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POLARIZED VIRTUAL PHOTON STATES IN THE THEORY OF FIRST-ORDER QED PROCESSES IN A LINEARLY POLARIZED WAVE FIELD

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The theory of first-order processes in the fine structure constant involving a polarized virtual (intermediate) photon state in the field of a linearly polarized electromagnetic wave has been developed. The obtained results constitute a part of the theory of second-order processes with respect to the fine-structure constant, whose probabilities can be expressed in terms of the probabilities of first-order processes.

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INTRODUCTION

The rapid advancement of laser technology has made it possible to conduct experiments aimed at an investigation of quantum electrodynamical (QED) processes in fields whose intensities are comparable to the critical (Schwinger) field [1, 2]. Despite the success of experimental studies, the theory of second-order processes remains undeveloped. This is primarily due to the fact that, within the framework of the standard approach [3] in calculating probabilities, the resulting expressions are excessively complex and cumbersome for analytical studies. Consequently, to compare theoretical predictions with experimental data, various approximations are employed, such as the equivalent photon method [4, 5], the resonance approximation (see, e.g., [6, 7]), and the light-front approximation [8], among others.

The authors propose a theory of second-order processes with intermediate photon states, which makes it possible to express their probabilities in terms of the probabilities of first-order processes that describe polarization effects associated with the intermediate photon state. In this framework, a separate problem arises – the extension of the theory of first-order processes to virtual photon states.

There are four first-order processes in the field of an electromagnetic wave that can occur within the wave field:

a) Emission of a photon by an electron during scattering in the field of an electromagnetic wave (Fig. 1,a).

b) Annihilation of an electron–positron pair in the field of a monochromatic electromagnetic wave (Fig. 1,b).

c) Creation of an electron–positron pair by a photon in the field of a monochromatic electromagnetic wave (Fig. 1,c).

d) Absorption of a photon by an electron in the field of a monochromatic electromagnetic wave (Fig. 1,d).

The processes will be considered in the field of a linearly polarized monochromatic electromagnetic wave:

$$A = a e_x \cos\varphi, \qquad (1)$$

where $k = (\omega, \mathbf{k})$ is the wave vector; $a = F/\omega$; where ω and F are the amplitude and frequency of the wave field, respectively; $\varphi = (kx)$ is the phase of the wave.

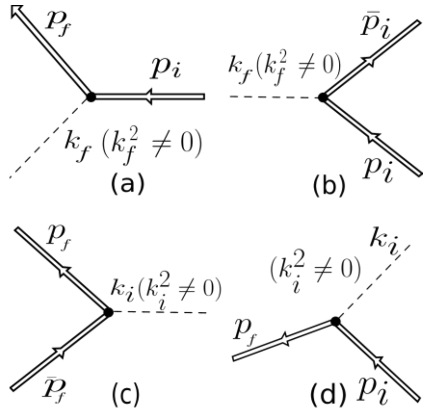

Fig. 1. Feynman diagrams of first-order processes in the wave field

Calculations will be performed for the process of photon emission by an electron. Results for other processes can be obtained by simple substitutions due to the cross-invariance of these processes.

Fig. 2 presents the diagrams of second-order processes with respect to the fine-structure constant involving an intermediate photon state, which can be described by a single diagram.

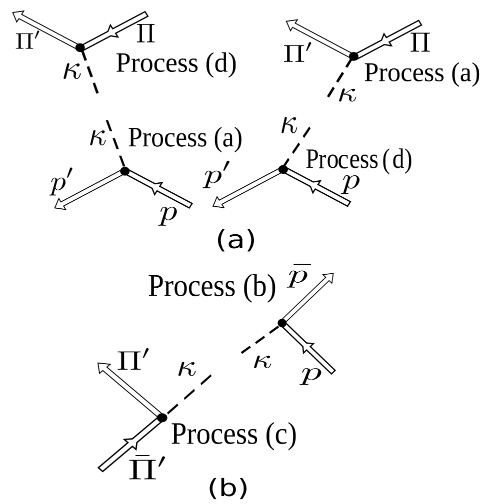

Fig. 2. Feynman diagrams of second-order processes with an intermediate photon state, represented by a single diagram: a – electron-muon scattering; b – conversion of an electron-positron pair into a muon pair

Although these diagrams can be drawn from 1st order diagrams (see Fig. 1), which also works for amplitudes, it cannot be done for process probabilities since the calculation of probability using standard way leads to the procedure of obtaining a spur. As a result components related to the 1st order subprocesses are mixed and the possibility of writing the probability of the 2nd order process in terms of the 1st order probabilities disappears.

