https://doi.org/10.46813/2025-158-061

HEATING AND ACCELERATION OF ELECTRONS BY SEVERAL PLANE TRANSVERSE ELECTROMAGNETIC WAVES

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The dynamics of electrons in the field of several plane transverse electromagnetic waves is considered. It is shown that the fields of such waves can effectively (resonantly) transfer energy to particles in almost any interval of the momentum space. This possibility opens new possibilities for particle acceleration, as well as for plasma heating, especially for the effective heating of the ion component of the plasma. The latter possibility is determined by the fact that in the field of several waves, modes with dynamic chaos are easily realized, especially in the area where Langmuir ion-acoustic waves are easily excited. These waves quickly and effectively transfer their energy to ions.

PACS: 41.60.Cr, 52.20.-j; 05.45.-a

INTRODUCTION

When studying the processes of interaction of plasma particles with electromagnetic wave fields, the conditions of resonant interaction play a special role. Usually, this refers to resonances in which the energy exchange occurs proportionally to the first power of the wave strength parameter ($\varepsilon = \mathrm{eE/mc\omega}$). Note that this parameter is often called the nonlinearity parameter. These are primarily Cherenkov resonances and cyclotron resonances. Let us recall that for 10-centimeter radiation this parameter is equal to one at field strengths of the order of 10^5 V/cm. For laser radiation ($\lambda = 10^{-5}$ cm) this is already more 10^{10} V/cm. All media in such fields are quickly destroyed. Therefore, such fields are mainly used in pulse processes. A striking example of the use of such fields are the schemes of inertial thermonuclear fusion [1 - 4]. It can be said that the modern trend of research is aimed at increasing the wave intensities that are used for acceleration, as well as at studying fast processes. It should also be noted that nonlinear resonances are proportional to the square of the wave strength parameter. Such as those used in free-electron laser (FEL) schemes or in inverted free-electron laser (IFEL) acceleration schemes. The forces acting on the particles in these schemes are proportional to the square of the wave force parameter. In other schemes of acceleration or heating of plasma such resonances are practically not used.

In this paper, we draw attention to the fact that such resonances have useful features that allow them to be successfully used in new applications. It is shown below that the branches of these resonances can be located almost anywhere in the momentum space. This feature allows for the effective excitation of the required plasma waves, such as, for example, Langmuir ion-sound waves. The latter, as is known, effectively and quickly transfer their energy to ions.

Below we will see that such excitation can be realized in the dynamic chaos mode. In this case, the energy of external high-frequency waves (laser waves) will quickly and effectively heat the ion component of the plasma.

STATEMENT OF THE PROBLEM AND BASIC EQUATIONS

Consider a charged particle that moves in the fields of N+1 plane transverse electromagnetic waves, which in the general case has the following components

$$\vec{E} = \sum_{n=0}^{N} \mathrm{Re}(\vec{E}_n \exp(i\varphi_n)),$$
$$\vec{H} = \sum_{n=0}^{N} \mathrm{Re}\left(\frac{c}{\omega_n}\left[\vec{k}_n \vec{E}_n\right]\exp(i\varphi_n)\right), \quad (1)$$

where $\varphi_n = \omega_n t - \vec{k}_n \vec{r}$, $\vec{E}_n = E\vec{\alpha}_n$, $\vec{\alpha}_n = \left\{\alpha_x, i\alpha_y, \alpha_z\right\}_n$ – is polarization vector of the n – wave.

It is convenient to use the following dimensionless dependent and independent variables:

$$\vec{p} \to \vec{p}/mc,\ \vec{E}_n \to e\vec{E}_n / mc\omega_0 = \vec{\varepsilon}_n,$$
$$\tau = t\omega_0,\ \omega_n \to \omega_n / \omega_o,\ \vec{r} \to \vec{r}\omega_0 / c,$$
$$\vec{E}_n \to E_n\vec{\alpha} = E_n\left\{\alpha_x, i\alpha_y, \alpha_z\right\}.$$

The equations of motion in these variables will have the form:

$$\frac{d\vec{p}}{d\tau} = \mathrm{Re}\sum_{n=0}^{N}\left[\vec{\varepsilon}_n\left(1-\vec{l}\vec{v}\right)+\vec{l}_n\left(\vec{v}\vec{\varepsilon}_n\right)\right]\exp\left(i\varphi_n\right), \quad (2)$$

where $\vec{l}_n = \dfrac{\vec{k}_n c}{\omega_n}$; $\dot{\varphi}_n = \omega_n\left(1-\vec{l}\vec{v}\right)\vec{v}$.

