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MODEL ACCOUNTING OF THE INFLUENCE OF RADIATION-INDUCED DEFECTS AND EXTERNAL MAGNETIC FIELD ON THE AUTO-ELECTRONIC EMISSION UNDER VACUUM BREAKDOWN CONDITIONS

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Theoretical models are developed to describe field emission from metal surfaces with radiation-induced defects and in an external magnetic field. A defective layer is modelled by a rectangular potential barrier that modifies the tunneling probability through three components: the classical Fowler-Nordheim barrier, the exponential suppression factor due to the layer thickness, and the quantum interference factor. Resonant current oscillations depending on the defect depth, as well as its exponential decrease for thick layers, are revealed. For the magnetic field, a correction factor is proposed that takes into account changes in the trajectories and energy spectrum of electrons and quantum effects. The influence of both factors can lead to enhancement or suppression of emission.

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INTRODUCTION

The need to develop new materials resistant to vacuum breakdown, as stated in the updated European Strategy for Particle Physics (ESPPU, 2020) [1], has found practical implementation within the framework of a collaboration between CERN and IAP NAS of Ukraine. In accordance with the requirements of paragraph 3.7.2 of this strategy, aimed at achieving an electric field strength higher than that of oxygen-free copper, a series of experiments on high-voltage breakdown was initiated using DC facilities [2, 3].

To implement this research programme, an original experimental setup was created at IAP NAS of Ukraine [4], which allows simulating conditions of deep vacuum and high electric fields and performing a comprehensive analysis of the factors leading to vacuum breakdown in the materials of accelerator structures. One of the key areas of this research has been the study of the effect of ion-plasma surface modification on the breakdown resistance of materials [5, 6, 7].

Previous studies have shown that the probability of breakdown is significantly influenced by both near-surface defective layers formed as a result of radiation processes [5, 7] and by an external magnetic field that alters the motion of electrons after tunnelling [8]. In this context, field emission is considered a key mechanism of breakdown initiation in high-voltage vacuum systems.

The aim of this work is to further develop previous theoretical studies and to construct individual models of field emission that take into account the influence of the near-surface defect layer formed by radiation defects and the external magnetic field under vacuum breakdown conditions.

THEORETICAL MODEL OF THE INFLUENCE OF RADIATION DEFECTS ON TUNNEL EMISSION

Previous studies conducted at IAP NAS of Ukraine [7] have shown a clear correlation between the dose of copper ion implantation and breakdown characteristics, where an increase in the irradiation dose resulted in a significant improvement of the surface's breakdown stability. To gain a deeper understanding of this effect, potentially linked to the formation of radiation-induced defects (such as pores and vacancies), their influence on the field emission current density was examined. The core concept is that a defective layer at a depth z_d from the surface, with a thickness δ, modifies the overall transmission probability of the potential barrier for tunnelling electrons.

As proposed in [7, 9, 10], the defect-enriched layer can be modelled using an additional rectangular potential barrier in the near-surface region of the metal (Fig. 1).

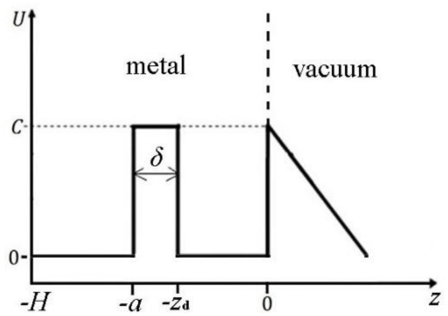

Fig. 1. Schematic representation of the model of a near-surface layer with radiation defects in the form of a rectangular potential barrier [7, 9, 10]

Unlike the approach in [7], which considered the effect of near-surface radiation defects via a single tunnelling transmission coefficient for the double barrier, this work proposes a modular representation of the total electron tunnelling probability. This form allows for the separate evaluation of the contributions from geometric current attenuation through the rectangular barrier and quantum interference effects arising at nanometre distances between the potential barriers.

The total field emission current density is given by the standard form:

$$j = e\int_0^{\infty} n(W)D(W)dW\ , \quad (1)$$

where *n(W)* is the energy distribution of electrons in the metal, and *D(W)* is the electron transmission coefficient.