However, it is possible to express the probabilities of 2nd-order processes in terms of 1st-order probabilities, but their theory must be extended to polarized intermediate states located outside the mass surface. For these reasons, obtaining such probabilities is a separate problem, which is the subject of this study.

AMPLITUDE

The amplitude of the process of photon emission by an electron in the field of an electromagnetic wave (see Fig. 1,a) has the form:

$$S_{fi} = -ie\int \overline{\Psi}_{p_f}\hat{e}_f^*\Psi_{p_i}\frac{\sqrt{4\pi}}{\sqrt{2\omega_f V}}e^{-i(k_f x)}d^4x, \quad (2)$$

where $p_{i,f} = (E_{i,f}, \mathbf{p}_{i,f})$ are the four-momenta of initial (i) and final electrons (f); $k_f = (\omega_f, \boldsymbol{k}_f)$ is the 4-momentum of emitted photon (we assume the photon to be virtual: $k_f^2 \neq 0$); V is the normalization volume; x is the radius 4-vector; e_f is the photon polarization 4-vector satisfying the conditions: $(k_f e_f) = 0$, $(e_f e_f^*) = -1$; hats above four-vectors denotes the scalar product of corresponding four-vector with the Dirac matrices: $\hat{a} = (\gamma_\mu a^\mu)$.

Note that unlike real, virtual photons are not transverse and therefore possess three polarization states [3].

In expression (2) Ψ_p is the wave function of an electron in the field of an electromagnetic wave [3].

The amplitude of the process of photon emission by an electron in the wave field, after integration over the four-coordinate x of the vertex, can be expressed as a sum of partial components:

$$S_{fi} = \sum_{l=-\infty}^{\infty} S_{fi}^{(l)}, \quad (3)$$

$$S_{fi}^{(l)} = C(e_f^* J)\delta^{(4)}(\tilde{p}_i + lk - \tilde{p}_f - k_f), \quad (4)$$

$$J_\mu = \bar{u}_{p_f} M_{l,\mu}(p_f, p_i) u_{p_i}, \quad (5)$$

where C is the normalization constant; u_{p_i} is the bispinor of a free electron and it is denoted

$$M_{l,\mu}(p_f, p_i) = L_l\gamma_\mu + \frac{m}{2(kp_i)}\widehat{D}_l k_\mu + \\ +\left(\frac{m^2}{8(kp_f)(kp_i)}B_l k_\mu - \frac{m}{2(kp_i)}D_{l,\mu}\right)\hat{k} + \\ +\frac{m}{4}\beta_- \widehat{D}_l \hat{k}\gamma_\mu,\ \beta_\pm = \frac{1}{(kp_f)} \pm \frac{1}{(kp_i)}, \quad (6)$$

$$L_l = \sum_{s=-\infty}^{\infty} J_{l-2s}(y) J_s(z), \quad (7)$$

$$D_l = e_x D_+,\ D_+ = \eta(L_{l+1} + L_{l-1}), \quad (8)$$

where $\eta = |e|a/m$ is the classical relativistic invariant parameter characterizing the intensity of the wave [9].

The arguments of the Bessel functions in expression (7) take the form:

$$y = 2l\eta\frac{m}{\tilde{m}}\sqrt{\rho(1-\rho-\beta_{\rm res})}\cos\psi, \quad (9)$$

$$z = l\eta^2\frac{m^2}{\tilde{m}^2}\frac{\rho}{4},\ \ \rho = \frac{\tilde{m}^2\beta_-}{2l}, \quad (10)$$

$$\psi = \angle(\vec{g}_{||}, \vec{e}_x),\ g = \frac{p_f}{(kp_f)} - \frac{p_i}{(kp_i)}, \quad (11)$$

$$\beta_{res} = \frac{k_f^2}{4\tilde{m}^2 v_l'},\ v_l' = \frac{l(kk_f)}{2\tilde{m}^2}. \quad (12)$$

where $\tilde{m} = m\sqrt{1+\eta^2/2}$ is the effective mass of an electron (positron) in the field (1).