It is convenient to add to the system of equations (2) an equation that describes the dynamics of the particle energy:

$$\frac{d\gamma}{d\tau} = \mathrm{Re}\sum_{n=0}^{N}\left[\left(\vec{\varepsilon}_n\vec{v}\right)\right]\exp\left(i\varphi_n\right). \quad (3)$$

For simplicity, but without losing the physical content of the processes under consideration, we will assume below that all waves are collinear, the waves are transverse ($\left(\vec{\varepsilon}_n\vec{l}_m\right)=0$) and linearly polarized. This means that the following relations are satisfied

$$\vec{l}_n = \left\{0,0,1\right\},\ \vec{\varepsilon}_n = \left\{\varepsilon_{xn}, 0.0\right\} = \left\{\varepsilon_n, 0.0\right\},$$
$$\varphi_n = \omega_n\left(\tau - \vec{l}_n\vec{r}\right),\ \vec{\alpha} = \left\{\alpha_x, 0, 0\right\}. \quad (4)$$

In this case, from the system of equations (2) we obtain the following equations for determining the components of the particle momentum

$$\begin{aligned}\frac{dp_z}{d\tau} &= \sum_{n=0}^{N}\left[\left(p_x\varepsilon_n\right)\right]\cos\left(\varphi_n\right),\\ \frac{dp_x}{d\tau} &= \sum_{n=0}^{N}\left[\varepsilon_n\left(1-\vec{l}_n\vec{v}\right)\right]\cos\left(\varphi_n\right).\end{aligned} \qquad (5)$$

This system has a rigorous analytical solution. To find it, we rewrite the last equation

$$\frac{dp_x}{d\left[\left(\tau-\vec{1}_n\vec{r}\right)\right]} = \sum_{n=0}^{N}\varepsilon_n\cos\left(\left(\tau-\vec{1}_n\vec{r}\right)\omega_n\right). \qquad (6)$$

This equation has the following solution

$$p_x = p_x(\kappa=0)+\sum_{n=0}^{N}\frac{\varepsilon_n}{\omega_n}\sin\left(\kappa_n\omega_n\right), \qquad (7)$$

where $\varphi_n=\omega_n\kappa_n$, $\kappa_n\equiv\left(\tau-\vec{1}_n\vec{r}\right)$.

Let's $p_x(\kappa=0)=0$. Now it is easy to obtain general equations for the energy and longitudinal momentum of particles

$$\frac{d\gamma^2}{d\tau}=\sum_{m=0}^{N}\sum_{n=0}^{N}\frac{\left(\varepsilon_n\varepsilon_m\right)}{\omega_m\omega_n}\left[\begin{bmatrix}\sin\left(\varphi_n+\varphi_m\right)\left(\omega_n+\omega_m\right)-\\-\sin\left(\varphi_n-\varphi_m\right)\left(\omega_n-\omega_m\right)\end{bmatrix}\right], \qquad (8)$$

$$\frac{dp_z}{d\tau}=\frac{1}{2}\sum_{m=0}^{N}\sum_{n=0}^{N}\frac{\varepsilon_m\varepsilon_n}{\omega_n\omega_m}\begin{Bmatrix}\sin\left(\varphi_n+\varphi_m\right)\left(\omega_n+\omega_m\right)-\\-\sin\left(\varphi_n-\varphi_m\right)\left(\omega_n-\omega_m\right)\end{Bmatrix}. \qquad (9)$$

Let's consider the case when there are only three waves. And two of them are propagating towards the third: $\vec{l}_0=1$, $\vec{l}_1=-1$, $\vec{l}_2=-1$, see Fig. 1.