To account for the defective layer, the transmission probability *D(W)* in this model is decomposed into a product of three independent factors:

$$D(W) = D_{F-N}(W)\cdot D_{rect}(W)\cdot R(W)\ . \quad (2)$$

The first term, $D_{F-N}(W)$ describes the classical tunnelling through the triangular Fowler-Nordheim barrier [11].

The second term, $D_{rect}(W)$, models the additional exponential suppression of the transmission probability due to a rectangular defect layer of thickness δ, corresponding to the standard rectangular barrier model [12]:

$$D_{rect}(W) = \exp\left(-\frac{2\sqrt{2m(C-W)}}{\hbar}\delta\right), \quad (3)$$

where C is the height of the potential barrier, δ is its thickness, m is the mass of the electron.

This factor decreases exponentially with the layer thickness δ, representing a purely geometric suppression of tunnelling through the additional potential region.

The third term, *R(W)*, incorporates quantum interference effects caused by the presence of nanoscale defects [10]:

$$R(W) = \left(2\sqrt{W}\sqrt{C-W}cos\left(\frac{\sqrt{2mW}}{\hbar}z_d\right) + (C-2W)sin\left(\frac{\sqrt{2mW}}{\hbar}z_d\right)\right)^{-2}, \quad (4)$$

where z_d is the depth of the defective layer.

It describes the interference of de Broglie waves between the interfaces of the defective layer and the metal-vacuum barrier, leading to resonance phenomena on the nanometre scale.

This factor exhibits a strong oscillatory dependence on the depth z_d particularly at small values. It reaches its maximum when the depth of the layer z_d corresponds to the condition under which the reflected electron waves reinforce each other ($z_d = \lambda_B/4$, where λ_B is the de Broglie wavelength), which leads to a sharp increase in the emission current. Outside these resonant conditions, *R(W)* decreases drastically, significantly reducing the overall tunnelling probability.

Essentially, the analytical expression for the total transmission probability *D(W)* is structurally similar to that presented in our previous work [7]. Its key advantage lies in the explicit separation of contributions, allowing for separate analysis of the two key mechanisms of defect influence: the exponential current suppression with increasing effective defect layer thickness δ, and the resonant amplification or attenuation of the current depending on the defect depth z_d due to interference.

For large distances z_d ($z_d \gg \lambda_B$), where resonant interference conditions break down, the oscillations in *R*(*W*) are smoothed out and its value approaches unity. In this case, the influence of the defective layer reduces solely to the exponential suppression of tunnelling via the factor $D_{rect}(W)$.

The proposed model thus describes two characteristic behavioural regimes:

- The defective layer has well-defined geometrical parameters – i.e., its thickness and distance from the surface are constant on the nanometre scale – enabling resonant enhancement of the emission current due to phase matching of the electron wave functions;
- Defects are distributed randomly or the layer thickness varies across the plane; the phase conditions are disrupted, the interference effect vanishes, and only exponential current suppression via the additional barrier remains.

Within the Fermi level approximation ($W \approx W_F$), the field emission current density can be approximated as:

$$\frac{j_{def}}{j_{F-N}} \approx D_{rect}(W_F)\cdot R(W_F), \quad (5)$$

which enables the analysis of the competition between tunnelling suppression (due to layer thickness δ) and possible resonant enhancement (due to the position z_d) without performing full integration.

Numerical simulations based on this model, presented as two-dimensional dependencies (Fig. 2), confirm that in the absence of resonant conditions or for a sufficient defect layer thickness (δ > 0.2…0.3 nm), an exponential decrease in emission current is observed.