From the conservation law of the 4-momentum (the argument of the δ-function in expression (4)) for k_f^2 we obtain:

$$k_f^2 = 2\tilde{m}^2\left(1 + \frac{u_l}{2} - (P\tilde{p}_f)\right), \quad (13)$$

where $P = (P_0, \vec{P}) = \tilde{p}_i + lk$.

The maximum value of k_f^2 is taken for $\tilde{E}_f = P_0/\sqrt{1+u_l}$, $\angle(\vec{P}, \overrightarrow{\tilde{p}_f}) = 0$ and is equal to:

$$k_f^2 = 2\tilde{m}^2\left(1 + u_l/2 - \sqrt{1+u_l}\right). \quad (14)$$

Note that based on the conservation laws of 4-momentum, there are no restrictions on the minimum value of k_f^2. However, considering that the theory is meaningful only within the paradigm of second-order processes, for which all particles except the intermediate ones must be on the mass shell, this leads to a lower bound for k_f^2.

To simplify the calculations we expand the four-by-four matrix $(e_f^* M_l)$ in a basis set of four-by-four matrices [3]. As resulting we obtain:

$$(e_f^* M_l) = \hat{b} - i\beta\gamma^5\hat{k}, \quad (15)$$

where

$$b = L_l e_f^* + \frac{m}{4}\beta_+ D_+ (e_x e_f^*), \quad (16)$$

$$\beta = \frac{m}{4}\beta_- D_+ (e_y e_f^*). \quad (17)$$

PROBABILITY

The differential probability of a photon emission by an electron in a wave field per unit of time can be expressed in terms of partial components:

$$d\mathrm{w}_{fi} = \sum_{l=1}^{\infty} d\mathrm{w}_{fi}^{(l)}, \quad (18)$$

where the differential partial probability $d\mathrm{w}_{fi}^{(l)}$ is defined by the expression:

$$d\mathrm{w}_{fi}^{(l)} = \left|S_{fi}^{(l)}\right|^2 \frac{V^2 d^3k_f d^3p_f}{T(2\pi)^6} = \mathrm{w}_0 W d\Gamma, \quad (19)$$

where T is an infinite time interval; w_0, $d\Gamma$ are given by the expressions:

$$\mathrm{w}_0 = e^2/(8\pi\tilde{E}_i),\ \ W = \left|(e_f^* J)\right|^2, \quad (20)$$

$$d\Gamma = \frac{d^3k_f d^3p_f}{\omega_f \tilde{E}_f}\delta^{(4)}(\tilde{p}_i + lk - \tilde{p}_f - k_f). \quad (21)$$

To describe the polarization properties of the intermediate (virtual) state, we consider an orthonormal basis in 4-dimensional space:

$$e_0 = n_f,\ e_1,\ e_2, e_3 = \lambda\frac{k}{(kn_f)} + n_f, \quad (22)$$

$$n_f = k/\sqrt{|k_f^2|},\ \lambda = e_0^2 = \pm 1, \quad (23)$$

$$e_1 = e_x - \frac{(k_f e_x)}{(kk_f)}k,\ e_2 = e_y - \frac{(k_f e_y)}{(kk_f)}k. \quad (24)$$

where e_1, e_2 coincide with the 4-vectors of polarization of a real photon [9]; e_3 corresponds to the additional polarization of the virtual photon.

In the expansion of the 4-vector J (5) over the basis (22), there are three components:

$$J = -(Je_1)e_1 - (Je_2)e_2 - \lambda(Je_3)e_3. \quad (25)$$

The contribution of the vector e_0 is absent due to the orthogonality of the photon and electron currents:

$$\left(k_f J\right) \equiv \left(n_f J\right) = 0. \tag{26}$$

Taking into account the expansion (25) for W (20) we obtain:

