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METHOD OF VERIFICATION OF THE RELIABILITY AND ACCURACY OF THE MATHEMATICAL MODEL OF MULTIFACTOR ASSESSMENT

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One of the most important stages of building any mathematical model is checking the degree of its adequacy to the simulated real process or phenomenon. Therefore, model verification is an obligatory element of its synthesis and identification. The quality of the model is determined by the accuracy of the selection and approximation of only the useful signal, which means the possibility of correct description of any input data sequence. The article considers one of the possible approaches to creating a mathematical model of multifactor assessment under conditions of uncertainty.

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The classical theory of structural-parametric identification of models assumes as a necessary condition for solving the problem the presence of two sets: the number of measured values of input influences X and the reactions of the studied system Y, which are determined during a series of active or passive experiments.

The quality of the synthesized model, regardless of whether it belongs to the classes of direct or indirect analogy, is determined by the accuracy of approximation of the original experimental data sequence Y. At the same time, it is known from the theory of function interpolation that by increasing the degree and number of terms of the interpolation polynomial, any sequence of data can be described absolutely accurately. However, this does not mean that a high-quality model will be obtained. This is due to the fact that experimental data are usually a complex composition of a useful signal and harmful "noise", which is the result of the influence of random uncontrolled interference on the system. Therefore, the quality of the model is determined not by the accuracy of approximation of a specific sequence of output data Y, but by the accuracy of selection and approximation of only the useful signal, which means the possibility of correct description of any input data sequence. Thus, the model must have the properties of a filter [1].

A mandatory condition for the synthesis of such a model is the use of the principle of external complementation. It is implemented as follows: the entire set of initial experimental data is divided into two subsets - training and testing. The first of them is used for model synthesis, and the second – as independent data to check its quality. This approach allows us to synthesize a model of optimal complexity, i.e. find an approximating polynomial of minimal complexity adequate to real data.

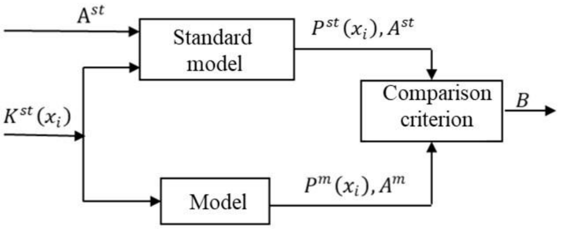
As is known, any sequence L of experimental data can be accurately approximated by a polynomial with $L+1$ terms by solving a system of normal algebra equations. However, such an approximation does not mean that an adequate high-accuracy model with good predictive properties has been synthesized. This is due to the fact that experimental data contain measurement and other uncontrolled random errors. Therefore, a polynomial of high complexity approximates not only the useful signal, but also the random errors of experimental data. It turns out that with increasing complexity of the model structure, the accuracy of approximation of the test sequence of experimental data first improves, then error reaches a certain minimum, and then becomes constant or worsens due to the inclusion of "harmful" random components. The model that gives a minimum error and approximation of the test sequence is called the model of "optimal complexity" [2].

The fundamental feature of the problems of identifying models of intellectual activity, such as decision-making, multifactorial assessment, pattern recognition, classification, is that the assessments (feelings) on the basis of which a person makes a final decision cannot often be directly measured quantitatively. Only indirect information can be obtained. The most common approach to solving this problem is the method of expert assessment, based on encouraging the expert to realize, structure, formalize and, if necessary, measure in quantitative or qualitative scales the process and individual stages of the conclusion [2, 3]. Without dwelling on the well-known shortcomings of the method, we note that the opinion of another expert or a group of those acts as an external supplement for verifying the expert's opinion. As a result, the quality of the identified model is determined by the degree of consistency of their opinions. Attempts to obtain a consistent (point) opinion of experts make the results of the examination incorrect or even false in many cases. Therefore, interval assessments that take into account the spread of experts' opinions are more correct. The range of the interval determines, firstly, the "accuracy of the assessment" of the values within the interval. In this case, conducting complex examinations, implementing a sequence of local assessment procedures leads to the accumulation of interval uncertainty and to the loss of constructiveness. This means that the value of the obtained intervals is so large that the assessments lose practical significance.

An alternative is to use the classical method of comparator identification, the basis of which is the opinion of one or a group of experts. The examination is organized in such a way that it provides more accurate and adequate information. However, this eliminates the need to assess the accuracy and adequacy of models synthesized taking into account the data of the method.

The difficulty lies in the fact that in most cases the number of alternatives on the basis of which the comparative experiment is conducted is relatively small (close to the decision-maker person (DPM)) and it is practically impossible to divide it into two representative subsets. Even in cases where a test subset can be identified, the information is qualitative in nature (linear order of alternatives) and therefore is suitable only for assessing adequacy, but not for quantitative assessments of model accuracy.

