{E}_{sr}^{\ 2}$, $K=\mathrm{E}_w^{\ 2}/\mathrm{E}_{sr}^{\ 2}>1$));

3 – absence (disruption) of generation.

Fig. 2 shows the boundaries between the modes depending on the parameters changed in the calculations: injection velocity and reflection coefficient at the right edge of the waveguide.

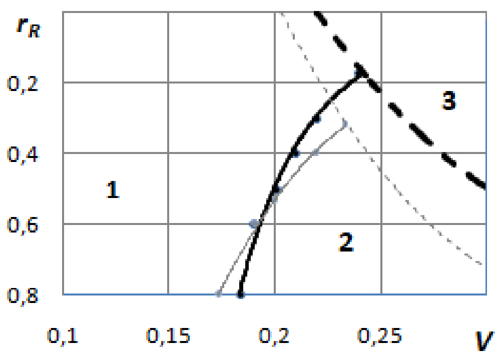

Fig. 2. Boundaries between generation modes in case of reflected wave correction according to equations (8). Thin lines indicate the boundaries of regions in the absence of such correction according to (13) ($\mathrm{E}_{w+}=\mathrm{E}_{ref+}$, $\mathrm{E}_{w-}=\mathrm{E}_{ref-}$). In region 1, the generation of the oscillators' own field (superradiance mode) dominates, in region 2 – the generation of the resonator field of the traditional type, caused by reflection processes from the ends, in region 3, a breakdown of generation is observed

It is important to note that with the correction of the reflected wave amplitude caused by taking into account the effect of the active zone oscillators in the volume of the resonator (8), and in the absence of such correction (13), a zone of predominant generation of the total particle radiation field (zone 1), i.e. superradiance, a zone of resonator field generation (zone 2), caused by noticeable reflection from the ends of the system and a zone of generation breakdown (zone 3) due to the violation of oscillator synchronization due to the dominance of oscillators with a random phase in the active zone arise.

For example, Fig. 2 shows that in a short waveguide at r_R< 0.2…0.3 the generation of the particle's own field (V < 0.23) dominates, i.e. the superradiance mode. At somewhat higher velocities at r_R< 0.2…0.3 the generation is disrupted (zone 3) without transition to the resonator field excitation mode. At r_R>0.2…0.3 the transition from the dominant generation of the particle's own fields (zone 1) to the resonator field generation mode (zone 2) is possible, which with a further increase in velocity leads to a disruption of the generation (in zone 3).

Let us consider the processes of field excitation in the mode of the oscillators' action in the active zone on these reflected waves (5). In Fig. 3, the parameters are shown as time dependences of the average squares of the total proper field of particles, and the resonator field in mode 1 – where the generation of the proper field of oscillator particles dominates, that is, in essence, the superradiance mode is realized.

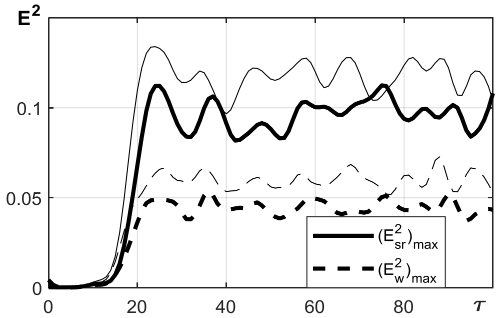

Fig. 3. Time dependence of the maximum squares of the particle field (solid line) and waveguide field (dotted line) in mode 1 ($V = 0.15$, $r_R = 0.6$). Bold lines are in the case of correction of reflected waves, according to equations (8). Thin lines are the particle field and the reflected field in the absence of such correction, according to (13)

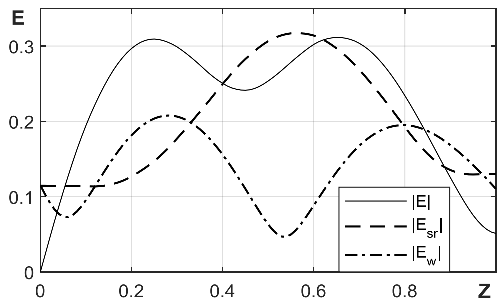

Fig. 4. Distributions of the amplitude moduli of the total field, the particle field and the waveguide field along the waveguide length in mode 1 at the moment τ = 100 for the parameters $V = 0.15$, $r_R = 0.6$ in the case of correction of reflected waves, according to eq. (8)

In Fig. 4 for the same parameters ($V = 0.15$, $r_R = 0.6$) in the case of correction of reflected waves, according to equations (5), the distributions along the length of the waveguide of the amplitude moduli of the total field, the total proper field of the particles and the resonator field at the moment τ = 100 are shown. The resonator field is less than the total field of the particles, the coefficient $K = 0.527$.