$$W = \frac{1}{3}\mathrm{w}_{Unpolar} + \sum_{i=1}^{2}\sum_{j=i+1}^{3}\frac{\xi_1^{(ij)}}{3}\left(\mathrm{w}(e_i)-\mathrm{w}(e_j)\right) + \frac{\xi_2^{(ij)}}{2}\left(\mathrm{w}\left(e_{\pi/4}^{(ij)}\right)-\mathrm{w}\left(e_{-\pi/4}^{(ij)}\right)\right) + \frac{\xi_3^{(ij)}}{2}\left(\mathrm{w}\left(e_+^{(ij)}\right)-\mathrm{w}\left(e_-^{(ij)}\right)\right), \tag{27}$$

$$\mathrm{w}_{Unpolar} = |J|^2 = \frac{1}{4}\mathrm{Sp}\left\{\left(\hat{p}_f+m\right)M_{l,\mu}\left(\hat{p}_f+m\right)\overline{M_l^{\mu}}\right\}, \tag{28}$$

$$\mathrm{w}(e) = |(eJ)|^2 = \frac{1}{4}\mathrm{Sp}\left\{\left(\hat{p}_f+m\right)(eM_l)\left(\hat{p}_i+m\right)\overline{(eM_l)}\right\}. \tag{29}$$

In the expression (27) $\xi_1^{(ij)}$, $\xi_2^{(ij)}$, $\xi_3^{(ij)}$ – generalization of the Stokes parameters to the case of 3 polarizations:

$$\xi_1^{(ij)} = \left|\left(e_f^* e_i\right)\right|^2 - \left|\left(e_f^* e_j\right)\right|^2, \tag{30}$$

$$\xi_2^{(ij)} = 2\Re\left(\left(e_f^* e_i\right)\left(e_f e_j\right)\right), \tag{31}$$

$$\xi_3^{(ij)} = 2\Im\left(\left(e_f^* e_i\right)\left(e_f e_j\right)\right). \tag{32}$$

Vectors $e_{\pm\pi/4}^{(ij)}$ correspond to polarization at an angle $\pm 45°$, $e_{\pm}^{(ij)}$ correspond to circular polarizations in the plane $e_i e_j$ (there are three such planes: e_1e_2, e_1e_3, e_2e_3):

$$e_{\pm\pi/4}^{(ij)} = \frac{e_i \pm e_j}{\sqrt{2}},\ e_{\pm}^{(ij)} = \frac{e_i \pm i e_j}{\sqrt{2}}. \tag{33}$$

For an unpolarized photon (22), the calculations yield:

$$\frac{\mathrm{w}_{Unpolar}}{2m^2} = -\left(1 + 2v_l'\frac{\tilde{m}^2}{m^2}\beta_{res}\right)L_l^2 + \eta^2\left(1+2v_l'\rho\right)\left(D_+^2 - D_2 L_l\right). \tag{34}$$

A general formula for calculation (29) can be written in the case of polarization vectors lying in the plane of polarization e_1e_2:

$$\mathrm{w}(e) = 2\left|(p_i e^*)L_l + m(e^* e_1)\eta D_+\right|^2 - 2l\left(kk_f\right)\beta_{\mathrm{res}}L_l^2 + \frac{m^2\eta^2}{2}\frac{\left(kk_f\right)^2}{(kp_i)(kp_f)}\left(D_+^2 - D_2L_l\right), \tag{35}$$

$$D_2 = 2L_l + L_{l+2} + L_{l-2}. \tag{36}$$

For effects related to linear polarization in the plane of e_1e_2 we obtain:

$$\frac{W(e_1)-W(e_2)}{2m^2} = \eta^2\left(D_+^2 - D_2L_l\right) - \left(1 + 2\tau^2 + \left(\frac{\tilde{m}}{m}\right)^2\left(\frac{1}{\rho} + v_l'\right)\beta_{\mathrm{res}}\right)L_l^2. \tag{37}$$

$$\tau^2 = \frac{-g^2}{m^2\beta_-^2}\sin^2\psi = \left(\frac{\tilde{m}}{m}\right)^2\frac{1-\rho-\beta_{\mathrm{res}}}{\rho}\sin^2\psi, \tag{38}$$

For effects related to polarization at an angle $\pm 45°$ in the plane e_1e_2 we obtain:

$$\frac{W\left(e_{\pi/4}^{(ij)}\right) - W\left(e_{-\pi/4}^{(ij)}\right)}{2m^2} = 2\frac{\tilde{m}}{m}\sqrt{\frac{1-\rho-\beta_{\mathrm{res}}}{\rho}}\sin\psi \times \left(\eta D_+ + \frac{\tilde{m}}{m}L_l\sqrt{\frac{1-\rho-\beta_{\mathrm{res}}}{\rho}}\cos\psi\right)L_l. \tag{39}$$

This expression is an odd function of the azimuthal angle ψ and vanishes upon integration over it.