It is these circumstances that determine the need to form a special quantitative external complement, which is used to assess the quality of the synthesized model (Figure).



Scheme of model accuracy verification

The general method for checking the adequacy of the final model is as follows.

1. Let n – power of the set of admissible alternatives $X = (x_1, x_2, \dots, x_n)$ and m – power the set of their partial factors $K(x_i) = \{k_1(x_i), k_2(x_i), \dots, k_m(x_i)\}$, $x_i \in X$, $i = 1, \dots, n$.

2. For each alternative a reference tuple (K^{st}) of normalized values of partial characteristics is randomly formed (for example, using a generator of uniformly distributed pseudorandom numbers in the interval [0, 1]).

$$K^{st}(x_i) = \left(k_j^{st}(x_i)\right), i = 1, \dots, n,\ j = 1, \dots, m. \quad (1)$$

Normalization means that all partial characteristics are reduced to a dimensionless form and the condition

$$0 \le k_j^{st}(x_i) \le 1, i = \overline{1,n}, j = \overline{1,m}$$

is satisfied.

As a result, we obtain a rectangular table of the following form.

Normalized partial characteristics of alternatives

	$K_1^{st}(x_i)$	$K_2^{st}(x_i)$	. . .	$K_m^{st}(x_i)$
x_1	$K_1^{st}(x_1)$	$K_2^{st}(x_1)$	. . .	$K_m^{st}(x_1)$
x_2	$K_1^{st}(x_2)$	$K_2^{st}(x_2)$	. . .	$K_m^{st}(x_2)$
. . .	. . .	. . .	. . .	. . .
x_n	$K_1^{st}(x_n)$	$K_2^{st}(x_n)$	. . .	$K_m^{st}(x_n)$

3. The structure of the operator of the evaluation model P (alternative utility function) is set, for example, some polynomial

$$P(x_i) = F[K(x_i), A], i = \overline{1,m}. \quad (2)$$

4. A tuple $A^{st} = \left(a_{1,}^{st}\ a_{2,\dots,}^{st}\ a_n^{st}\right)$, of reference dimensionless coefficients (parameters) of the polynomial (2) is randomly generated for which the following conditions are met: $0 \le a_l^{st} \le 1$, $l = 1,2, \dots, L$, where L – number of polynomial terms.

5. The set of alternatives $X = \{x_1, x_2, \dots, x_n\}$ is checked for non-domination, i.e. for the absence of alternatives that, with all or part of the other things being equal, have private characteristics, and then we exclude all nominated alternatives from the set *X*. Thus, the initial set of alternatives *X* in the general case can be reduced.

6. The values of the utility functions $P^{st}(x_i) = F[k^{st}(x_i), A^{st}]$ are calculated for all alternatives $x_i \in X$, $i = 1, \dots, m$, and, based on them, on the set X we establish the equivalence relation $x_1 {\sim} x_2 {\sim} \dots {\sim} x_n$, non-strict $x_1 \ge x_2 \ge \dots \ge x_n$ or partially linear order of decreasing values $P^{st}(x_i)$, and the best alternative $x_s \in X$ is also determined. This completes the formation of the reference situation.

7. Based on the information about the order relation on the set *X* or the best alternative $x_s \in X$ a comparator identification model is formed:

$$\{P(x_1) = P(x_2)\} = (\ge)\{P(x_3) = P(x_4) = P(x_5)\}$$
$$> (\ge)\{P(x_{n-1}) = P(x_n)\}.$$

In the general case, such a system will consist of

$$H_n = \sum_{t=1}^{T} n_t\,!\,(n_t - 2)!/2 \text{ equations,}$$

where T – is the number of equivalent subsets, and

$$C_n^2 = n!/2!\,(n-2)! \text{ inequalities.} \quad (3)$$

Complete information about the structures of the sets X and V can usually be obtained only under the conditions of an active, i.e. specially planned experiment with a specially trained expert or group of experts. If such an experiment is impossible or undesirable, a passive experiment is conducted, which consists in observing the behavior of a specially untrained expert in natural conditions. As a rule, in this case it is not possible to obtain complete information about the structures of the sets *X* and *V*. For example, if the choice process is studied, then in the process of a passive experiment, by recording the choice of the DPM, it is possible to obtain information only about the best alternative (the extreme element of the set *X*), but not about the structure of preferences on the entire set of available alternatives.

This case involves conducting either an active or passive comparator experiment, during which the DPM chooses the only best alternative $x_s \in X$, s=$\overline{1,n}$ from the set $X = \{x_1, x_2, \dots, x_n\}$.