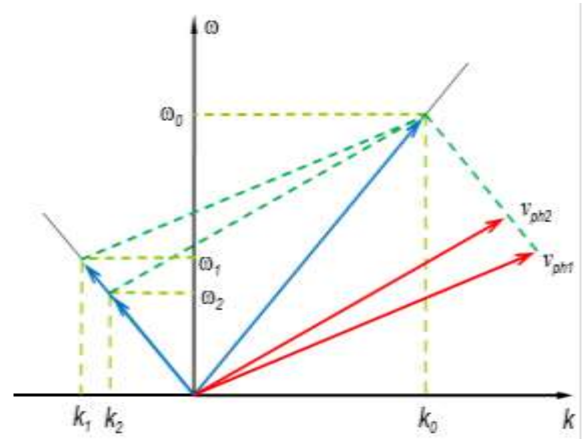

Fig. 1. Dispersion diagram of interacting waves

This figure shows three quick ($v_{ph}\sim c$), falling on transparent plasma, waves ($\omega_0,\omega_1,\omega_2$; $\omega_j>\omega_p$). Moreover, two of them (ω_1,ω_2) spread towards the third (ω_0). The interaction of each counter-pair generates two slow virtual waves. The expressions for the frequencies and phases of these two virtual waves take the form

$$\begin{aligned}\Delta\omega_{0i}&=1-\omega_i,\\ \left(\varphi_0-\varphi_i\right)&=\Delta\omega_{0i}\tau-\left(1+\omega_i\right)z,\ i=\{1,2\}.\end{aligned} \qquad (10)$$

If a particle moves in one virtual wave, then the system of equations that describes its dynamics can be represented as

$$\begin{aligned}\frac{dp_z}{d\tau}&=a_1\sin\psi,\\ \frac{d\gamma}{d\tau}&=\frac{a_1}{\gamma}\sin\psi,\\ \frac{d\psi}{d\tau}&=\left(v_{ph2}-v_z\right),\end{aligned} \qquad (11)$$

where $a_1\equiv-\dfrac{\left(\varepsilon_0\varepsilon_1\right)}{\omega_1\omega_0 2}\left(1-\omega_1\right)$,

$$\psi\equiv\left(\varphi_0-\varphi_1\right)=\frac{\Delta\omega_{01}}{\Delta k_{01}}\tau-z=v_{ph}\tau-z.$$

The dynamics of the system of equations (11) is equivalent to the dynamics of a mathematical pendulum

$$\frac{d^2\psi}{d\tau^2}+\Omega_1^2\sin\psi=0, \qquad (12)$$

where $\Omega_1^2=\left(1-\dfrac{v_{ph1}}{\gamma}\right)\dfrac{|a_1|}{\gamma}$.

Similarly, if a particle moves in the field of the second virtual wave, then its dynamics will also be determined by the equation of the mathematical pendulum (12), in which it is necessary to replace with

$$a_2\equiv-\frac{\left(\varepsilon_0\varepsilon_2\right)}{\omega_2\omega_0 2}\left(1-\omega_2\right),\ \omega_0=1,\ \omega_{1,2}<1.$$

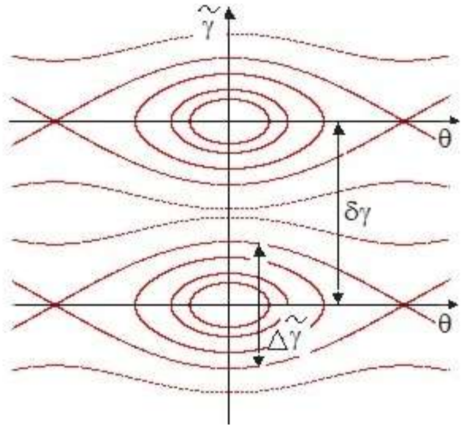

Fig. 2. Phase plane. $\Delta\tilde{\gamma}$ – resonance width;
$\delta\gamma$ – distance between resonances

If the separatrices of nonlinear pendulums intersect (Fig. 2), then the dynamics of the particles will become chaotic. This condition will be the condition

$$\left(v_{ph2}-v_{ph1}\right)<2\left(\Omega_1+\Omega_2\right), \qquad (13)$$

where $\Omega_2^2=\left(1-\dfrac{v_{ph2}}{\gamma}\right)\dfrac{|a_2|}{\gamma}$.