In Figs. 5 and 6 for the parameters $V = 0.23$, $r_R = 0.6$ similar dependencies are shown in mode 2, where the generation of the resonator field dominates in the case of correction of reflected waves, according to equations (5). As can be seen from the figure, the resonator field exceeds the proper field of the particles, the coefficient $K = 1.5$.

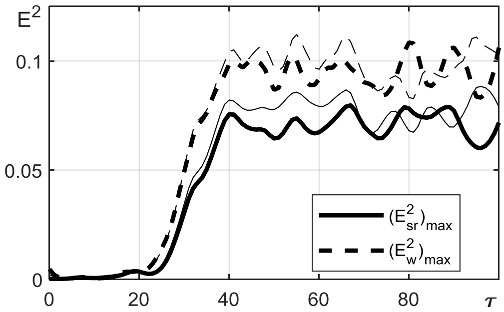

Fig. 5. Time dependence of the maximum squares of the particle field (solid line) and waveguide field (dotted line) over the waveguide volume in mode 2 ($V = 0.23$, $r_R = 0.6$). Bold lines are in the case of correction of reflected waves, according to equations (8). Thin lines are the particle field and the reflected field in the absence of such correction, according to (13)

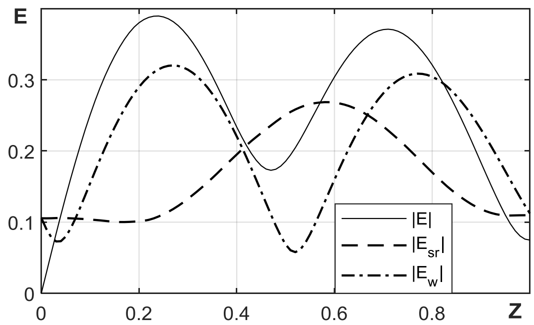

Fig. 6. Distributions along the waveguide length of the amplitude modules of the total field, the oscillator field and the waveguide field in mode 2 at the moment τ = 100 for the parameters $V = 0.23$, $r_R = 0.6$ in the case of correction of reflected waves, according to eq. (8)

CONCLUSIONS

This paper shows that the generation process in a short resonator can be disrupted by rapid injection of oscillator particles with random phases due to insufficient time for phase synchronization, which is aggravated by a weak increase in the field amplitude. If the injection rate of new particles is low, then the conditions for dominant excitation of the total eigenfield of particles are realized, which corresponds to superradiance modes. The amplitudes of the resonator field are small in this case. With increasing reflection from the ends and at a higher injection rate, the resonator field exceeds the total eigenfield of the oscillator particles in the active zone. That is, the resonator field generation mode is realized.

It is also important to note that the zones of dominance of superradiance modes and traditional resonator generation, as well as the zone of field excitation breakdown, are always formed under different conditions of energy exchange between reflected waves and oscillators in the resonator volume, described by relations (8) and (13). Note that even with a decrease in the direct effect of oscillators in the volume on reflected waves, which meets conditions (13), the zones of different types of generation and its breakdown shift almost weakly. This is due to the presence of a total field of oscillators, which is capable of acting as an intermediary between the oscillators in the active zone and the waves reflected from the ends of the resonator, forming the resonator field.

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МОДЕЛЮВАННЯ РЕЖИМІВ НАДВИПРОМІНЮВАННЯ ТА ФОРМУВАННЯ РЕЗОНАТОРНОГО ПОЛЯ

Є.В. Поклонський, В.М. Куклін

Показано, що сумарне поле рухомих активних елементів-осциляторів в об'ємі резонатора здатне за рахунок відбиття від торців формувати резонаторне поле. При високій швидкості інжекції частинок з випадковою фазою пригнічуються умови їхньої фазової синхронізації, і генерація зривається. У разі помітного відбиття полів та виконання умов синхронізації, випромінювачів за відносно великої швидкості їх інжекції резонаторне поле помітно перевищує сумарне поле випромінювачів, та формується традиційний режим генерації резонаторного поля. У разі відкритого резонатора з малими коефіцієнтами відбиття від торців амплітуди сумарного випромінювання осциляторів перевищують амплітуди резонаторного поля, та реалізується режим надвипромінювання. Представлено на площині «швидкість інжекції – коефіцієнт відбиття» зони, де спостерігається зрив генерації, виникає генерація резонаторного поля і генерація в умовах надвипромінювання.