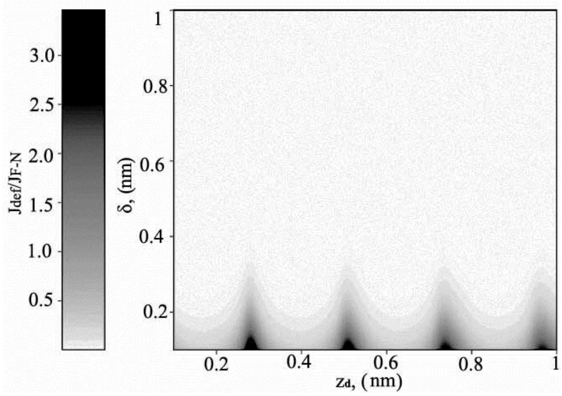

Fig. 2. The ratio of the emission current density from the modified surface j_{def} to the Fowler-Nordheim current j_{F-N} depending on the layer thickness δ and the depth z_d [7]

This qualitatively and quantitatively explains the experimentally observed improvement in the breakdown stability of copper electrodes after high-dose copper ion implantation: the formation of a dense defect layer with sufficient effective thickness leads to significant suppression of field emission – one of the key initiators of vacuum breakdown.

Therefore, the proposed model demonstrates that radiation-induced defects can play a dual role: under certain parameters, they enhance emission efficiency, whereas under other conditions, they suppress it, consistent with the experimental observations reported in [7].

THE EFFECT OF A WEAK MAGNETIC FIELD ON THE AUTOELECTRONIC EMISSION CURRENT: A SEMI-EMPIRICAL MODEL

As established in our previous joint research with CERN, reported in [8], weak magnetic fields (up to 0.54 T) oriented parallel to the cathode surface show no statistically significant effect on the vacuum breakdown probability (breakdown rate) under short-pulse conditions and with small interelectrode gaps. However, experiments conducted at IAP NAS of Ukraine using a system with larger gaps and under DC conditions revealed a significant reduction of 10…20% in the breakdown voltage in the presence of a magnetic field with an induction of 0.33 T. This indicates a complex mechanism of the magnetic field's influence, suggesting that its primary effect manifests not during the electron tunnelling stage from the cathode, but rather during their subsequent motion across the interelectrode gap.

In [8], a theoretical approach to generalizing the Fowler-Nordheim equation for magnetic fields was proposed. In this work, we develop this idea further by introducing a semi-empirical model that allows quantifying the contribution of the magnetic field to the change in emission current post-tunnelling. The model is based on introducing a multiplicative correction factor $M(B)$ to the expression for the field emission current density, which accounts for the modification of the electron flow towards the anode due to the magnetic field:

$$j = e\int_0^{\infty} n(W)D(W)M(B)dW. \quad (6)$$

The multiplier *M(B)* is the product of three independent components, each of which describes a separate physical mechanism:

$$M(B) = M_{traj}(B)\cdot M_{energy}(B)\cdot M_{quantum}(B). \quad (7)$$

The first and most significant component, $M_{traj}(B)$, accounts for the geometric effect of electron path elongation due to the Lorentz force. In the perpendicular field configuration $\vec{B}\perp\vec{E}$ used in our experiments, the electron trajectory becomes curved, increasing the probability of collisions with and ionization of residual gas atoms. This contribution is modelled by exponential decay:

$$M_{traj}(B) = exp\left(-\alpha\cdot\frac{B}{B_c}\right), \quad (8)$$

where the parameter α=0.03 is empirical and was determined from experimental data [9], corresponding to approximately a 10% increase in path length at B = 0.33T. Here, $B_c = mv/eR$ represents a critical magnetic field, with R being the characteristic radius of curvature of the electron trajectory, v the electron velocity, m the electron mass, and e the elementary charge.

The second component, $M_{energy}(B)$, describes how the magnetic field affects the energy distribution of electrons in the cathode within the plane perpendicular to $\vec{B}$, leading to a modification of the density of states near the Fermi level. This quantum-statistical effect causes a contraction of the electron energy distribution and, consequently, a slight reduction in the number of electrons capable of tunnelling. For fields well below the quantization limits, this contribution is described by a quadratic dependence:

$$M_{energy}(B) = 1-\beta\cdot\left(\frac{B}{B_C}\right)^2, \quad (9)$$

where the parameter =0.015 reflects the more subtle influence of the field on the electron energy distribution, resulting in a minor contraction of the density of states.

The chosen value of β yields only a 0.5% decrease in $M_{energy}(B)$ at B = 0.54 T, but allows for estimating the effects at higher fields.