Effects associated with circular polarization in the e_1e_2, plane do not manifest:

$$W\left(e_+^{(12)}\right) - W\left(e_-^{(12)}\right) = 0. \tag{40}$$

For the planes e_1e_3, e_2e_3 calculations are performed directly. Thus, for polarization effects in the plane e_1e_3 we obtain:

$$\frac{W(e_1)-W(e_3)}{2m^2} = \eta^2\left(1+v'_l\rho\right)\left(D_+^2 - D_2L_l\right) - \left(1 + \tau^2 - \left(\frac{\tilde{m}}{m}\right)^2\left(\frac{2}{\rho} + v'_l\right)\beta_{res}\right)L_l^2, \tag{41}$$

$$\frac{W\left(e_{\pi/4}^{(13)}\right)-W\left(e_{-\pi/4}^{(13)}\right)}{2m^2} = \lambda\left(\frac{\tilde{m}}{m}\right)^2\sqrt{\frac{|\beta_{res}|}{v'_l}}\frac{u_l+u'_l}{2} \times \left(L_l^2\sqrt{\frac{1-\rho-\beta_{res}}{\rho}}\cos\psi + \frac{m}{\tilde{m}}\eta L_l D_+\right), \tag{42}$$

$$W\left(e_+^{(13)}\right) - W\left(e_-^{(13)}\right) = 0, \tag{43}$$

$$u_l = \frac{2l(kp_i)}{\tilde{m}^2}, u'_l = \frac{2l\left(kp_f\right)}{\tilde{m}^2}. \tag{44}$$

For polarization effects in the plane e_2e_3 we obtain:

$$\frac{W(e_2)-W(e_3)}{2m^2} = \left(\tau^2 - \left(\frac{\tilde{m}}{m}\right)^2\frac{\beta_{\mathrm{res}}}{\rho}\right)L_l^2 + \eta^2 v'_l\rho\left(D_+^2 - D_2L_l\right), \tag{45}$$

$$\frac{W\left(e_{\pi/4}^{(ij)}\right)-W\left(e_{-\pi/4}^{(ij)}\right)}{2m^2} = \frac{\lambda}{2}\left(\frac{\tilde{m}}{m}\right)^2\sqrt{\frac{|\beta_{\mathrm{res}}|}{v_l'}\frac{1-\rho-\beta_{\mathrm{res}}}{\rho}} \times \left(u_l + u_l'\right)L_l^2\sin\psi. \tag{46}$$

This expression is an odd function of the azimuthal angle ψ and vanishes upon integration; however, since these results will be used for second-order processes, in which they enter quadratically, they need to be retained

$$W\left(e_+^{(13)}\right) - W\left(e_-^{(13)}\right) = 0. \tag{47}$$

Expressions (34)–(47) are obtained for the process of photon emission by an electron in an electromagnetic wave with such a level of detail that they can be rewritten for other first-order processes to obtain the corresponding expressions for these processes through simple substitutions.

a) Photon emission by an electron during scattering in an electromagnetic wave field.

Using the conservation laws for ~~the~~ parameters that determine the probability (27), we obtain:

$$\rho = \frac{u'}{u_l},\ u' = \frac{(kk_f)}{(kp_f)}, \tag{48}$$

$$v_l' = \frac{u_l}{4}\frac{u'}{1+u'},\ u_l' = \frac{u_l}{1+u'}. \tag{49}$$

b) Annihilation of an electron-positron pair in a monochromatic electromagnetic wave field.