In equations (11) and (12) only the basic relatively slow terms are considered. In the general case, as follows from the systems of equations (8) and (9), equations (11) and (12) must be supplemented by additional, rapidly changing terms $F(\tau)$:

$$\frac{d^2\psi}{d\tau^2}+\Omega_1^2\sin\psi=F(\tau). \qquad (14)$$

This is especially important for the case where the interaction of only two high-frequency waves is considered. In previous works it was shown [3, 4] that if the parameters of the wave forces of the interacting waves are sufficiently large ($\varepsilon_{1,2} > 0.3$), then the dynamics of the particles becomes chaotic.

It is of interest to estimate the energy that particles receive in such regimes. To carry out such an estimate, we will assume that chaotic dynamics can be described by white noise. In this case, we find:

$$\left\langle (\Delta\gamma)^2 \right\rangle \approx \frac{\varepsilon^4 \cdot (\Delta\omega)^2}{\gamma_0^2}\tau . \tag{15}$$

Here we assumed $\varepsilon_0 = \varepsilon_1 = \varepsilon_2 = \varepsilon$ and $\Delta\omega_{01} = \Delta\omega_{02} = \Delta\omega$.

It is necessary to pay attention to two features of the formulas obtained above. The first is that all formulas are valid for any values of the wave force parameters. As far as the authors know, in all FEL schemes and IFEL schemes this value is a small parameter. The second feature is that the occurrence of modes with dynamic chaos in FEL and IFEL schemes is undesirable [5 - 7]. These modes must be suppressed. For the purposes of heating a given group of particles (for our purposes), these modes are necessary and useful.

The system (2) was studied numerically in the case where there are three waves. In this case, a combination of waves was used, which is shown in Fig. 1, i.e. one wave propagated in the positive direction, and the other two in the opposite direction.

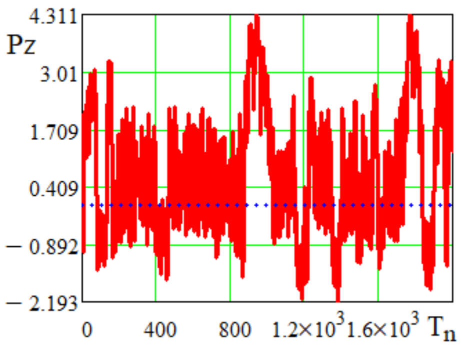

Fig. 3. Dependence of the magnitude of the longitudinal impulse on time for the following parameter values: $\varepsilon_1 = 1$, $\varepsilon_2 = 0.5$, $\varepsilon_3 = 0.5$, *and initial conditions:* $p_x = p_y = 0.01$, $p_z = 0.005$, $x = y = 0$, $z = 1$

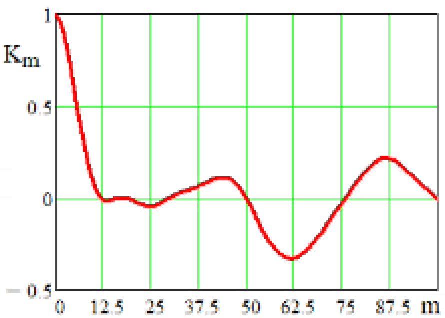

Fig. 4. Correlation function obtained with the same parameter values as in Fig. 3

Such a situation during the interaction of waves and particles, with a sufficiently large amplitude of the waves, leads to chaotic dynamics. An example of such dynamics is shown in Fig. 3, as well as the calculation of the correlation function in Fig. 4, which quickly decreases, indicating chaotic dynamics.

It is worth noting that chaotic dynamics in this system can also arise when two waves propagate in opposite directions interact. However, this occurs with a higher field intensity of the interacting waves. An example of dynamics in the field of two waves is shown in Fig. 5 and the correlation function of this dynamics is shown in Fig. 6.

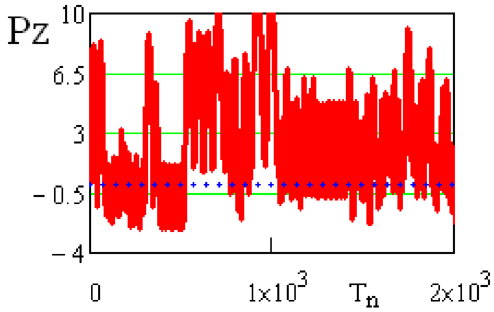

Fig. 5. Dependence of the magnitude of the longitudinal impulse on time for the following parameter values: $\varepsilon_1 = 2$, $\varepsilon_2 = 1$, *and initial conditions:* $p_x = p_y = 0.01$, $p_z = 0.005$, $x = y = 0$, $z = 1$