At the same time, depending on how confident the DPM is in his choice, it is possible to establish the relation of either strict or non-strict preference of the chosen alternative over the others:

$$x_s > (\ge) x_i\,,\ x_s, x_i \in X, i = 1, \cdots, n, s \ne i. \quad (4)$$

It is necessary to note the special value of this case, since information about the DPM's choice is obtained during a passive experiment (for example, when a consumer buys a certain product). This means that there is no need to specially prepare the experiment and the consumer may not even know about its conduct and, therefore, will behave naturally. Moreover, information about purchases can be obtained from sales reports, which allows saving time and money for conducting the experiment and providing a large sample of observations. Therefore, the case under consideration is characterized by cheapness and the greatest degree of objectivity, but is the least informative.

Although the DPM is able to make its choice exactly once, it cannot formalize its preferences on a set of alternatives. Therefore, this case is characterized by a lower degree of confidence of the DPM in its preferences than in cases where it establishes a preference relation (strictly or non-strictly) on the entire set of alternatives.

If the DPM is completely confident in its choice, the comparator

$$D_2(\gamma_q,\gamma_r)=\begin{cases}1, \text{if } \gamma_q>\gamma_r\\ 0, \text{if } \gamma_q<\gamma_r\end{cases}, \quad (5)$$

or

$$D_3(\gamma_q,\gamma_r)=\begin{cases}1, \text{if } \gamma_q\geq\gamma_r\\ 0, \text{if } \gamma_q<\gamma_r\end{cases}, \quad (6)$$

$$E_2(x_q,x_j)=D_2[P(x_q),P(x_r)], \forall x_q,x_r\in X, \quad (7)$$

$$E_3(x_q,x_j)=D_3[P(x_q),P(x_r)], \forall x_q,x_r\in X \quad (8)$$

is implemeted.

The predicates $D_2(\gamma_q,\gamma_r)$ and $E_2(x_q,x_j)$ have the properties of transitivity and antireflexivity and therefore are predicates of strict order. Accordingly, predicate $D_2(\gamma_q,\gamma_r)$ establishes a precedence relation on V', and predicate $E_2(x_q,x_j)$ establishes a strict preference relation on X'.

Predicates of non-strict order $D_3(\gamma_q,\gamma_r)$ and $E_3(x_q,x_j)$, possessing the properties of reflectivity, transitivity and antisymmetry, establish a precedence relation on V' and non-strict preference relation on X' respectively.

As the comparators described above, it can be both the expert himself, conducting an introspective analysis of his preferences and registering them, and an outside observer, if the expert's behavior has external manifestations, can act.

Depending on the features of the intellectual process being studied, the goals of modeling, and the possibility of conducting an active or passive experiment, it may be necessary to simultaneously present the expert with more than two situations. It is assumed that in this case the analysis procedure and its final result are similar to those described above and consist in establishing binary relations between the elements of the set *X* and *V*.

CONCLUSIONS

The obtained information is the initial information for solving the problem of identifying the operator P. For each pair of equivalent alternatives $x_q\sim x_r;\ x_q, x_r\in X, q\neq r,$ з (8) it follows

$$P(x_q) = P(x_r) \quad (9)$$

for the following, when the strict preference relation $(x_q > (<)\ x_r)$ id established from (7) it follows

$$P(x_q > (<)P\ x_r). \quad (10)$$

The formalization of the predicates (5), (6) is based on the principles of utility theory and radical choice.

The total number of relations of the form (9), (10) depends on the power of *X* and its structure, i.e. the number of equivalence classes and the number of elements in each of them.

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In a particular case, the set *X* may:

– not contain subsets of equivalent alternatives with more than one element, i.e. be a linearly ordered set;

– contain several linearly ordered subsets of equivalent alternatives, some or all of which contain more than one element;

– contain one subset of equivalent alternatives, which includes all comparative elements.

The first case corresponds only to the relations in the form of inequalities (10), (11);

the second – only to the equality (9);

the last – to the composition of equalities and inequalities (9) – (11).

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МЕТОД ВЕРИФІКАЦІЇ ДОСВІДНОСТІ І ТОЧНОСТІ МАТЕМАТИЧНОЇ МОДЕЛІ БАГАТОФАКТОРНОГО ОЦІНЮВАННЯ

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Одним із найважливіших етапів побудови будь-якої математичної моделі є перевірка ступеня її адекватності модельованому реальному процесу чи явищу. Тому верифікація моделі є обов'язковим елементом її синтезу та ідентифікації. Якість моделі визначається точністю виділення та апроксимації тільки корисного сигналу, що означає можливість коректного опису будь-якої вхідної послідовності даних. Розглянуто один із можливих підходів до створення математичної моделі багатофакторного оцінювання в умовах невизначеності.