It is important to note that the characteristic field B_C appearing in both expressions has different physical interpretations. In the context of the geometric factor $M_{traj}(B)$, it directly determines the electron trajectory curvature. Conversely, in the energy factor $M_{energy}(B)$, this field serves merely as a convenient scaling parameter for describing the effect on the density of states. This distinction explains why the strengths of these two independent effects, quantified by the parameters α and β, differ in magnitude.

The third component, $M_{quantum}(B)$, incorporates oscillations in the density of states associated with the formation of Landau levels (the Shubnikov-de Haas effect). Although at room temperature these effects are strongly suppressed by thermal smearing of the Fermi-Dirac distribution, we include an oscillatory term for model completeness, which can become significant in low-temperature experiments or for materials with a large effective mass:

$$M_{quantum}(B) = 1+\gamma\cdot\cos\left(\frac{B_q}{B}\right), \quad (10)$$

where Bq ∼ 0.1 T is the characteristic oscillation field and γ is the oscillation amplitude.

At room temperature, the contribution of quantum oscillations is negligible, and the chosen amplitude γ = 0.005 corresponds to maximum current variations of no more than ± 0.5% at $B \lesssim$ 0.1 T. The oscillation period is set by B_q = 0.1T, corresponding to the low-field regime where quantum effects may become observable.

The parameter B_q determines the period of quantum oscillations arising from the discretization of electron energy levels in a magnetic field (Landau levels). Theoretically, observing these effects requires the Landau energy $\hbar\omega_c$ to exceed the thermal energy kT, corresponding to fields $B > B_q$. Such fields are prohibitively high at room temperature. Therefore, in our model, B_q serves as an empirical parameter that sets the field scale at which the cos(B_q/B) periodicity models potential weak oscillations.

For clarity, Table summarizes the model components, their operating ranges, and physical nature.

Fig. 3 shows the behavior of each component *Mi* (*B*) as a function of the magnetic field *B*. As shown in the figure, in the range of B ∼ 0.3…0.5 T, corresponding to our experiments, the dominant contribution comes from the factor $M_{traj}(B)$, while the other components remain nearly constant.

Substituting the parameter estimates into expression (7) yields the total effect for the experimental conditions: M(0.54T) ≈ 0.984 and M(0.33T) ≈ 0.993. This corresponds to a very small direct reduction of the field emission current by 1.6 and 0.7%, respectively, which is consistent with the absence of a statistically significant effect in the CERN experiments [8].

Components of the semi-empirical model M(B) and their characteristics

Multiplier	Analytical expression	Working range	Physical mechanism	Behaviour
$M_{traj}(B)$	$exp(-\alpha B/B_c)$	$B \gtrsim 0.2$ T	Electron path elongation; increased ionization probability	Moderate exponential decay
$M_{energy}(B)$	$1-\beta(B/B_c)^2$	$B \gtrsim 1$ T	Modification of density of states near Fermi level	Gradual quadratic attenuation
$M_{quantum}(B)$	$1+\gamma cos(B_q/B)$	$B \lesssim 0.1$ T ($B_q/B \gg 1$)	Density of states oscillations (Landau levels)	Rapid oscillations at low *B*, damped at high *B*

The model also explains the degradation in performance observed at IAP NASU [8]. The primary mechanism is the trajectory lengthening, which, without significantly affecting electron emission from the cathode, dramatically increases the probability of ionization processes within the vacuum gap. This ultimately leads to breakdown at a lower voltage.

It should be noted that the proposed model is limited to describing the field emission current. In the pre-breakdown regime, ionic and plasma components can contribute significantly to the total current but are not considered in this analysis. The model's validity is confirmed by its asymptotic reduction to the classical Fowler-Nordheim equation at small magnetic fields ($B \ll Bc$).

Thus, the developed semi-empirical model can be used for the qualitative and quantitative analysis of weak magnetic field effects. It highlights that when designing high-gradient accelerator structures, especially those with extended interelectrode gaps, it is necessary to consider the geometrical effect of the magnetic field on electron trajectories, not merely its potential suppressive effect on emission.