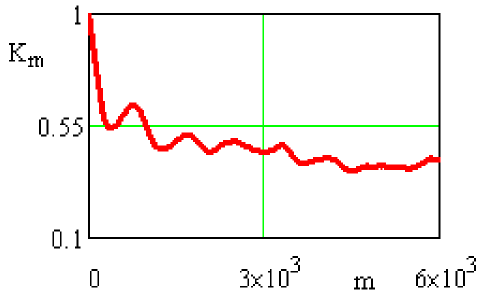

Fig. 6. Correlation function obtained with the same parameter values as in Fig. 5

CONCLUSIONS

Let us highlight the most important results of the study. The primary finding is the proof that, in the presence of several high-frequency waves, new branches of slow, virtual waves can emerge on the dispersion diagram (see Fig. 1). The phase velocities of these waves are practically unlimited. Consequently, by exciting these waves with external high-frequency waves, it becomes possible to selectively transfer the energy of the external waves to a specific region of the pulse space, particularly to the domain of ion-acoustic oscillations.

It is well established that electrons with such characteristics transfer their energy to ions most efficiently. This provides an additional mechanism for the rapid heating of the ionic component of the plasma.

It should be noted that the considered branches of oscillations were used earlier in the schemes of free-electron lasers (FEL) and in the schemes of acceleration on inverted free-electron lasers (IFEL). However, as was shown in the works [3 - 5], the efficiency of such schemes was significantly limited by the occurrence of stochastic instability. Moreover, such instability arose already at the parameter of the wave strength, which is quite moderate for laser radiation ($\varepsilon > 0.3$). For the purposes of heating (for our purposes), this disadvantage turns out to be an advantage. It is also significant that all the obtained formulas have no restrictions on the magnitude of the wave strength. This allows the plasma particles to be heated quickly.

Let's estimate the energy obtained and the time required for this. Let's consider solid fuel target with density $n \sim 10^{22}\,\text{cm}^{-3}$. The frequency of laser radiation should be greater than $10^{15}\,\text{s}^{-1}$. Such lasers exist $\omega \sim 5 \cdot 10^{15}\,\text{s}^{-1}$.

Using formula (15) we find that the plasma is heated to temperatures of the order of 10 keV in $10^{-14} \ldots 10^{-13}$ s. The cross section of the Coulomb interaction of particles can be estimated by the well-known formula $\sigma = \left(3 \cdot 10^{-18} / \text{W}(\text{keV})\right) \text{cm}^2$ [8]. The frequency of particle collisions in this case will be equal to $f = n\sigma v \approx 10^{13}$ s^{-1}. Thus, the electrons will transfer their energy to the ions in times shorter than 10^{-10} s. During this time, the target will not have time to fly apart (the speed of the flyaway is of the order of the speed of ion sound). Known instabilities also do not have time to be excited. These are, of course, rough and very optimistic estimates. One thing is clear to the authors: the considered scheme of energy transfer to plasma particles is quite promising for further study.

ACKNOWLEDGEMENTS

The authors thank G.V. Sotnikov for helpful discussions of the article and valuable comments.

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Article received 09.06.2025

НАГРІВАННЯ ТА ПРИСКОРЕННЯ ЕЛЕКТРОНІВ ДЕКІЛЬКОМА ПЛОСКИМИ ПОПЕРЕЧНИМИ ЕЛЕКТРОМАГНІТНИМИ ХВИЛЯМИ

В.О. Буц, В.В. Кузьмін, Д.М. Ваврів

Розглянуто динаміку електронів у полі кількох плоских поперечних електромагнітних хвиль. Показано, що полями таких хвиль є можливість ефективно (резонансно) передавати енергію частинкам практично у будь-якому інтервалі простору імпульсів. Така можливість відкриває нові можливості для прискорення частинок, а також нагрівання плазми, особливо для ефективного нагрівання іонної компоненти плазми. Остання можливість визначається тим, що в полі кількох хвиль легко реалізуються режими динамічного хаосу, зокрема, в області імпульсів, де легко збуджуються ленгмюрівські іонні звукові хвилі. Останні швидко та ефективно віддають свою енергію іонам.