The following substitutions need to be made: $p_f \to -\bar{p}_i$. The differential probability has the form:

$$d\mathrm{w}_{fi}^{(l)} = -\mathrm{w}_0 W \times \delta^{(4)}\left(\tilde{p}_i + \tilde{\bar{p}}_i - |l|k - k_f\right)d^3k_f, \tag{50}$$

where $l \le 1$, the parameters that determine W (27) are given by:

$$\rho = \frac{v}{v_l},\ v = \frac{\left((kp_i)+(k\bar{p}_i)\right)^2}{4(kp_i)(k\bar{p}_i)}, \tag{51}$$

$$v_l = \frac{u_l+\bar{u}_l}{4},\ \ \bar{u}_l = \frac{2l(k\bar{p}_i)}{\tilde{m}^2}. \tag{52}$$

c) Formation of an electron-positron pair by a photon in a monochromatic electromagnetic wave field.

The following substitutions are required:

$$p_i \to -\bar{p}_f,\ k_f \ \to -k_i.$$

The differential probability takes the form:

$$dw_{fi}^{(l)} = -w_0 W \times$$
$$\times \delta^{(4)}\left(k_i + lk - \tilde{p}_f - \tilde{\bar{p}}_f\right) d^3p_f d^3\bar{p}_f. \quad (53)$$

where $l \geq 1$, he parameters that determine W (27) are given by:

$$\rho = \frac{\upsilon'}{\upsilon_l},\ \ \upsilon' = \frac{(kk_i)^2}{4(kp_f)(k\bar{p}_f)}, \quad (54)$$

$$\upsilon_l' \to \upsilon_l = \frac{l(kk_i)}{2\tilde{m}^2},\ \ 1 \leq \upsilon' \leq \upsilon_l, \quad (55)$$

$$u_l \to -\bar{u}_l',\ \bar{u}_l' = \frac{2l(k\bar{p}_f)}{\tilde{m}^2} =$$
$$= \frac{\upsilon_l}{2}\left(1 \pm \sqrt{1 - \upsilon'^{-1}}\right), \quad (56)$$

$$\bar{u}_l' = \frac{2l(kp_f)}{\tilde{m}^2} = \frac{\upsilon_l}{2}\left(1 \mp \sqrt{1 - \upsilon'^{-1}}\right). \quad (57)$$

d) Absorption of a photon by an electron in a monochromatic electromagnetic wave field.

The following substitutions are required: $k_f \to -k_i$.

The differential probability takes the form:

$$dw_{fi}^{(l)} = -w_0 W \times$$
$$\times \delta^{(4)}\left(\tilde{p}_i + k_i - |l|k - \tilde{p}_f\right) d^3p_f. \quad (58)$$

CONCLUSIONS

To construct an analytical theory of second-order processes in the fine-structure constant, the probabilities of first-order processes in a linearly polarized electromagnetic wave field were obtained for polarized photon states that fall off the mass shell. In contrast to the case of a real photon, the probability contains terms related to the additional polarization of the virtual (intermediate) photon along the vector e_3 (22). In addition to the extra term for the unpolarized (summed over polarizations) state, there are terms corresponding to linear polarization at an angle $\pm 45°$, circular polarizations in the planes e_1e_3, e_2e_3. For the case of linear polarization of the electromagnetic wave, effects associated with circular polarization do not appear. The probabilities describing effects related to polarization at an angle $\pm 45°$ are an odd function of the azimuthal angle ψ (11).

At the boundary $\beta_{res} = 0$, when the photon becomes real, the expressions for the probabilities (17) and (24) reduce to the well-known formula for the probability of polarized photon emission by an electron in a linearly polarized electromagnetic wave field [9].

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ПОЛЯРИЗОВАНІ ВІРТУАЛЬНІ ФОТОННІ СТАНИ В ТЕОРІЇ КЕД-ПРОЦЕСІВ ПЕРШОГО ПОРЯДКУ В ЛІНІЙНО-ПОЛЯРИЗОВАНОМУ ХВИЛЬОВОМУ ПОЛІ

О.І. Ворошило, О.В. Макаренко, О.А. Лебедь

Розвинуто теорію процесів 1-го порядку за сталою тонкої структури з поляризованим фотонним віртуальним (проміжним) станом у полі лінійно-поляризованої електромагнітної хвилі. Отримані результати є складовою теорії процесів 2-го порядку за сталою тонкої структури, ймовірністі яких можуть бути записані через ймовірності процесів 1-го порядку.