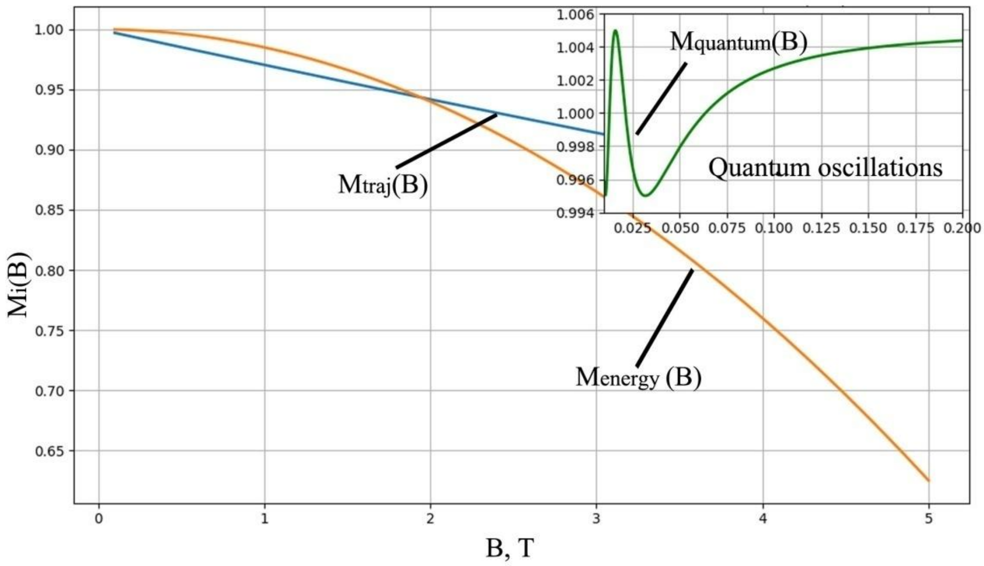

Fig. 3. Dependence of the correction factors $M_{traj}(B)$, $M_{energy}(B)$ and $M_{quantum}(B)$ on the magnetic field induction B

CONCLUSIONS

Theoretical models have been developed to analyze the influence of radiation-induced defects and weak magnetic fields on field electron emission. For defect description, a modular representation of the electron transmission probability has been proposed, enabling separate investigation of geometric current suppression and quantum interference effects. Radiation-induced defects demonstrate dual behavior: at small layer thicknesses ($\delta < 0.2$ nm), resonant emission enhancement up to 3 times is observed, while at $\delta > 0.2…0.3$ nm, exponential current suppression occurs. This explains experimental improvements in breakdown stability following high-dose implantation.

For magnetic field analysis, a semi-empirical model with multiplicative correction factor M(B) has been developed, incorporating changes in electron trajectories, energy spectrum, and quantum effects. In the 0.3…0.5 T range, the geometric factor M_traj dominates, increasing electron path length by approximately 10%. Direct emission current suppression is negligible (0.7…1.6%), with the primary influence mechanism being enhanced ionization probability due to trajectory elongation.

The developed models provide a practical toolkit for optimizing accelerator structure materials through controlled defect structure formation and appropriate magnetic field configuration.

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МОДЕЛЬНЕ ВРАХУВАННЯ ВПЛИВУ РАДІАЦІЙНО-ІНДУКОВАНИХ ДЕФЕКТІВ ТА ЗОВНІШНЬОГО МАГНІТНОГО ПОЛЯ НА АВТОЕЛЕКТРОННУ ЕМІСІЮ В УМОВАХ ВАКУУМНОГО ПРОБОЮ

О.Ю. Карпенко, В.А. Батурін

Розроблено теоретичні моделі для опису польової емісії з металевих поверхонь із радіаційно-індукованими дефектами та у зовнішньому магнітному полі. Дефектний шар моделюється прямокутним потенційним бар'єром, що модифікує ймовірність тунелювання через три компоненти: класичний бар'єр Фаулера-Нордгейма, фактор експоненційного придушення, зумовлений товщиною шару, та квантово-інтерференційний множник. Виявлено резонансні коливання струму залежно від глибини дефектів та його експоненційне зменшення для товстих шарів. Для магнітного поля запропоновано поправочний коефіцієнт, що враховує зміну траєкторій і енергетичного спектра електронів та квантові ефекти. Вплив обох факторів може призводити до посилення чи придушення емісії.